Automated recovery of modal components in shallow-water acoustic signals

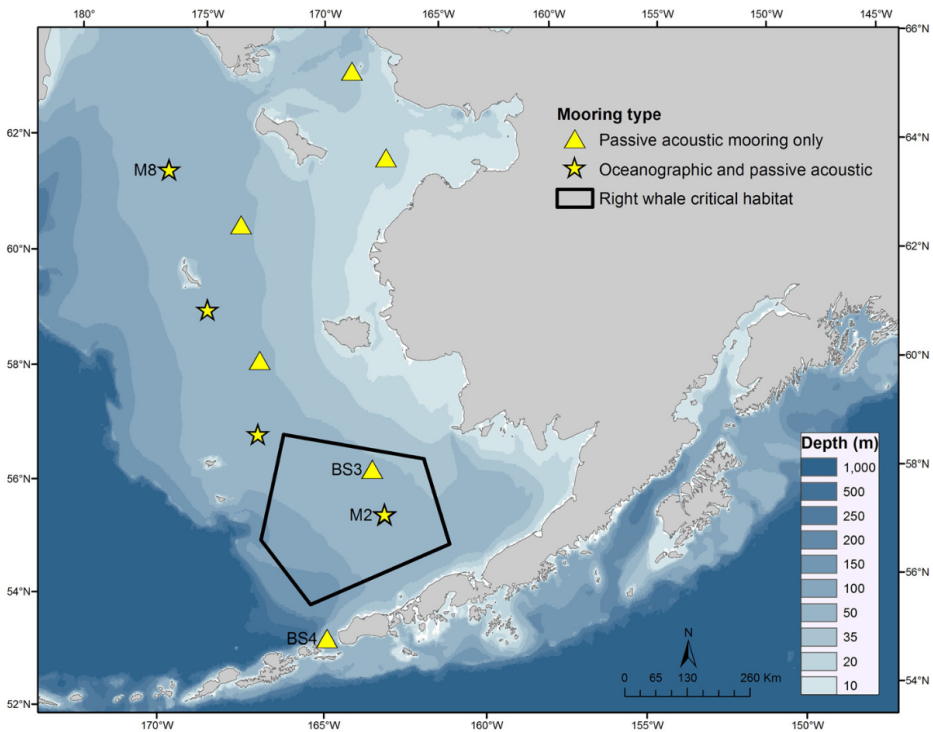
Angèle Niclas

Join work with Josselin Garnier and Julien Bonnel

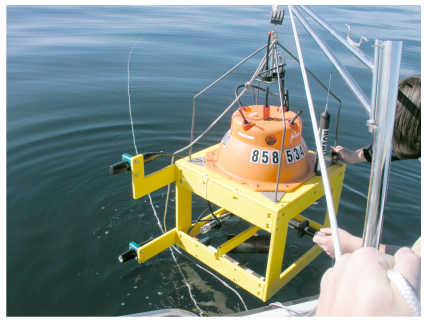
23th November 2026

Context

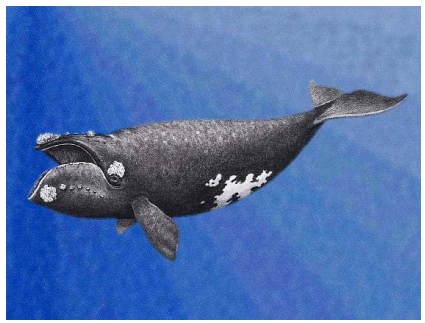
Goal: Infer oceanographic information (seabed, water column) and localize marine biology (fishes, whales) from acoustic signals.



Position of hydrophones in the Gulf of Alaska
Right whale critical habitat



Hydrophone



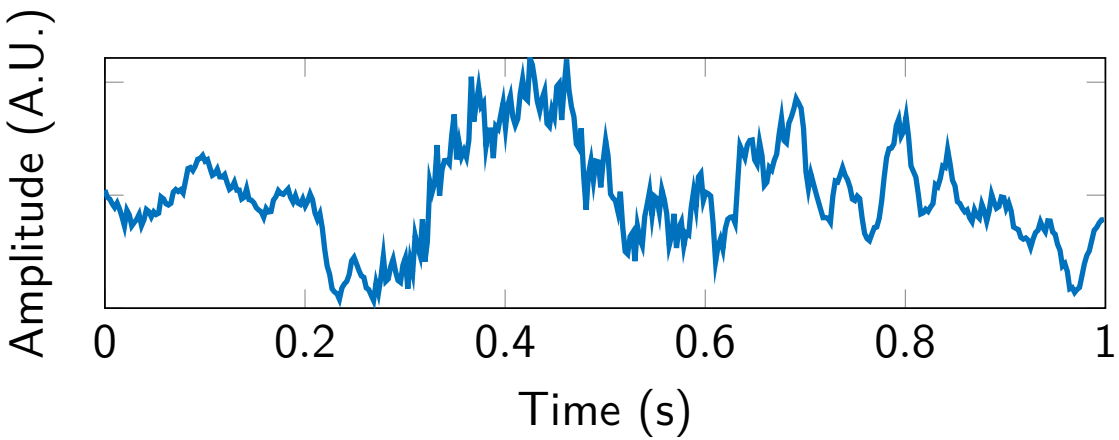
North Pacific right whale

Thode, A. et al. (2017). "Using non-linear time warping to estimate North Pacific right whale calling depths in the Bering Sea," The Journal of the Acoustical Society of America.

Right whale gunshot



Right whale gunshot



Measurements on the receiver

New England Aquarium Right Whale Research Program, https://www.youtube.com/watch?v=ICCsfeSJXVs

Challenges

Goal:

- ➤ Recover the distance between the source and the receptor
- ➤ Recover the properties of the medium (celerity, density, depth...)

Challenges:

- ✗ We lack information about the source of waves
- ✗ We only have one measurement point
- ✗ We may have a lot of noise from the background

Contents

1. Wave propagation in shallow water
   - Modal decomposition
   - Spectrograms
   - Warping
2. Automated physic-based warping
   - Optimal t_0
   - Estimated dispersion curves
   - Parameters estimation
3. Experimental data
   - Right Whale gunshot
   - Combustive sound source

Outline

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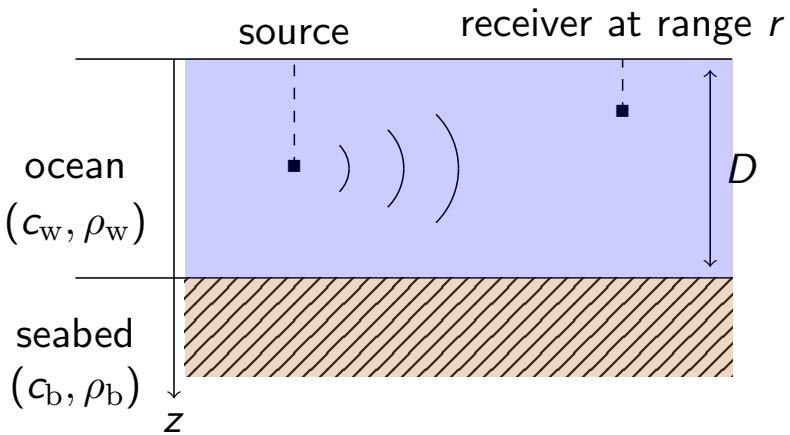
Modeling of the problem



Pekeris waveguide

Pekeris, C. L. (1948). Theory of propagation of explosive sound in shallow water.

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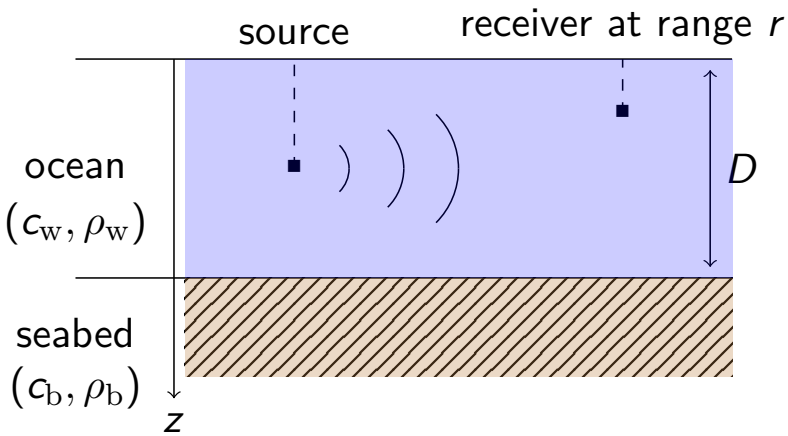

Pekeris waveguide

Propagation equation:

The acoustic wave u satisfies

$$\rho \nabla \cdot \left(\frac{1}{\rho} \nabla u \right) - \frac{1}{c^2} \partial_{tt} u = -s,$$

where

- c is the celerity
- ρ is the density
- s is the source term

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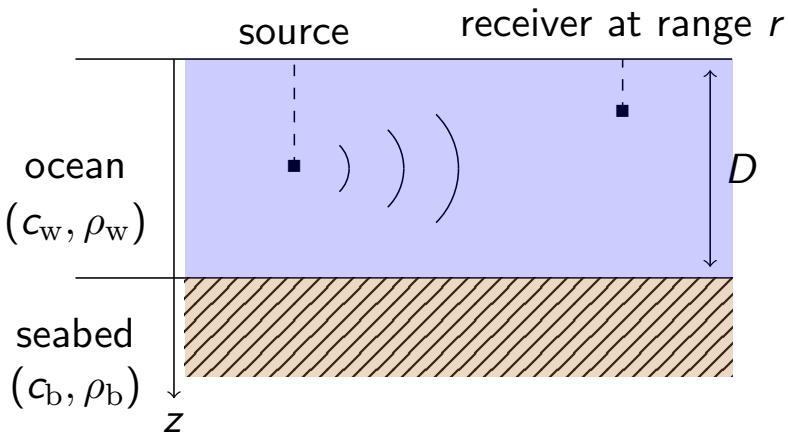

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Boundary conditions:

- $u_{|z=0} = 0$
- u is outgoing (Sommerfeld)
- Continuity of u and $\partial_z u / \rho$ at $z = D$

Pekeris, C. L. (1948). Theory of propagation of explosive sound in shallow water.

Modal decomposition

➤ We consider the Fourier transform $\hat{u}$. At a frequency ω, we look for eigenmodes Ψ_n satisfying, for an eigenvalue λ_n,

$$\rho(z)\frac{\mathrm{d}}{\mathrm{d}z}\left(\frac{1}{\rho(z)}\frac{\mathrm{d}}{\mathrm{d}z}\Psi_n(z)\right)+\frac{\omega^2}{c^2(z)}\Psi_n(z)=\lambda_n\Psi_n(z). \qquad (1)$$

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Modal decomposition:

- The spectrum is of the form $(-\infty, a)\cup\{\lambda_1,\lambda_2,\ldots,\lambda_N\}$.
- $(\Psi_n)_{n=1,\ldots,N}\cup(\Psi_\gamma)_{\gamma\in(-\infty,\alpha)}$ is an orthonormal basis of $L^2(-\infty,0)$

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- We denote $k_n=\sqrt{\lambda_n}$, and

$$\hat{u}(\omega,r,z_r)=\alpha\sum_{n=1}^{N}\Psi_n(z_s,\omega)\Psi_n(z_r,\omega)\frac{1}{\sqrt{rk_n(\omega)}}e^{irk_n(\omega)} + \int_{-\infty}^{a(\omega)}\Psi_\gamma(z_s,\omega)\Psi_\gamma(z_r,\omega)\frac{1}{\sqrt{r\sqrt{\gamma}(\omega)}}e^{ir\sqrt{\gamma}(\omega)}. \quad (2)$$

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- We denote $k_n=\sqrt{\lambda_n}$, and **if** $r \gg c/\omega$ **then**

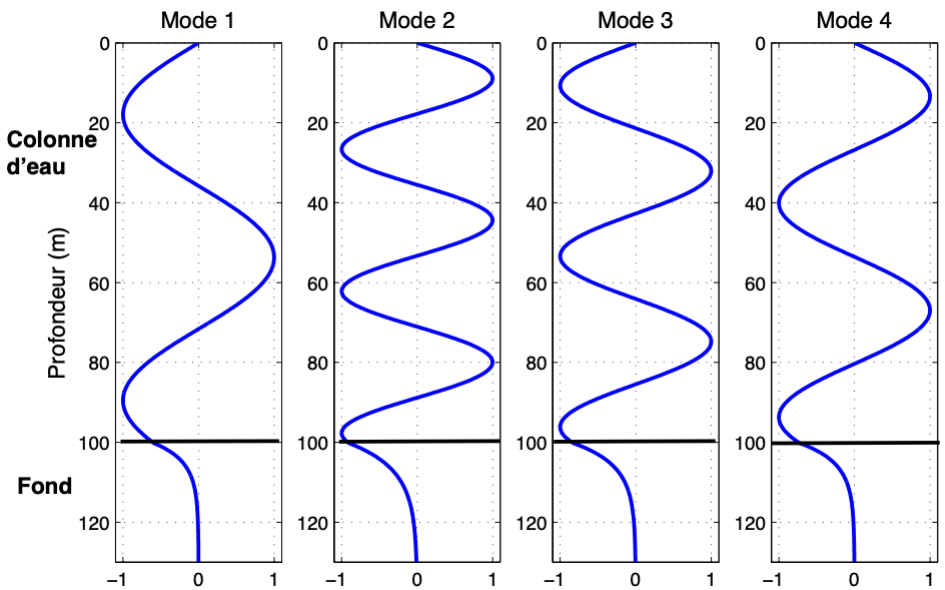
$$\hat{u}(\omega,r,z_r)=\alpha\sum_{n=1}^{N}\Psi_n(z_s,\omega)\Psi_n(z_r,\omega)\frac{1}{\sqrt{rk_n(\omega)}}e^{irk_n(\omega)} + \mathcal{O}\left(\frac{1}{\omega r}\right). \quad (2)$$

Magnanini, R. and Santosa, F. (2000). "Wave propagation in a 2-d optical waveguide", SIAM J. Appl. Math.

Modes in Pekeris waveguides

- **Eigenmodes:** In $(-D, 0)$, $\Psi_n(z,\omega) = \alpha_n(\omega)\sin(k_n(\omega)z)$
- **Eigenvalues:**

$$\tan\left(D\sqrt{\omega^2/c_w^2 - {k_n}^2}\right) = \frac{-\rho_b}{\rho_w}\sqrt{\frac{\omega^2/c_w^2 - {k_n}^2}{{k_n}^2 - \omega^2/c_b^2}}. \qquad (3)$$



Ψ_1, Ψ_2, Ψ_3 and Ψ_4 in a Pekeris waveguide for $\omega = 100$ Hz ($N = 4$)

Bonnel, J. (2010). Analyse de la dispersion acoustique UBF (0-150 Hz) pour la surveillance et la caractérisation du milieu marin.

A quick look at the inverse problem

- We need to find a way to separate each modal component in the signal

- From each modal component $\hat{u}_n$, we can only hope to recover $rk_n(\omega)$:

$$\hat{u}_n(\omega, r, z_r) = \underbrace{\alpha \Psi_n(z_s, \omega)}_{\text{unknown (source)}} \underbrace{\Psi_n(z_r, \omega) \frac{1}{\sqrt{r k_n(\omega)}}}_{\text{useless}} \underbrace{e^{ir k_n(\omega)}}_{\text{useful}}. \quad (4)$$

Bonnel, J. et al. (2012). "Single-receiver geoacoustic inversion using modal reversal". The Journal of the Acoustical Society of America.

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- Using k_n, we can recover the Pekeris parameters
- The various phase recovery attempts are too unstable to work in real-world scenarios

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➥ **Introduction of the spectrogram**

Bonnel, J. et al. (2012). "Single-receiver geoacoustic inversion using modal reversal". The Journal of the Acoustical Society of America.

Time/frequency analysis

Definition of the spectrogram:

$$S(t,\omega) = \left| \int_{\mathbb{R}} u(t) h(t-\tau) e^{-i\omega\tau} \mathrm{d}\tau \right|^2 . \tag{5}$$

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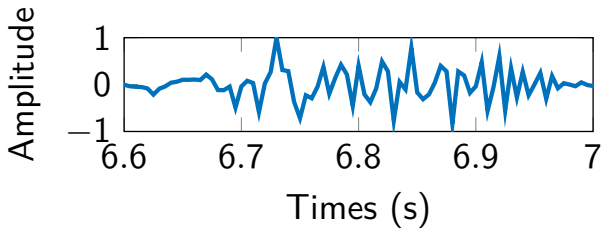
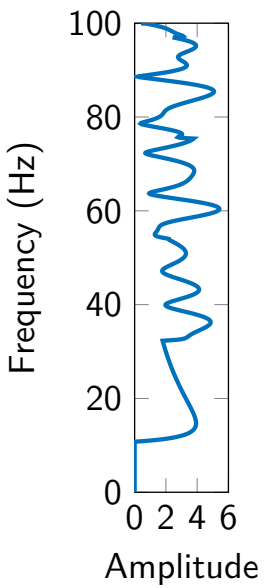
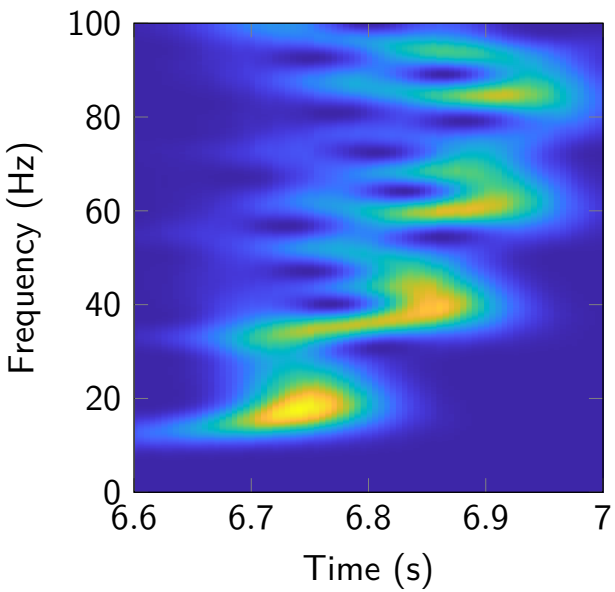


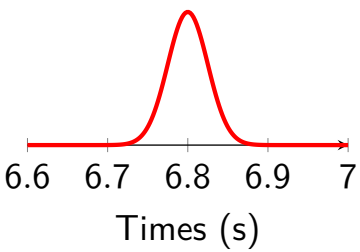




- **Choice of h:**

$$h(t) = e^{-\frac{\sigma^2 t^2}{2}}$$

with $\sigma = 40$ Hz



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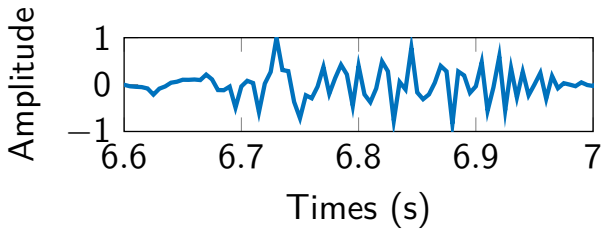
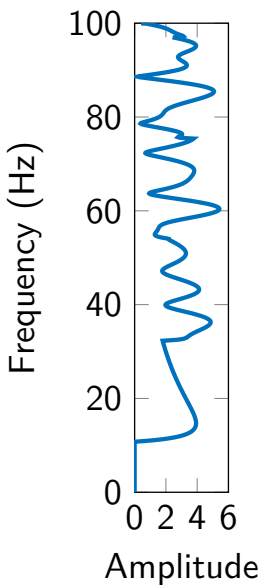
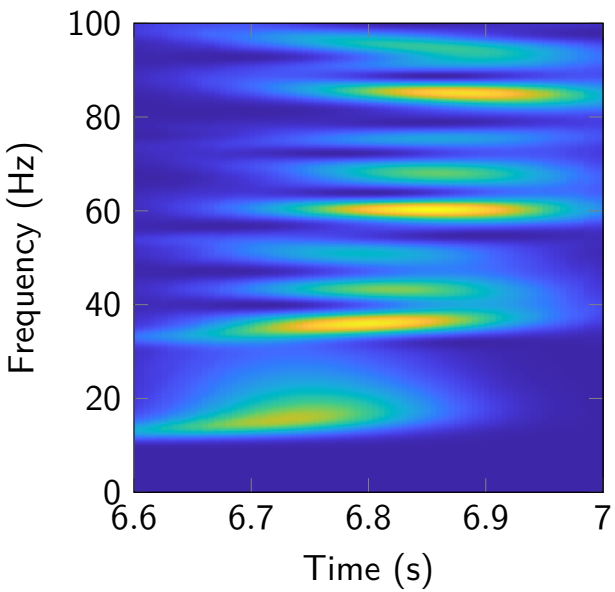


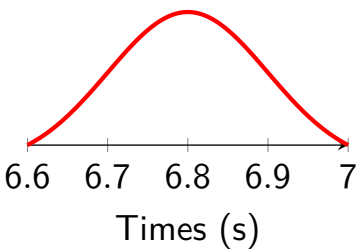




- **Choice of h:**

$$h(t) = e^{-\frac{\sigma^2 t^2}{2}}$$

with $\sigma = 10$ Hz



Time/frequency analysis

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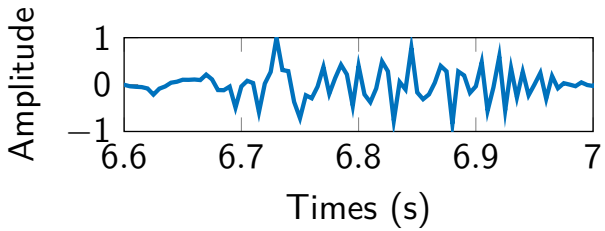

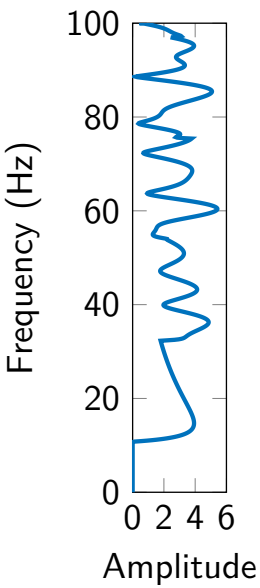

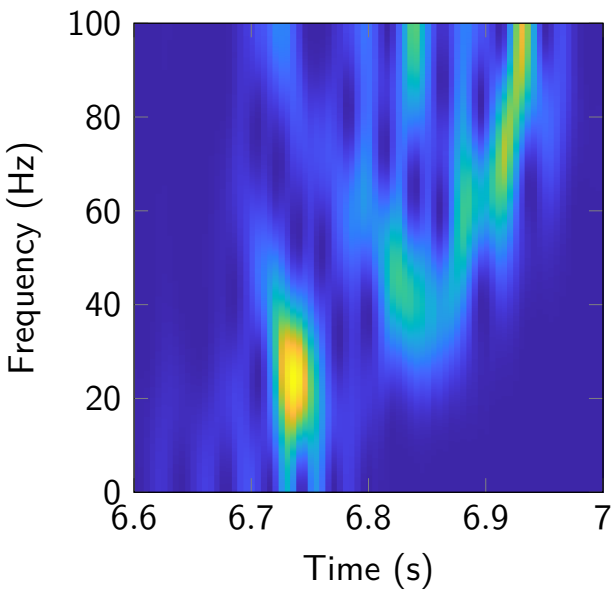

- **Choice of h:**

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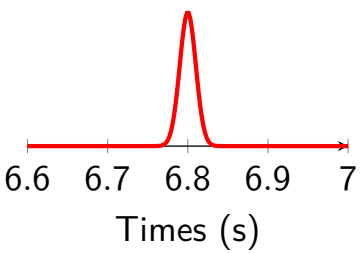
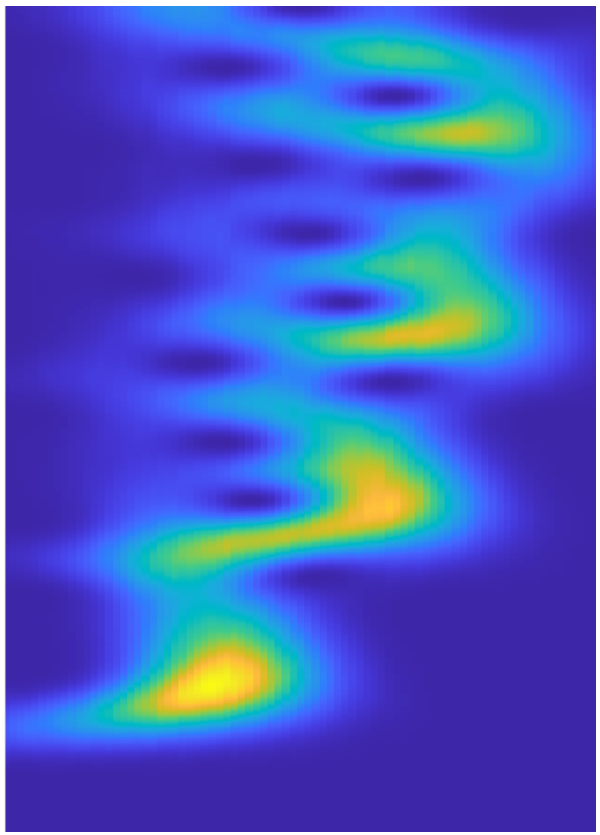
with $\sigma = 100$ Hz



Modal travel time

We define the modal travel time $t_n(\omega) = -rk_n'(\omega)$.



Spectrogram and modal travel times

- The energy in the spectrogram is concentrated around the curves $t_n(\omega)$

Jensen, F. B. et al. (2011). Computational ocean acoustics.

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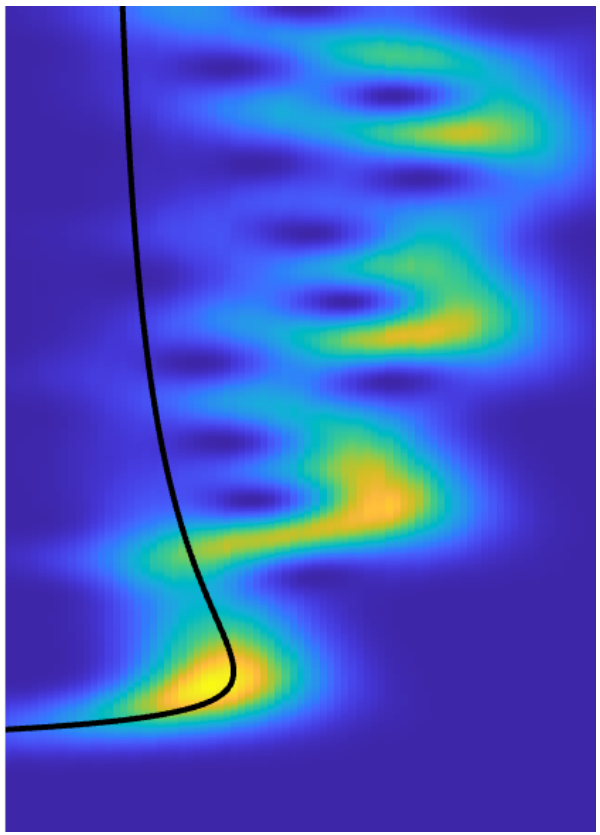

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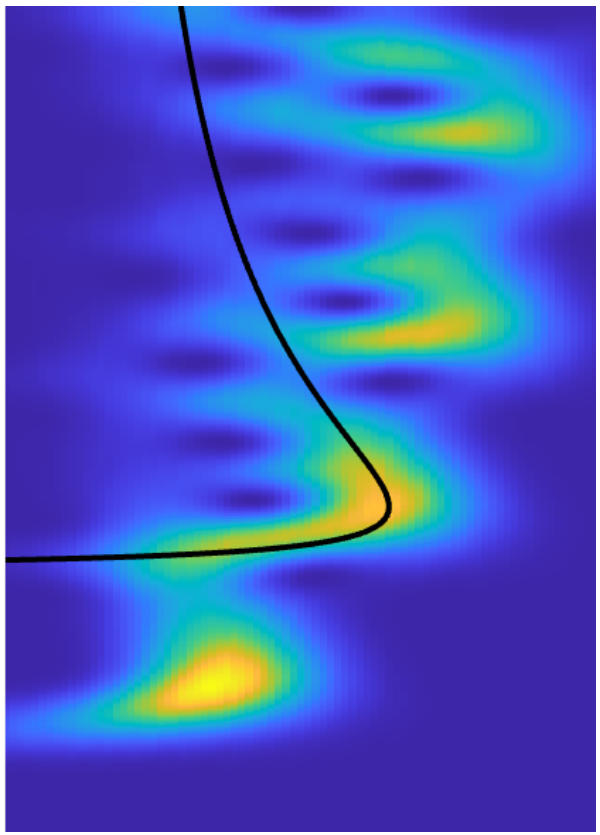

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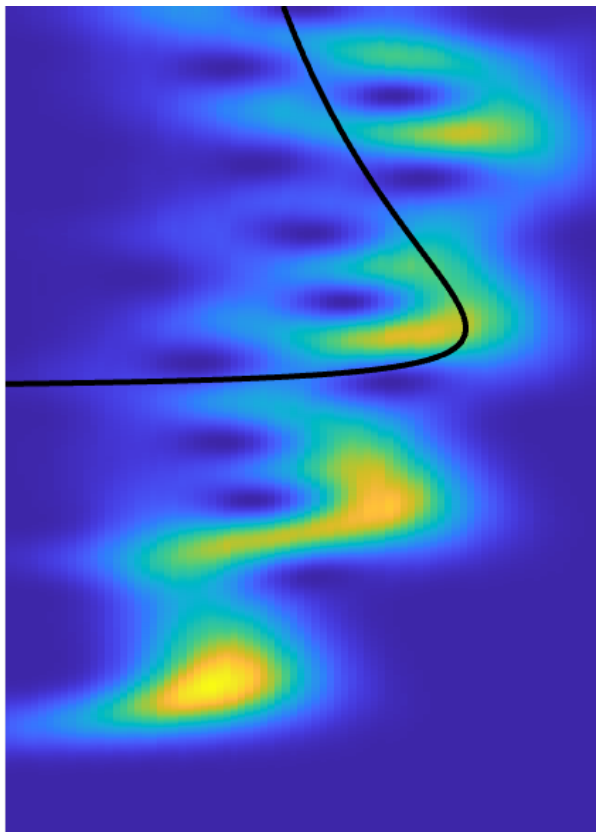

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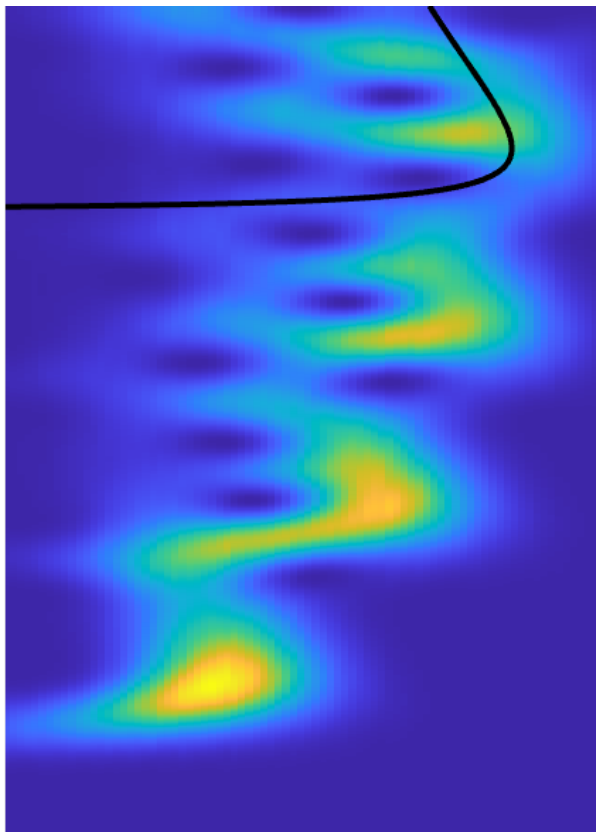

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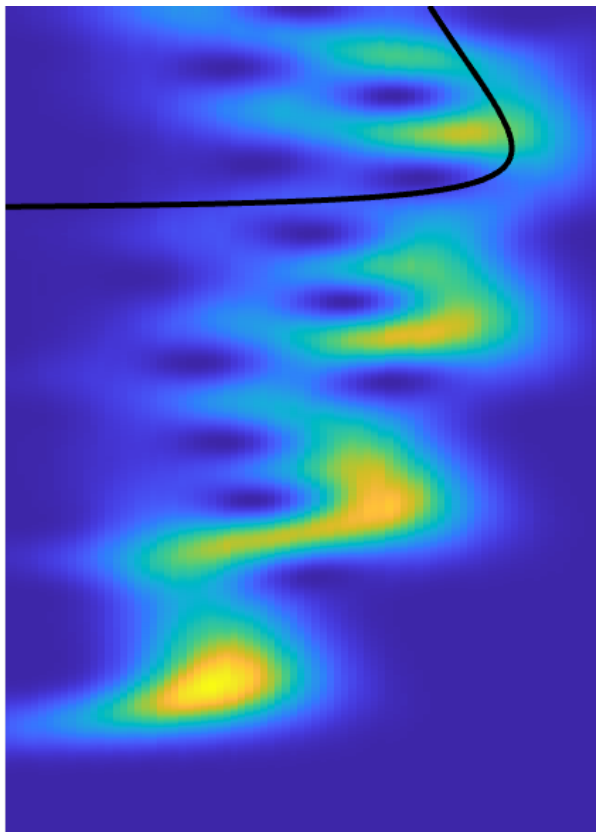


Spectrogram and modal travel times

- The energy in the spectrogram is concentrated around the curves $t_n(\omega)$
- We may use the spectrogram to recover each t_n
- We may use the spectrogram to separate each modal component

Jensen, F. B. et al. (2011). Computational ocean acoustics.

Non linear warping

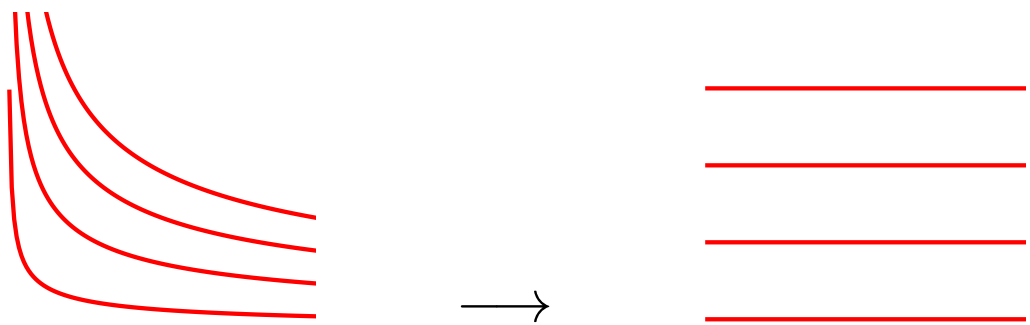
Non linear warping

- **Idea:** Change of variable $t \mapsto \psi(t)$ and new signal

$$\widetilde{u}(t) = \sqrt{\psi(t)}\, u(\psi(t)).$$

- **Justification:**

In a perfectly reflecting waveguide ($\partial_z u_{|z=D} = 0$), with the change of variable $\psi(t) = \sqrt{t^2 + r^2/c_w^2}$,



- **In practice:** We use $\psi(t) = \sqrt{t^2 + t_0^2}$

Niclas, A. and Garnier, J. (2024) "Automated approach for recovering modal components in shallow waters". The Journal of the Acoustical Society of America.

State of the art

Manual warping

Expert picks t_0 by hand

✓ High accuracy
✓ Physics-based

✗ User-dependent
✗ $\approx$ 2 min / signal

Van Komen D. F. et al. (2021). "Seabed type and source parameters predictions using ship spectrograms in convolutional neural networks "
Goldwater M. et al. (2023). "Machine-learning-based simultaneous detection and ranging of impulsive baleen whale vocalizations using a single hydrophone "

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Spectrogram treated as a **2D image**
Trained on 100k synthetic datas

✓ Fast inference

✗ Low accuracy on real data
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➥ Need an automated, physics-based method to find t_0.

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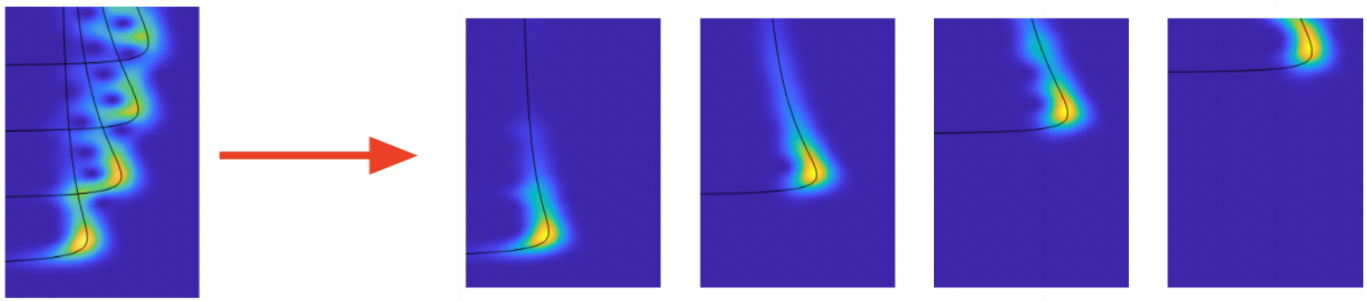
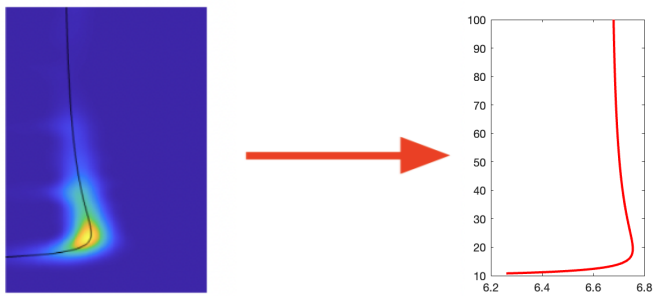
Three main steps

- Separate each modal component



- Find each modal travel time $\omega \mapsto t_n(\omega)$

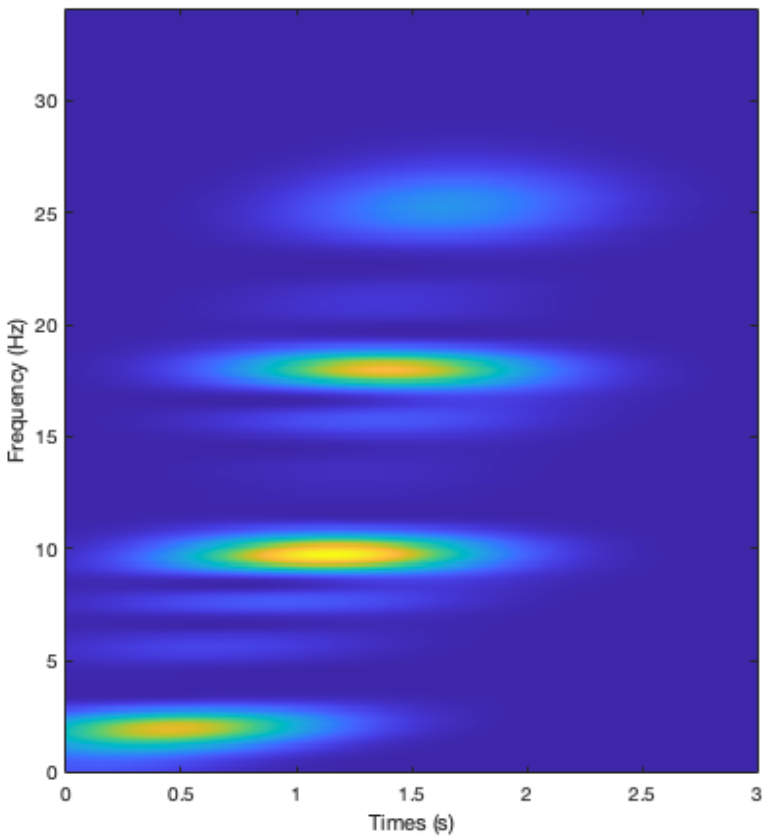


- Recover r and the parameters of the Pekeris waveguide

Mask of the spectrogram

For a fixed t_0, we use a watershed algorithm to separate the spectrogram in different drainage basins:

$$\widetilde{S}(t, \omega)$$



Meyer, F. (1994). "Topographic distance and watershed lines," Signal Processing.

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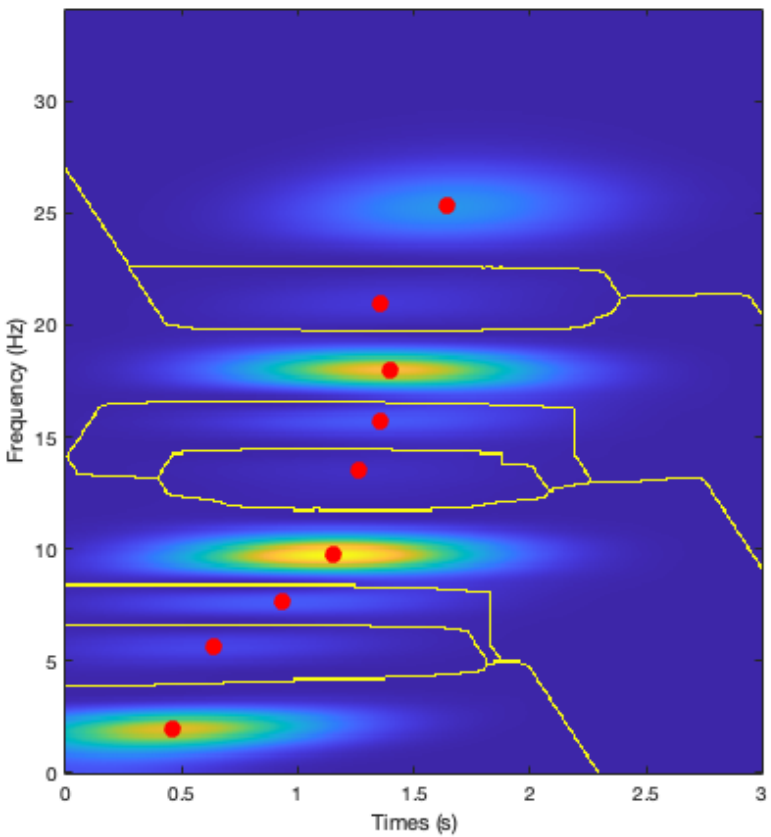

Meyer, F. (1994). "Topographic distance and watershed lines," Signal Processing.

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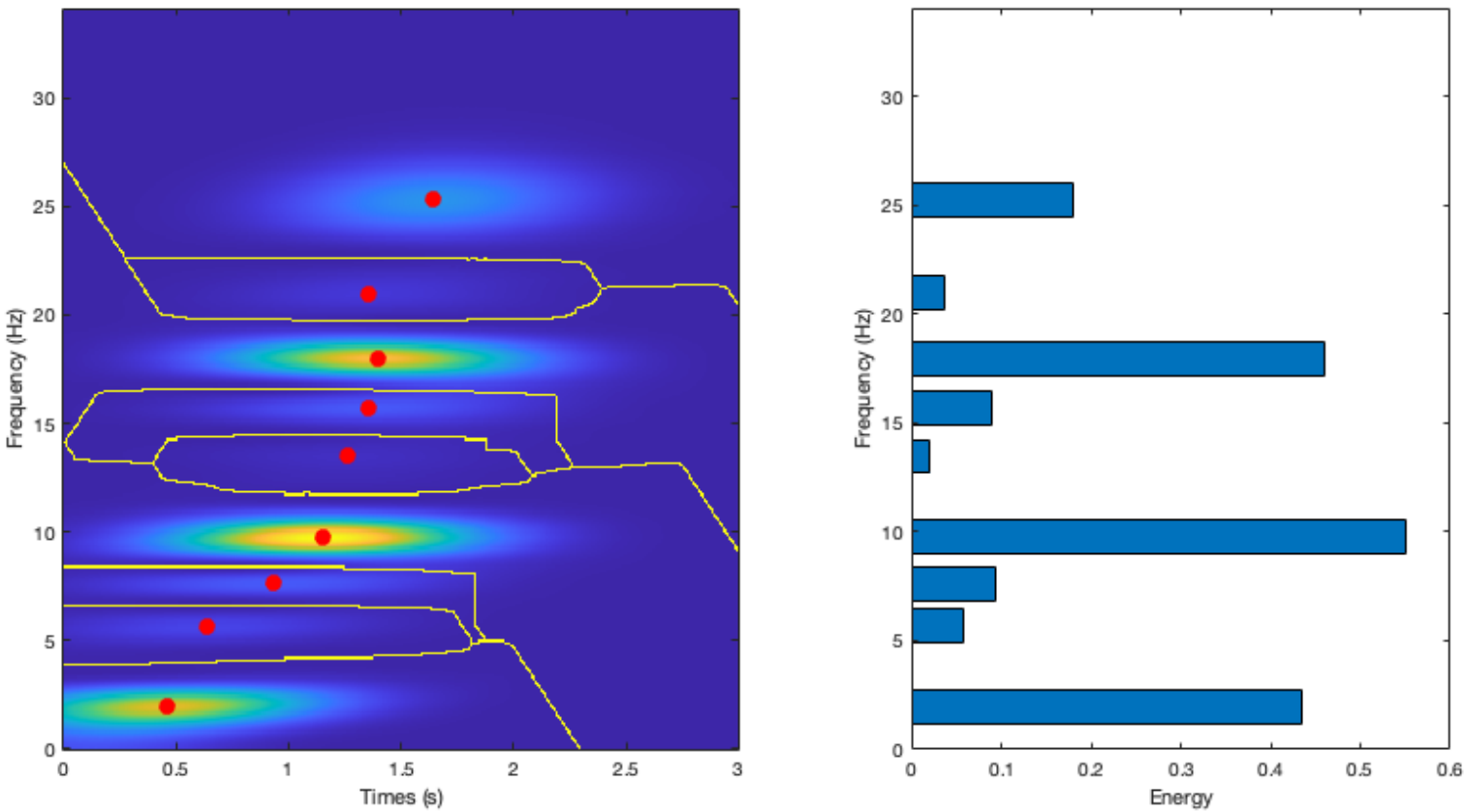
Maximum of $\widetilde{S}$ in each basin



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Mask of the spectrogram

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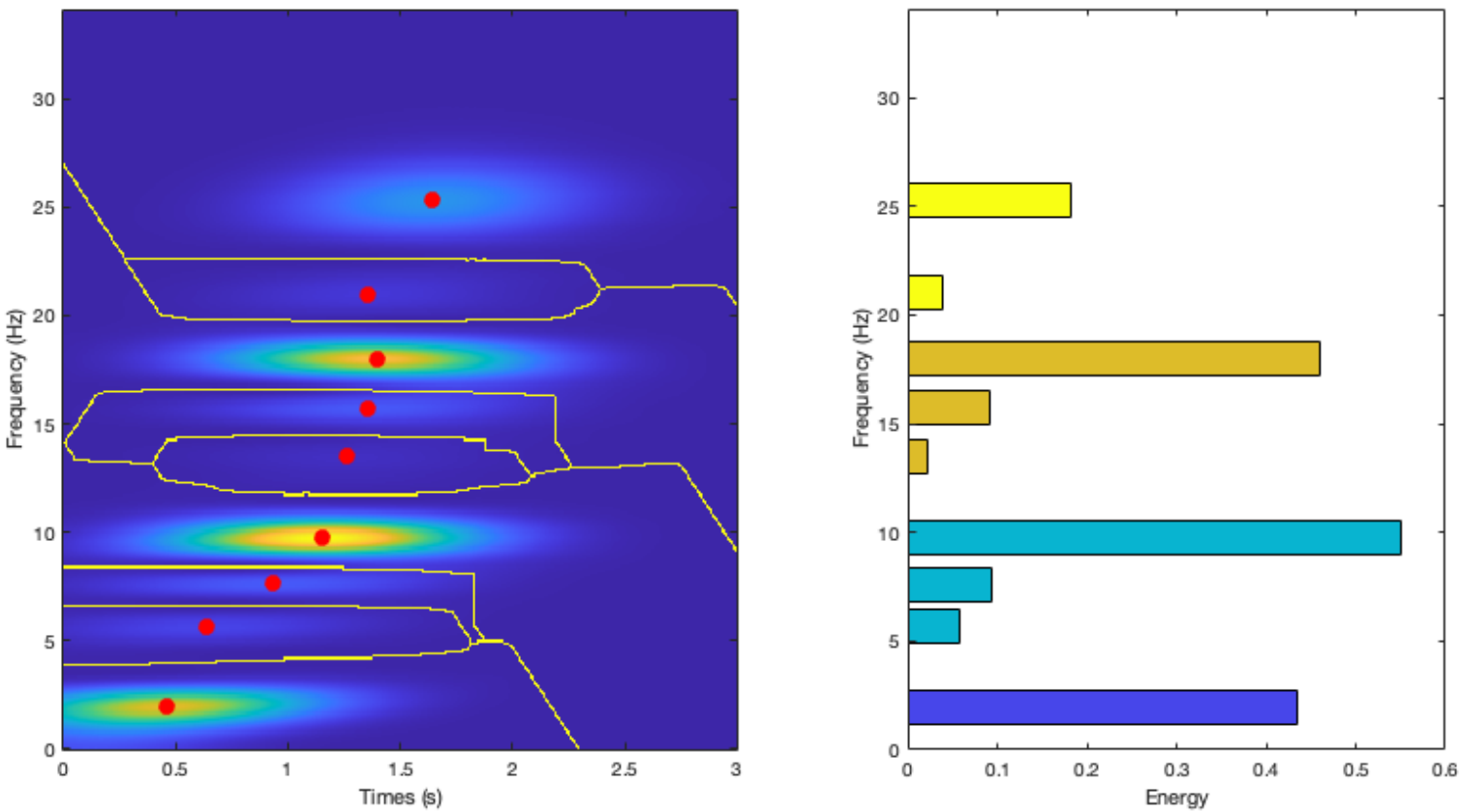
$\widetilde{S}(t, \omega)$ Maximum of $\widetilde{S}$ in each basin



Meyer, F. (1994). "Topographic distance and watershed lines," Signal Processing.

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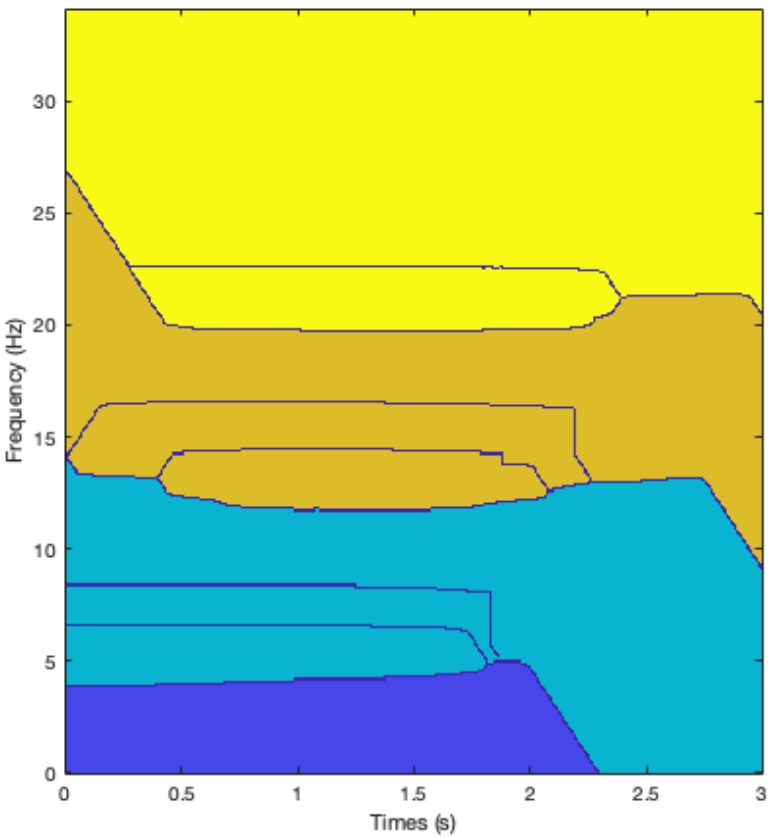
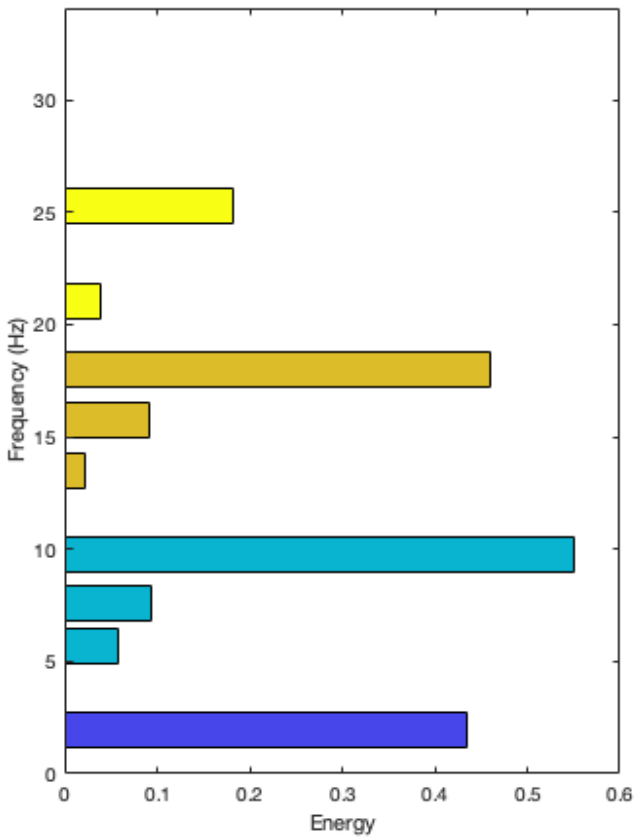
Mask

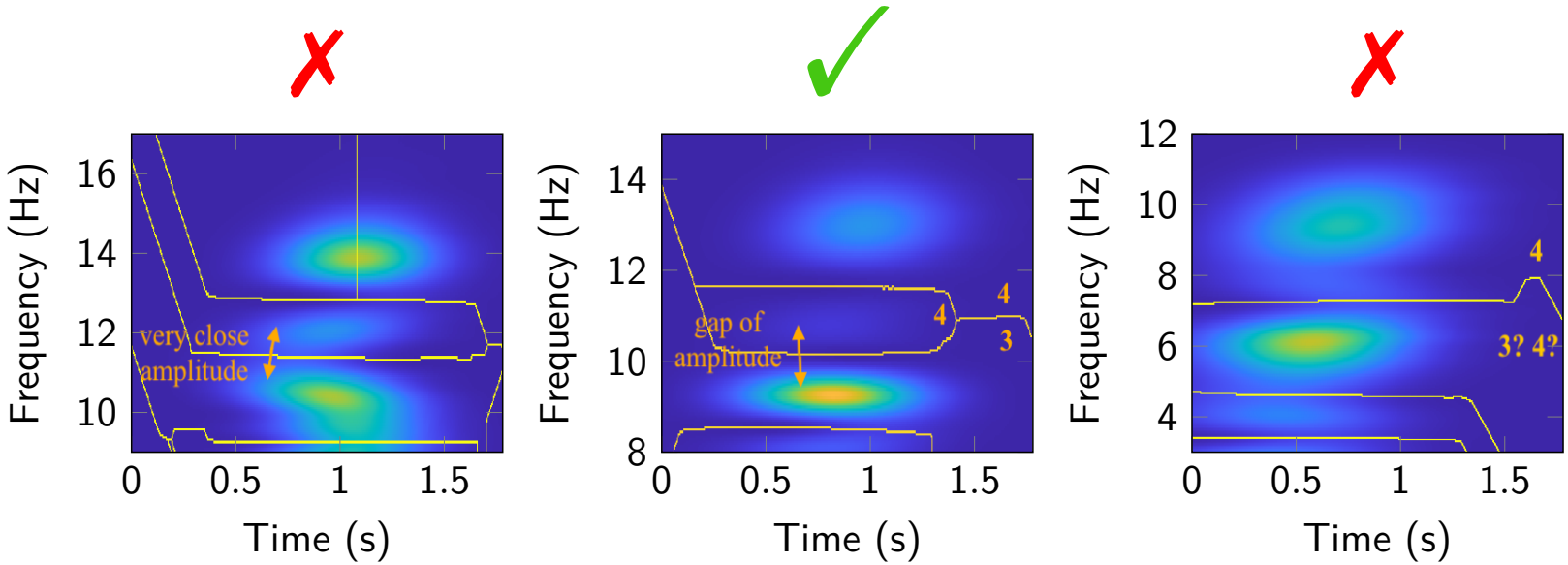


Maximum of $\widetilde{S}$ in each basin



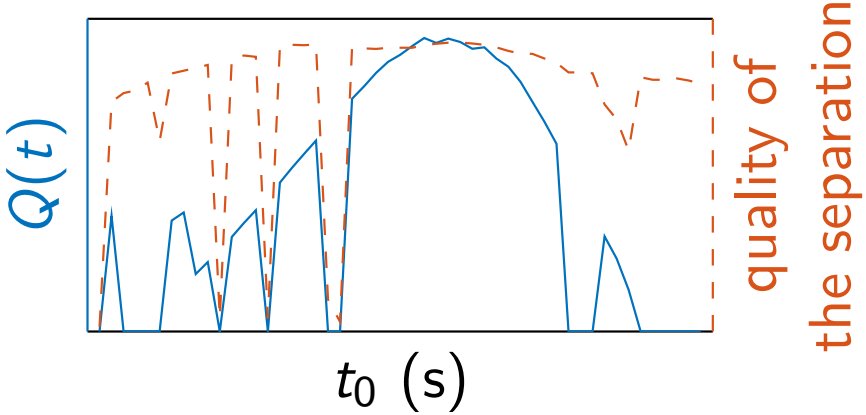
Meyer, F. (1994). "Topographic distance and watershed lines," Signal Processing.

Choice of t_0

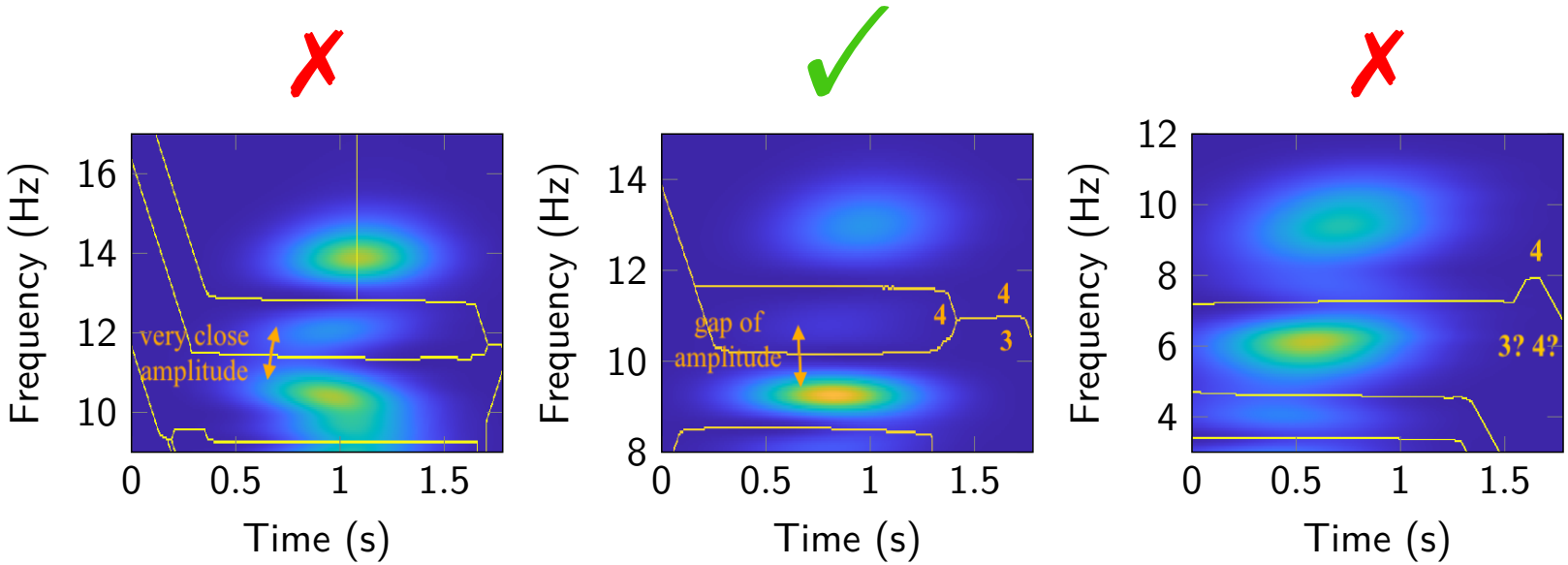


We introduce a quality factor $Q(t_0)$ such that:

- Q increases with the number of basins
- Q increases with the amplitude gap between each basin

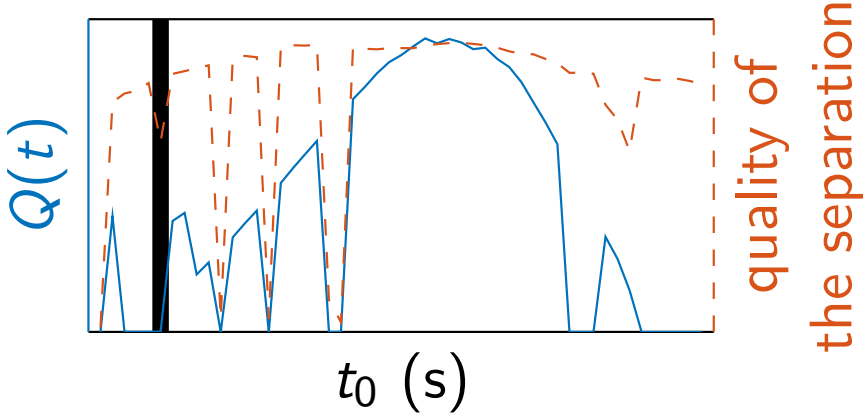


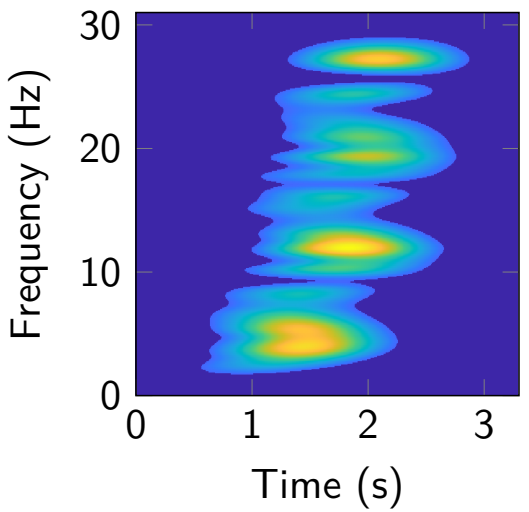
Choice of t_0



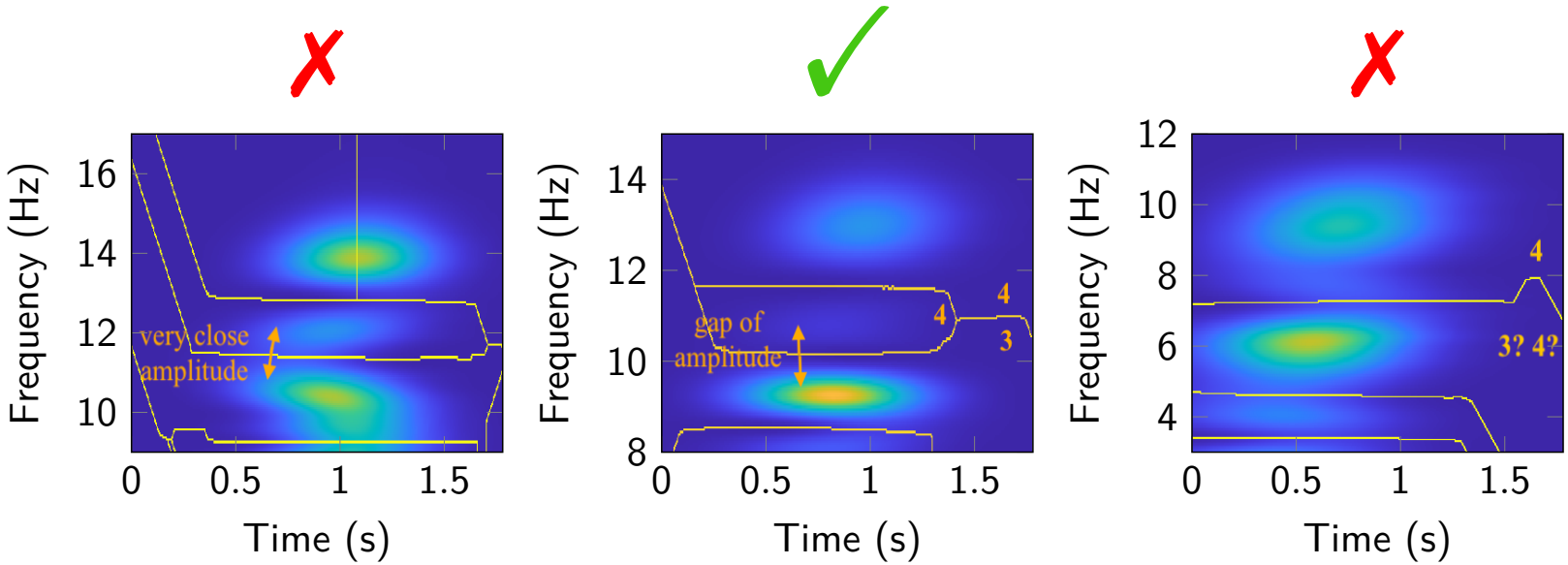
Example:

$t_0 = 6.5\text{s}, \sigma = 3\text{Hz}$



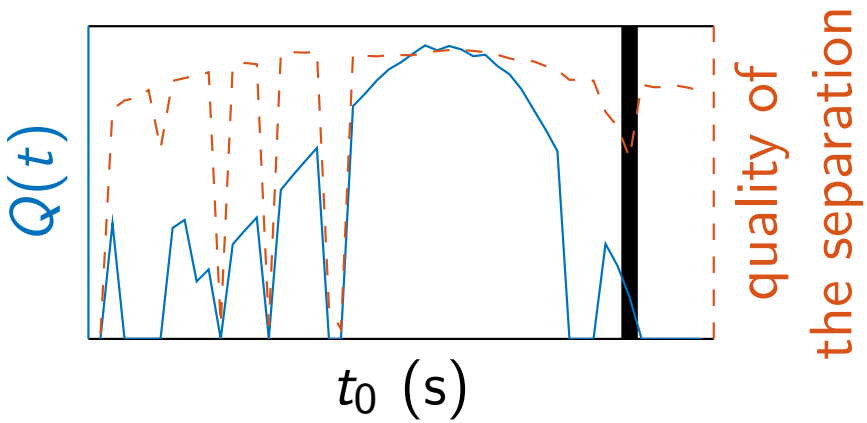


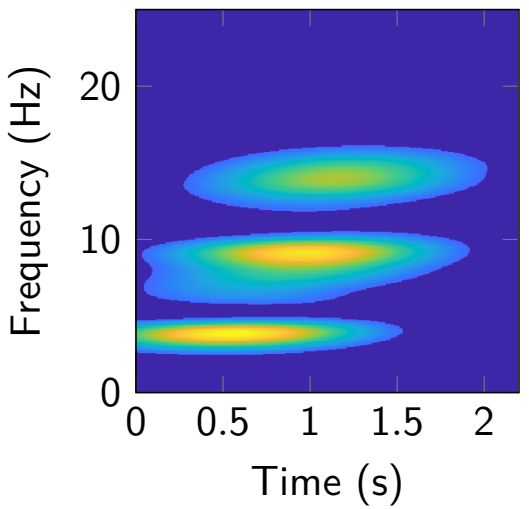
Choice of t_0



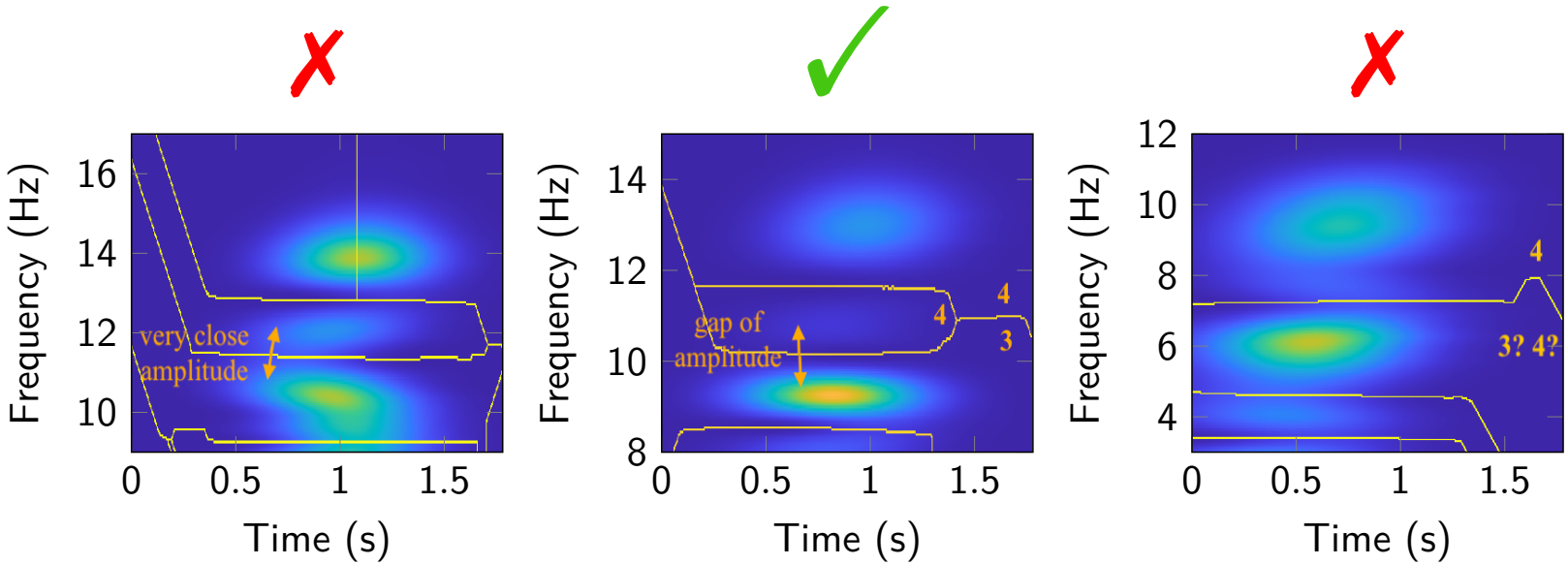
Example:

$t_0 = 6.76\text{s}, \sigma = 3\text{Hz}$



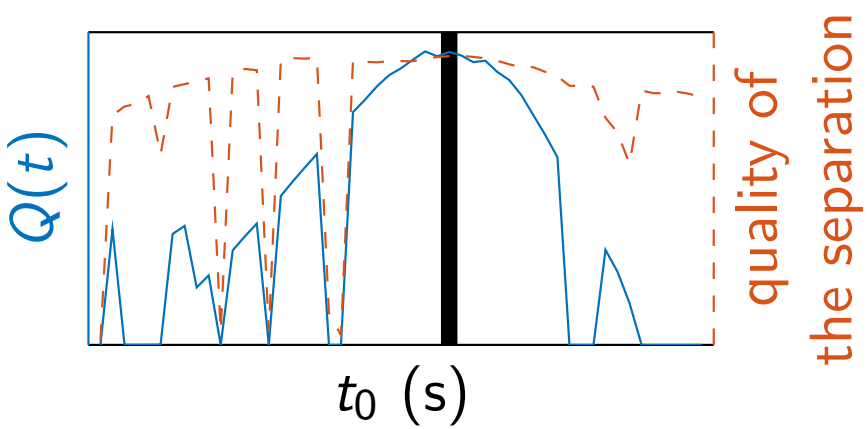


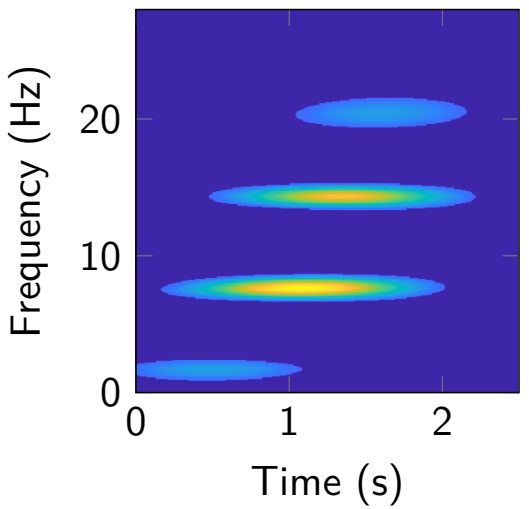
Choice of t_0



Example:

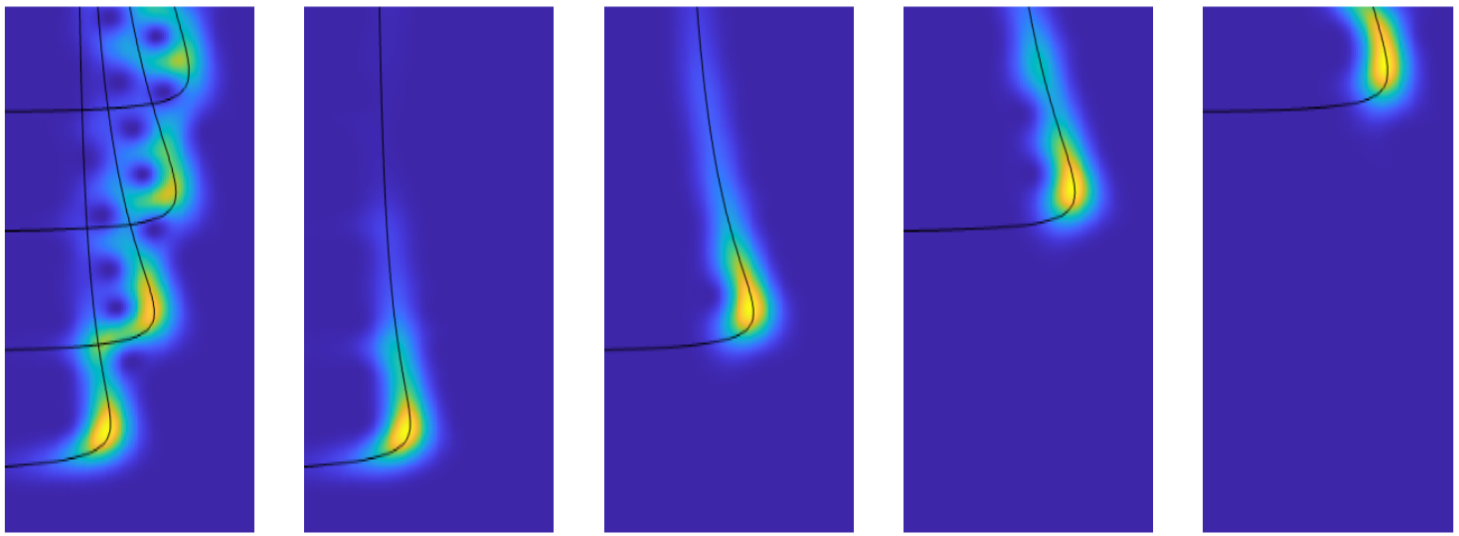
$t_0 = 6.8\text{s}, \sigma = 3\text{Hz}$





Modal separation

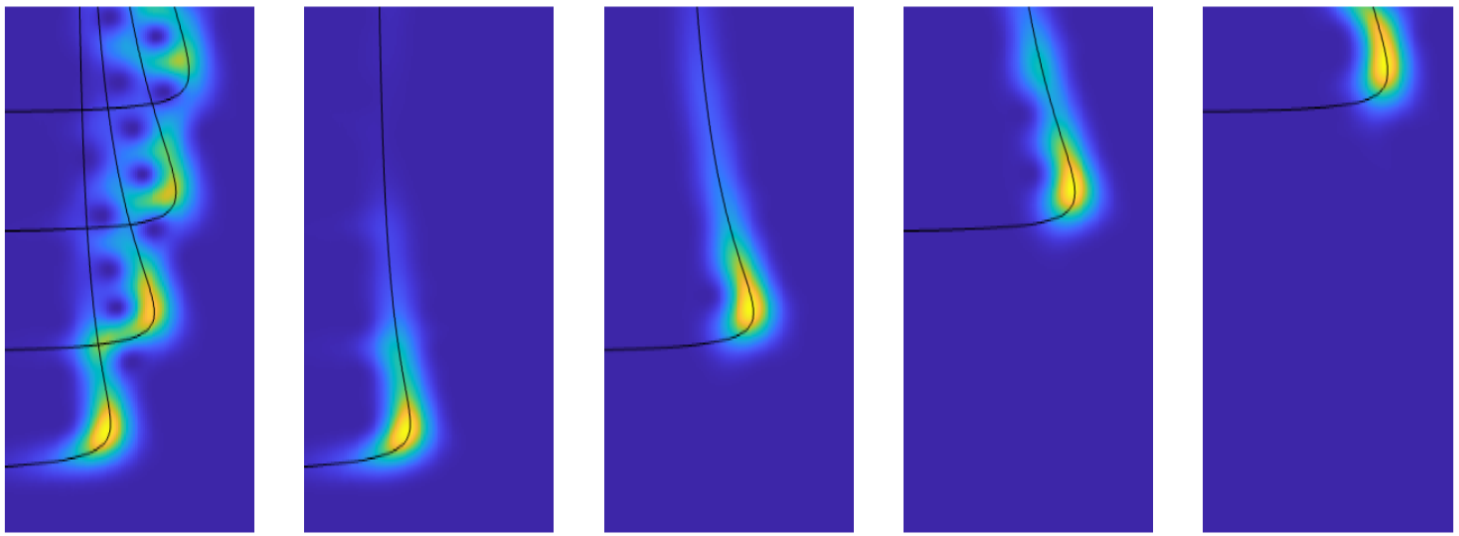
Final result:



Initial spectrogram and extracted modal components

Modal separation

Final result:



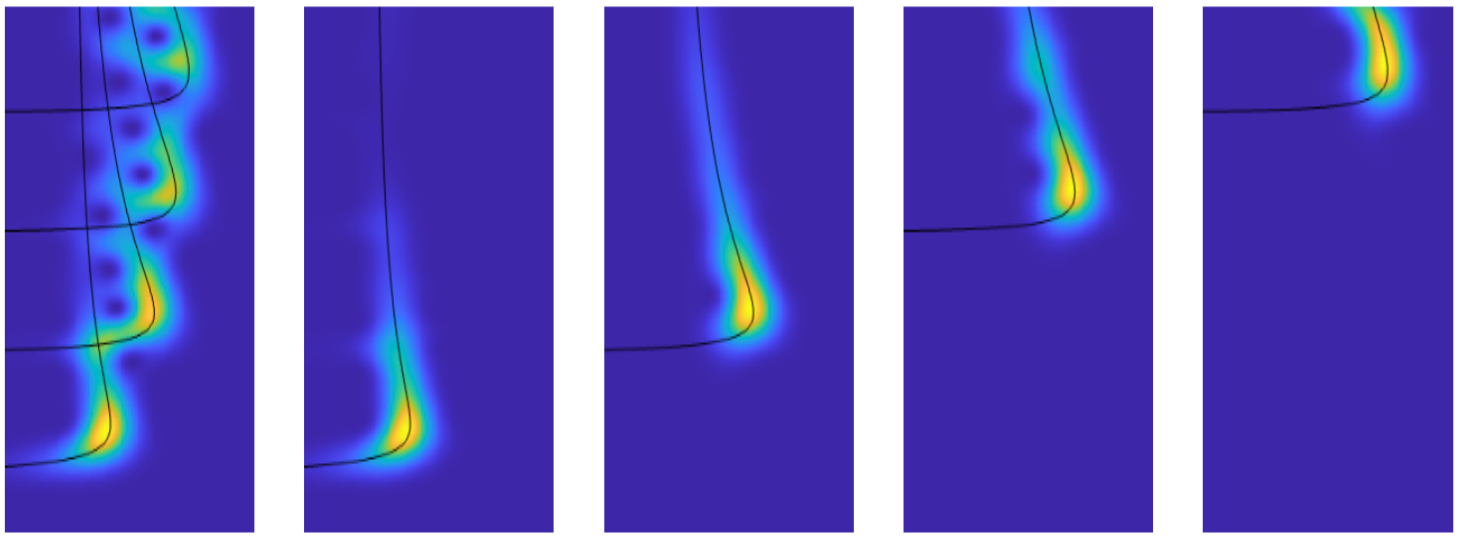
Initial spectrogram and extracted modal components

u_n	SNR $n=1$	SNR $n=2$	SNR $n=3$	SNR $n=4$
Automated algorithm	24	28	26	18
State of the art	15	10	7	1

Signal-to-Noise Ratio (SNR) of each recovered modal component

Modal separation

Final result:



Initial spectrogram and extracted modal components

- Automated method to separate the modal components (computation time around 10s vs. 4min)
- Improved quality of the separation

Link between S_n and t_n

Goal: Find the dispersion curves $\omega \mapsto t_n(\omega)$

- We known that $\hat{u_n}(\omega) = A_n(\omega)e^{irk_n(\omega)}$.

Link between S_n and t_n

Goal: Find the dispersion curves $\omega \mapsto t_n(\omega)$

- We known that $\hat{u_n}(\omega) = A_n(\omega)e^{irk_n(\omega)}$.
- Approximation of the spectrogram if the support of $\hat{h}$ is small:

$$S_n(t,\omega) = \frac{1}{4\pi^2}\left|\int_{\mathbb{R}} A_n(\xi+\omega)e^{irk_n(\xi+\omega)}\hat{h}(\xi)e^{i\xi t}\mathrm{d}\xi\right|^2. \quad (6)$$

$$A_n(\xi+\omega) \approx A_n(\omega), \qquad rk_n(\xi+\omega) \approx rk_n(\omega) - \xi t_n(\omega). \quad (7)$$

It leads to

$$\begin{aligned} S_n(t,\omega) &\approx \frac{1}{4\pi^2}|A_n(\omega)|^2\left|\int_{\mathbb{R}} \hat{h}(\xi)e^{i\xi(t-t_n(\omega))}\mathrm{d}\xi\right|^2 \\ &= |A_n(\omega)|^2 h(t-t_n(\omega))^2. \end{aligned} \quad (8)$$

Two competing methods

- Maximum method:

$$\partial_t S_n(t,\omega) = 0 \;\Leftrightarrow\; \partial_t h(t - t_n(\omega) = 0 \;\Leftrightarrow\; t = t_n(\omega). \quad (9)$$

- Mean method:

$$\frac{\int_{\mathbb{R}} t S_n(t,\omega)\mathrm{d}t}{\int_{\mathbb{R}} S_n(t,\omega)\mathrm{d}t} = t_n(\omega) \quad (10)$$

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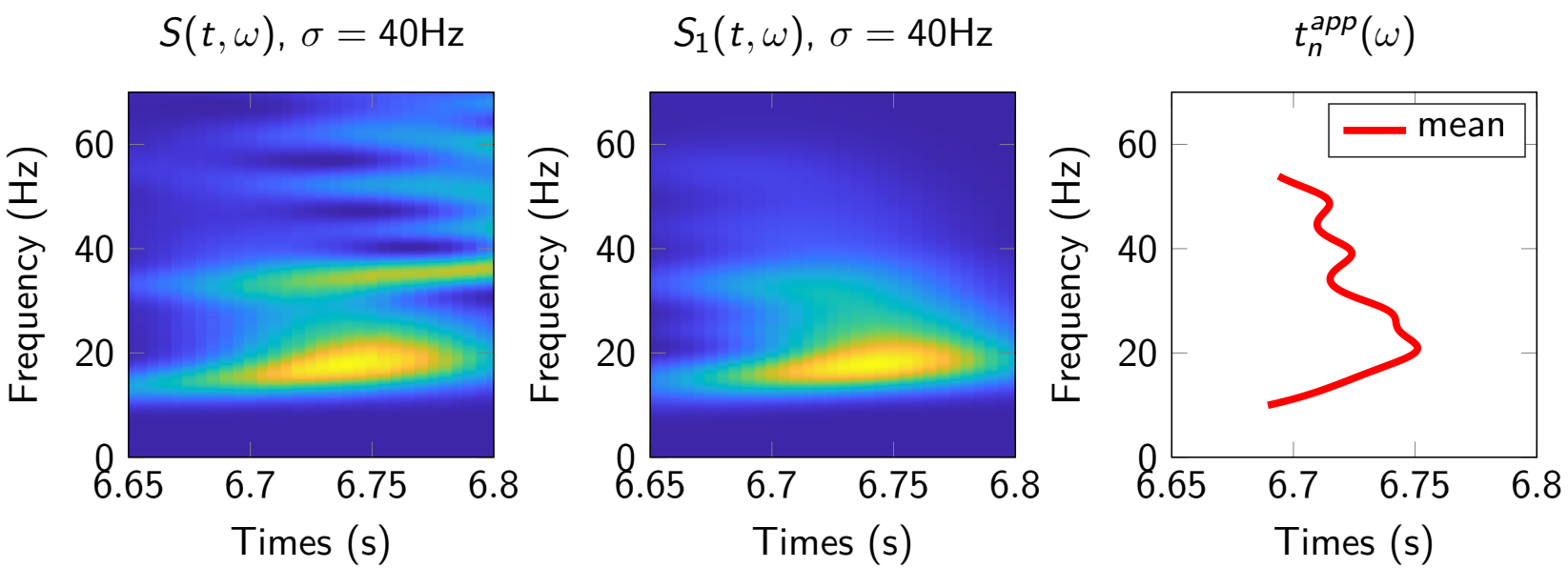

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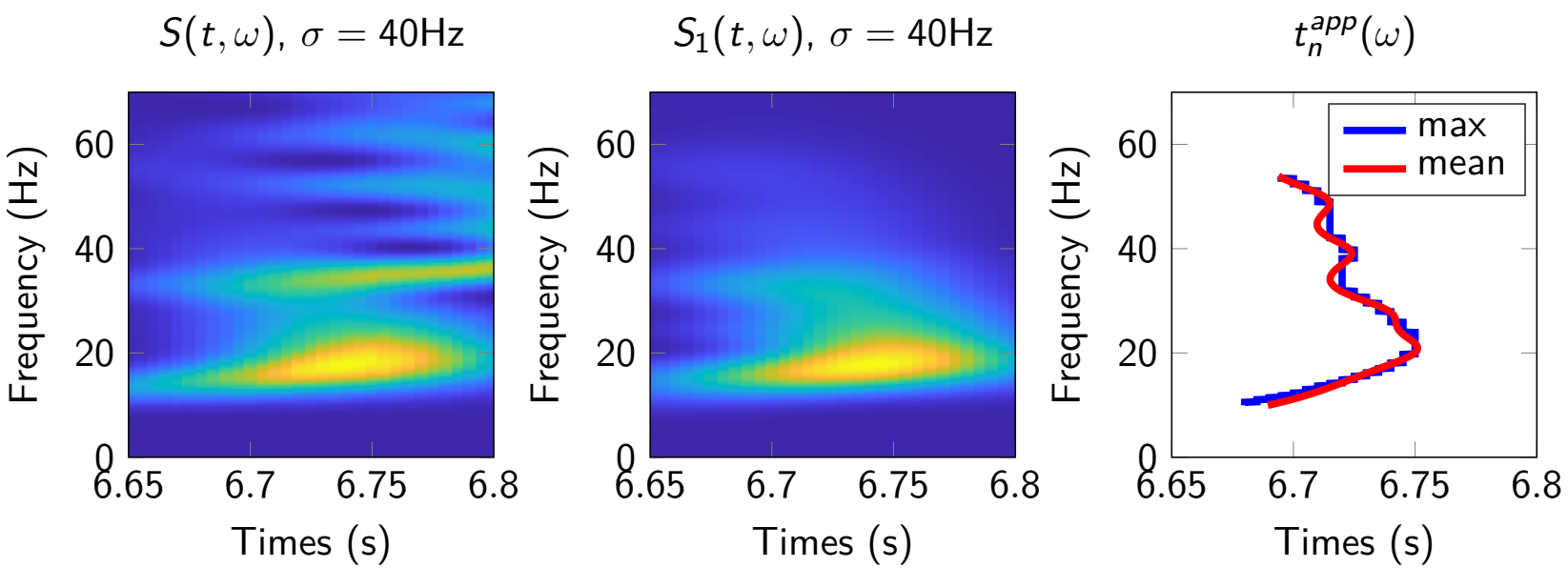

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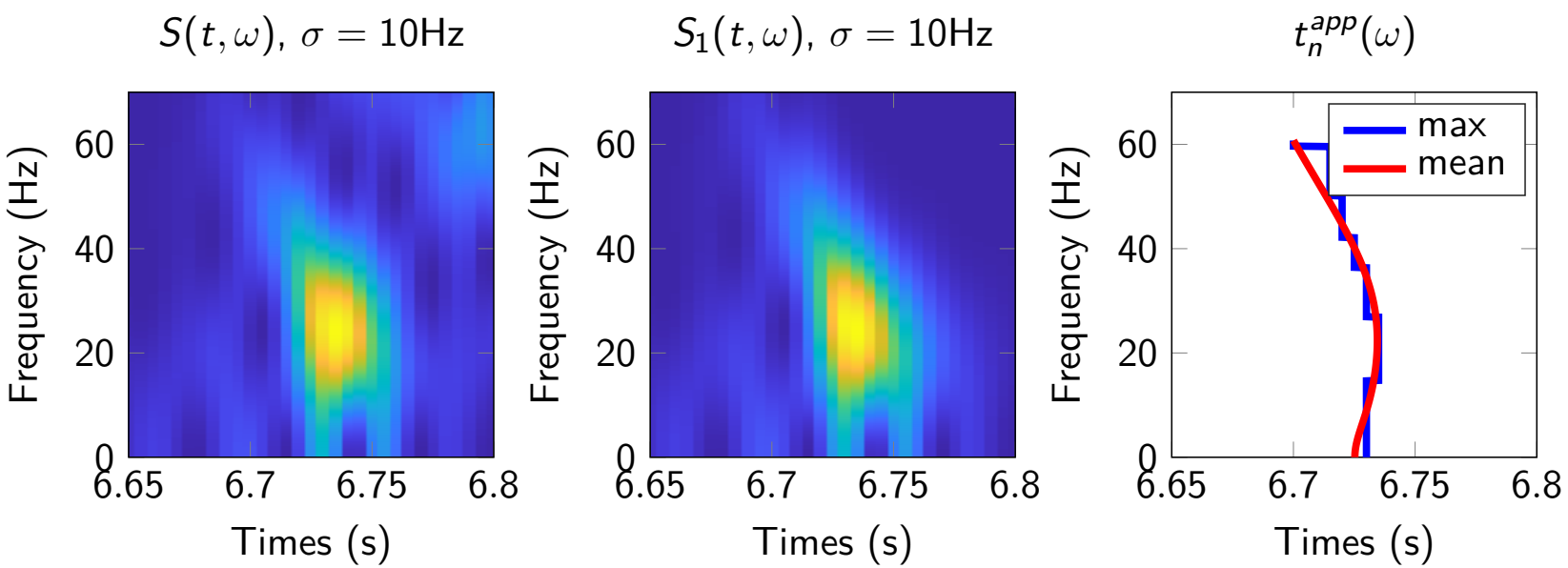

Stability of both methods

Let us consider a noised signal with an additive Gaussian noise of covariance δ, and a window function h parametrized by σ.
We denote by t_n^{app} the reconstruction of t_n.

Stability theorem

- Maximum method:

$$\mathbb{E}(|t_n^{\text{app}}(\omega) - t_n(\omega)|) \leq C_1 + C_2 \mathbf{1}_{\sigma > \sigma_{\text{lim}}(\omega)} + C_3 \frac{\delta}{\sigma^{3/2}}. \quad (11)$$

- Mean method:

$$\mathbb{E}(|t_n^{\text{app}}(\omega) - t_n(\omega)|) \leq D_1 + D_2 \sigma^2 + D_3 \frac{\delta}{\sigma}. \quad (12)$$

The expressions are optimal and verified numerically.

Niclas, A. and Garnier, J. (submitted) "Automated approach for source location in shallow waters".

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The expressions are optimal and verified numerically.

➥ Knowing δ, there exists an optimal choice of σ

Niclas, A. and Garnier, J. (submitted) "Automated approach for source location in shallow waters".

Stability of both methods

Let us consider a noised signal with an additive Gaussian noise of covariance δ, and a window function h parametrized by σ.
We denote by t_n^{app} the reconstruction of t_n.

Stability theorem - optimal choice

- Maximum method:

$$\mathbb{E}(|t_n^{\text{app}}(\omega) - t_n(\omega)|) \leq C_1 + C_2\delta. \tag{11}$$

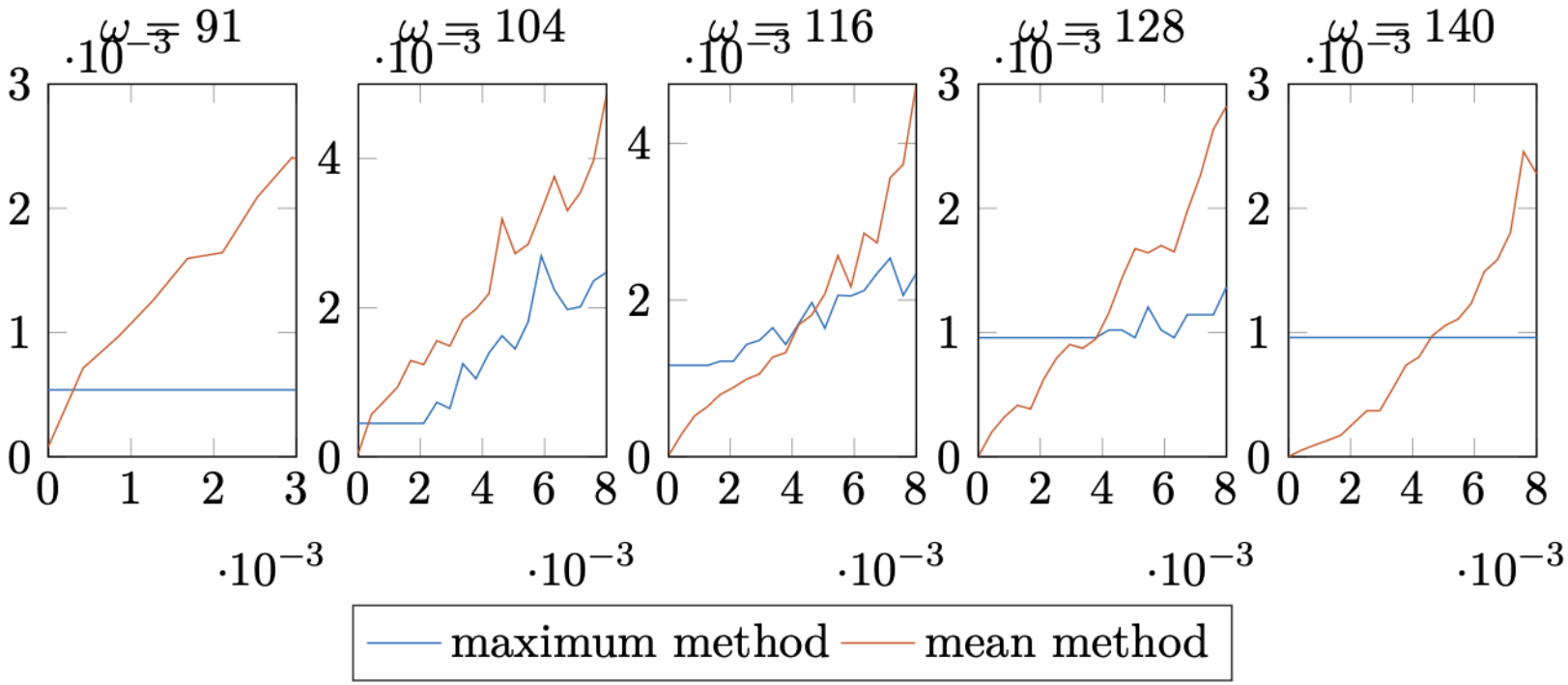
- Mean method:

$$\mathbb{E}(|t_n^{\text{app}}(\omega) - t_n(\omega)|) \leq C_1 - c + C_2\delta^{2/3}. \tag{12}$$

The expressions are optimal and verified numerically.

Niclas, A. and Garnier, J. (submitted) "Automated approach for source location in shallow waters".

Numerical comparisons

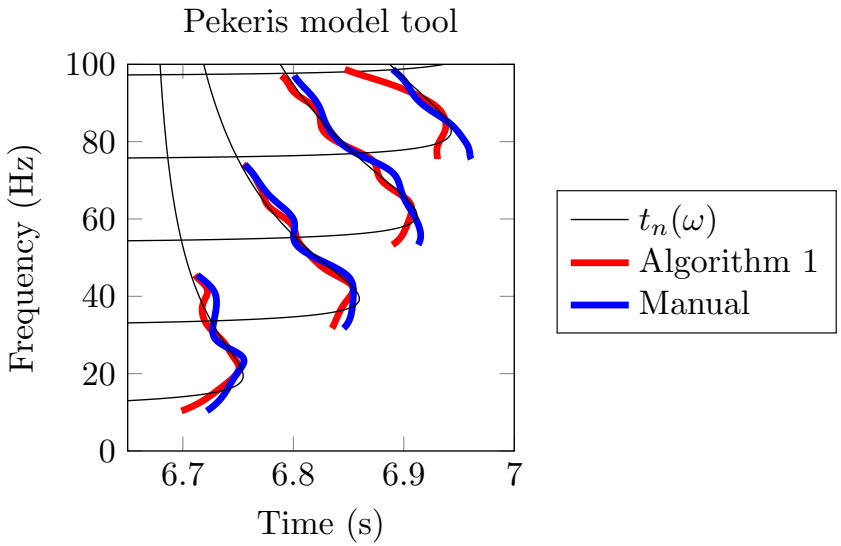


Different errors $\mathbb{E}(|t_n^{\text{app}}(\omega) - t_n(\omega)|)$ with respect to δ

- For small noise: mean method
- For medium/large noise: maximum method
- For the choice of σ: maximum method

Approximated dispersion curves

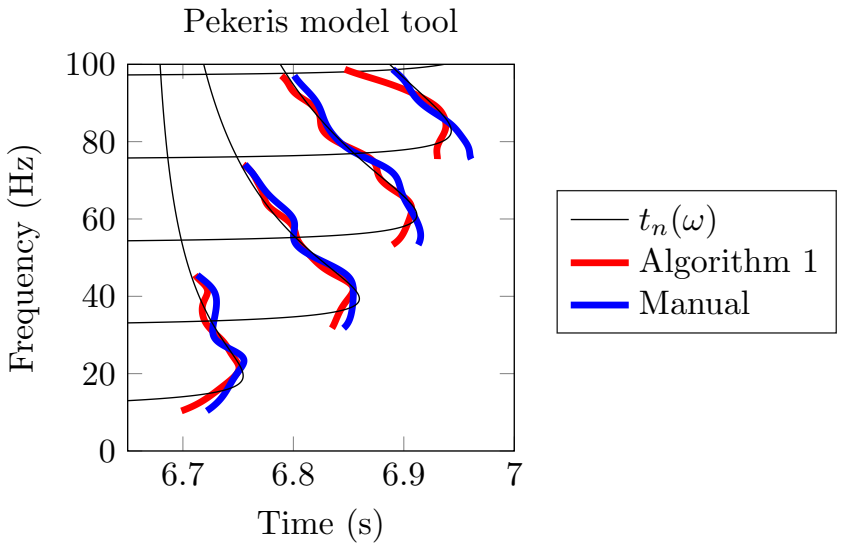
Final result:



Recovered dispersion curves $\omega \mapsto t_n(\omega)$

Approximated dispersion curves

Final result:



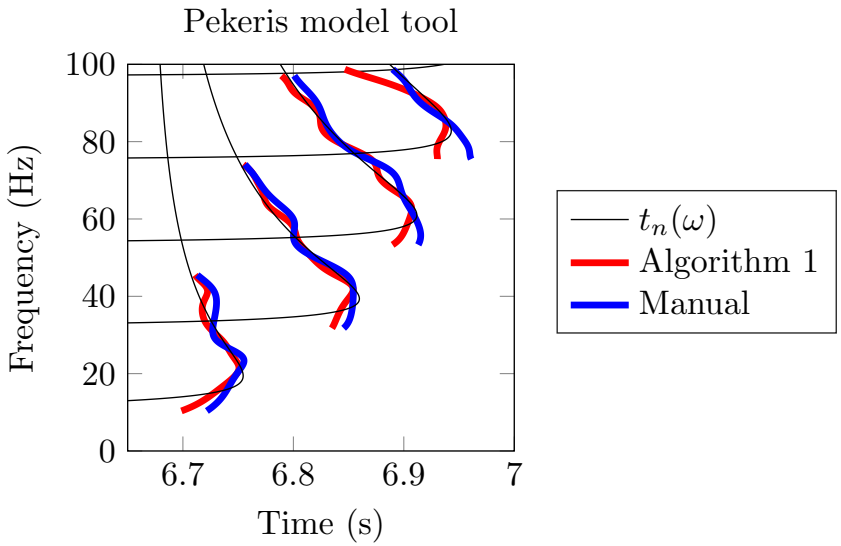
Recovered dispersion curves $\omega \mapsto t_n(\omega)$

t_n	SNR $n=1$	SNR $n=2$	SNR $n=3$	SNR $n=4$
Automated algorithm	34.3	34.6	32.9	39.6
State of the art	33.8	34	32.1	38.6

Signal-to-Noise Ratio (SNR) of each recovered t_n

Approximated dispersion curves

Final result:



Recovered dispersion curves $\omega \mapsto t_n(\omega)$

- Automated method to recover each t_n
- Mathematical justification of the choice of h
- Improved quality of the recovery of t_n

Bayesian approach for parameter estimation

We fix a prior on the values of c_w, ρ_w, ρ_b, D near $c_w^0, \rho_w^0, \rho_b^0, D^0$ and minimize the function

$$J(r, c_w, c_b, \rho_w, \rho_b, D, dt) = \|t_n^{\text{app}} + dt - t_n\|^2 + \alpha \left(10|c_w - c_w^0|^2 + 10|\rho_w - \rho_w^0|^2 + |\rho_b - \rho_b^0|^2 + |D - D^0|^2\right). \tag{13}$$

Issue:

- How to choose the coefficients in the Bayesian approach?

Results:

- error on r: $< 1\%$
- error on ρ_w: $< 1\%$
- error on c_b: $< 1\%$

Bayesian approach for parameter estimation

We fix a prior on the values of c_w, ρ_w, ρ_b, D near $c_w^0, \rho_w^0, \rho_b^0, D^0$ and minimize the function

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Issue:

- How to choose the coefficients in the Bayesian approach?

Results:

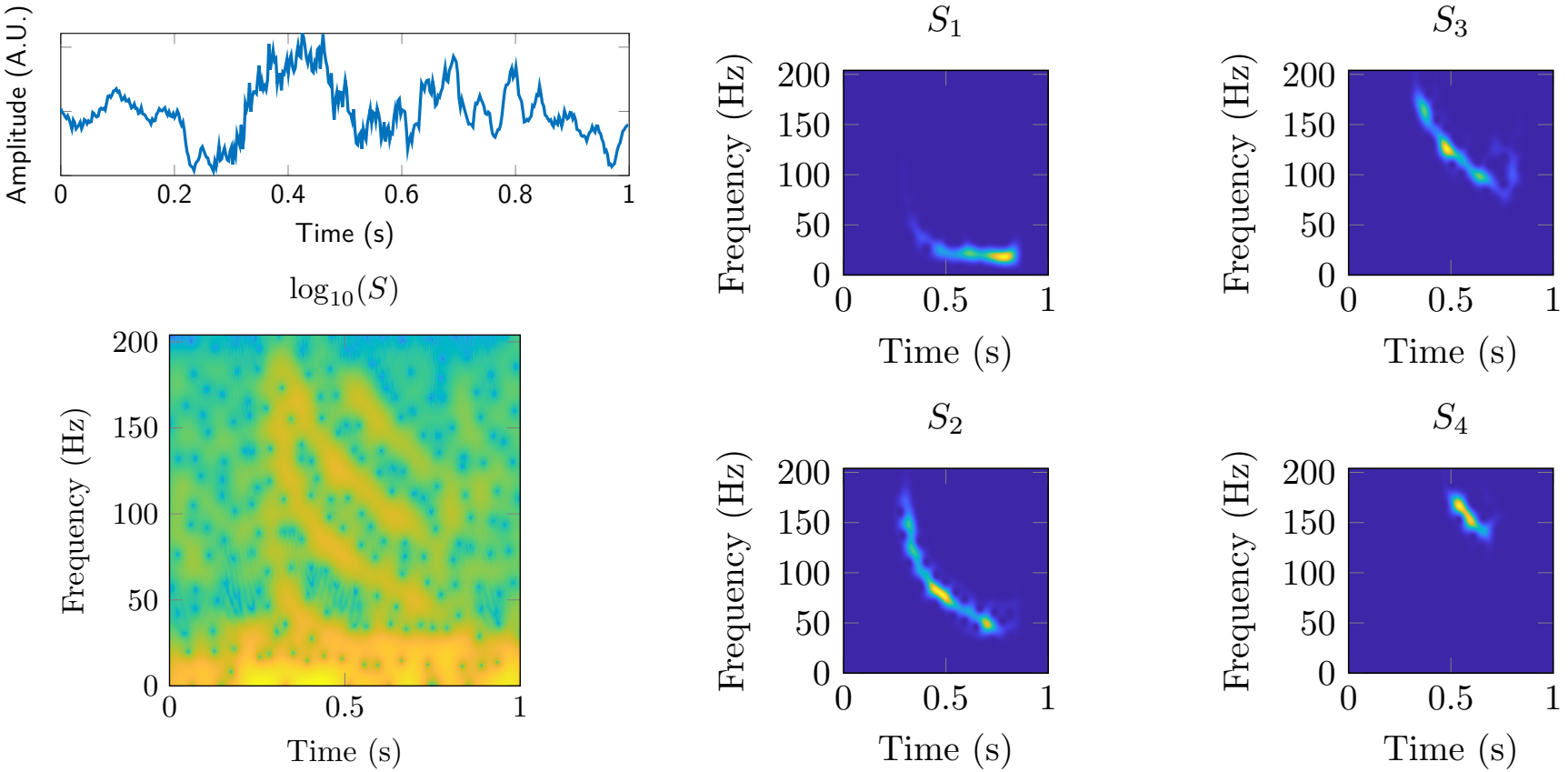
	SNR of r	SNR of c_b
Aut. Algorithm	124	92
State of the art	84	87

Signal-to-Noise Ratio (SNR) of each recovered parameter

Outline

1. Wave propagation in shallow water
   - Modal decomposition
   - Spectrograms
   - Warping
2. Automated physic-based warping
   - Optimal t_0
   - Estimated dispersion curves
   - Parameters estimation
3. Experimental data
   - Right Whale gunshot
   - Combustive sound source

Right Whales: modal components

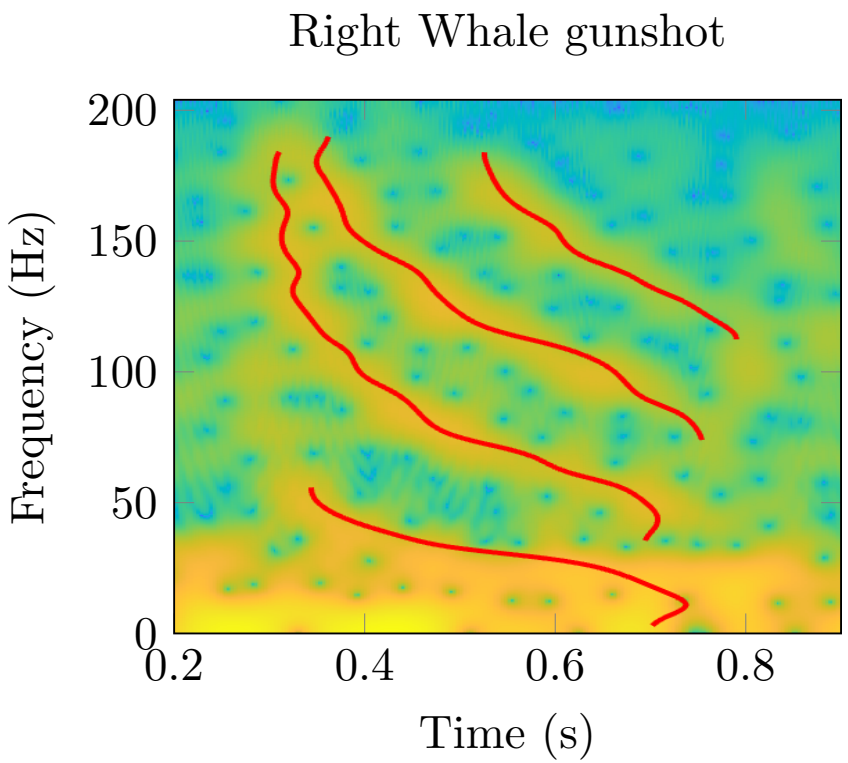


Signal, spectrogram and modal components of a right whale gunshot

Thode, A. et al. (2017). "Using non-linear time warping to estimate North Pacific right whale calling depths in the Bering Sea," The Journal of the Acoustical Society of America.
Niclas, A. and Garnier, J. (2024) "Automated approach for recovering modal components in shallow waters". The Journal of the Acoustical Society of America.

Identification of parameters

Approximated dispersion curves:

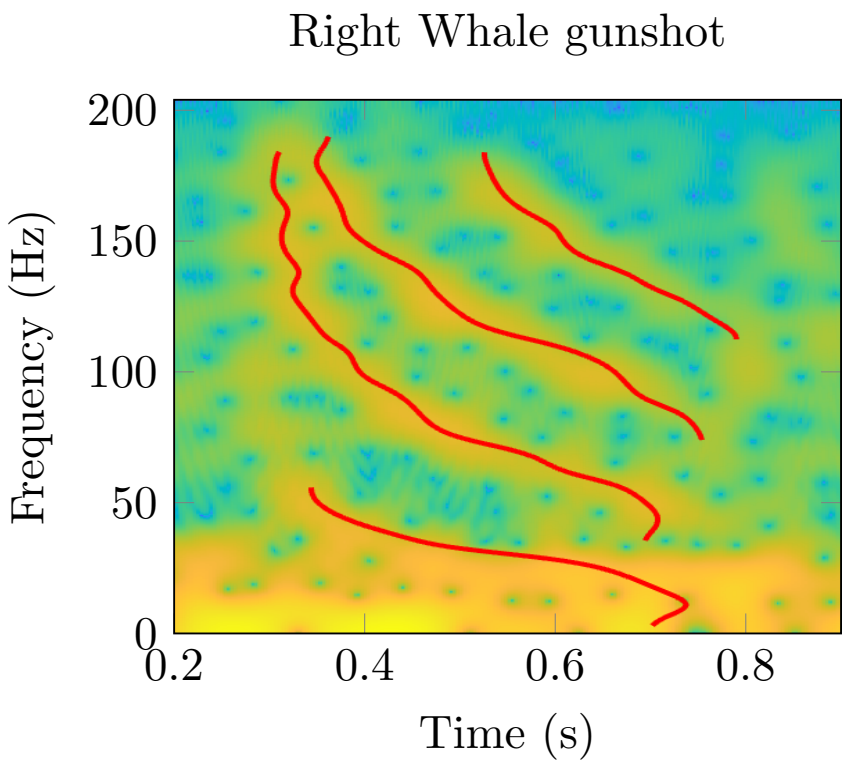


Associated parameters:

- $r = 8.81\text{km}$
- $c_b = 1727\text{m/s}$
- $c_w = 1449\text{m/s}$
- $\rho_b = 1481\text{kg/m}^3$
- $\rho_w = 1007\text{kg/m}^3$
- $D = 51.2\text{m}$
- $dt = 5.9\text{s}$

Identification of parameters

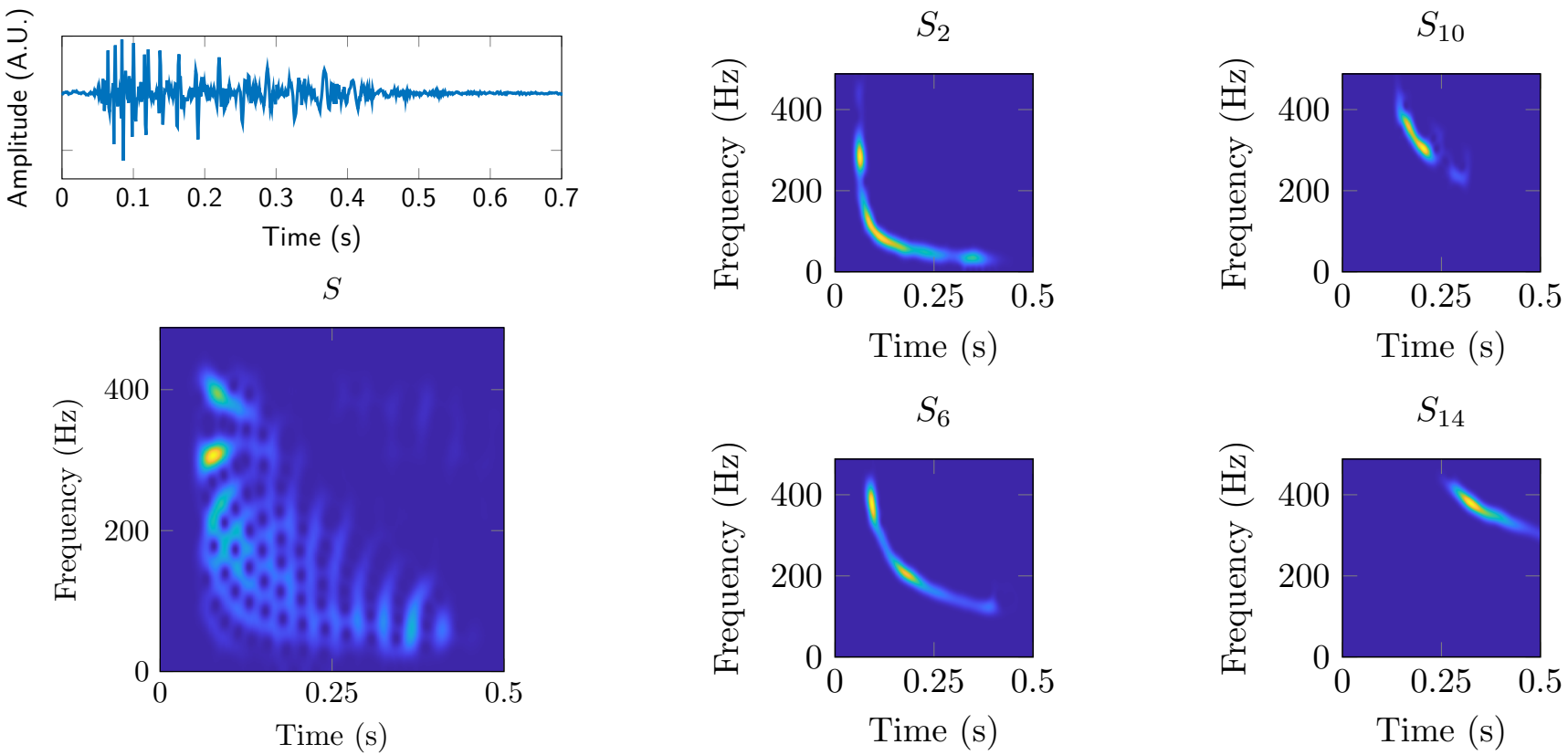
Approximated dispersion curves:



Associated parameters:

- $r = 8.81\text{km}$
  truth: $r = 8.75\text{km}$,
  relative error $< 1\%$
- $c_b = 1727\text{m/s}$
- $c_w = 1449\text{m/s}$
- $\rho_b = 1481\text{kg/m}^3$
- $\rho_w = 1007\text{kg/m}^3$
- $D = 51.2\text{m}$
- $dt = 5.9\text{s}$

CCS: modal components

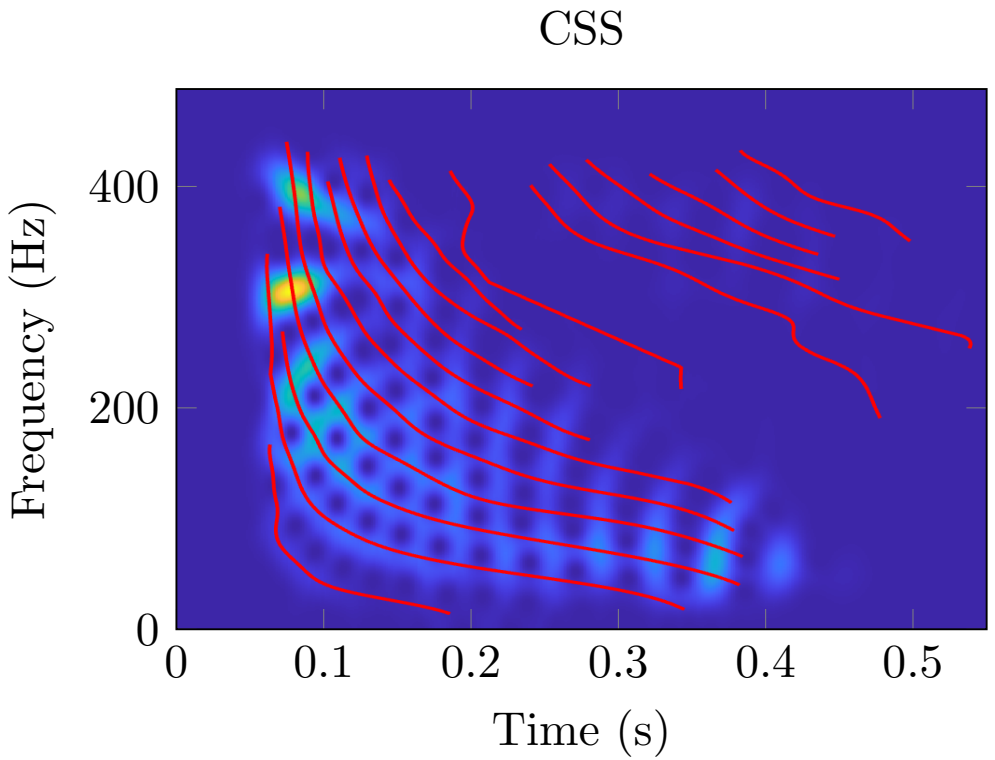


Signal, spectrogram and modal components of a combustive sound source

Bonnel, J. et al. (2018). "Geoacoustic inversion on the new england mud patch using warping and dispersion curves of high-order modes," The Journal of the Acoustical Society of America.
Niclas, A. and Garnier, J. (2024) "Automated approach for recovering modal components in shallow waters". The Journal of the Acoustical Society of America.

Identification of parameters

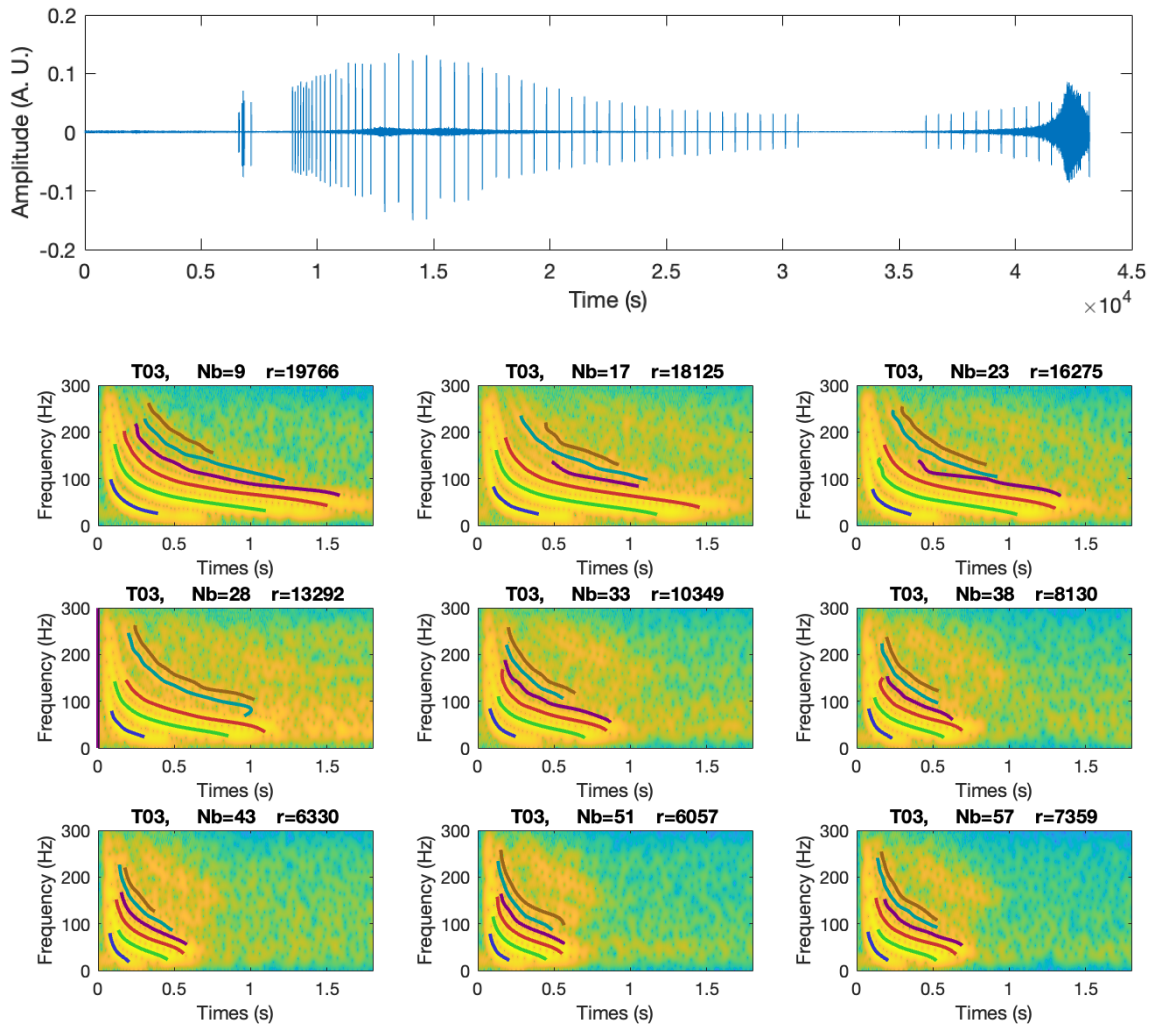
Approximated dispersion curves:



Associated parameters:

- $r = 4.8\text{km}$
- $c_b = 1649\text{m/s}$
- $c_w = 1476\text{m/s}$
- $\rho_b = 1282\text{kg/m}^3$
- $\rho_w = 996\text{kg/m}^3$
- $D = 73.1\text{m}$
- $dt = 3.2\text{s}$

Extraction of modal components



Signal and some extracted spectrograms and modal components of simulated gunshots

Summary

Summary:

- Automated method to recover the different modal components of a signal in a Pekeris waveguide
- Inverse problem using some prior information to find the parameters of the waveguide

Outlooks:

- Collaboration with Mark Goldwater and Julien Bonnel on applying machine learning to extracted modal components
- Adaptation to recover parameters in seismic imaging

Thank you for your attention!

