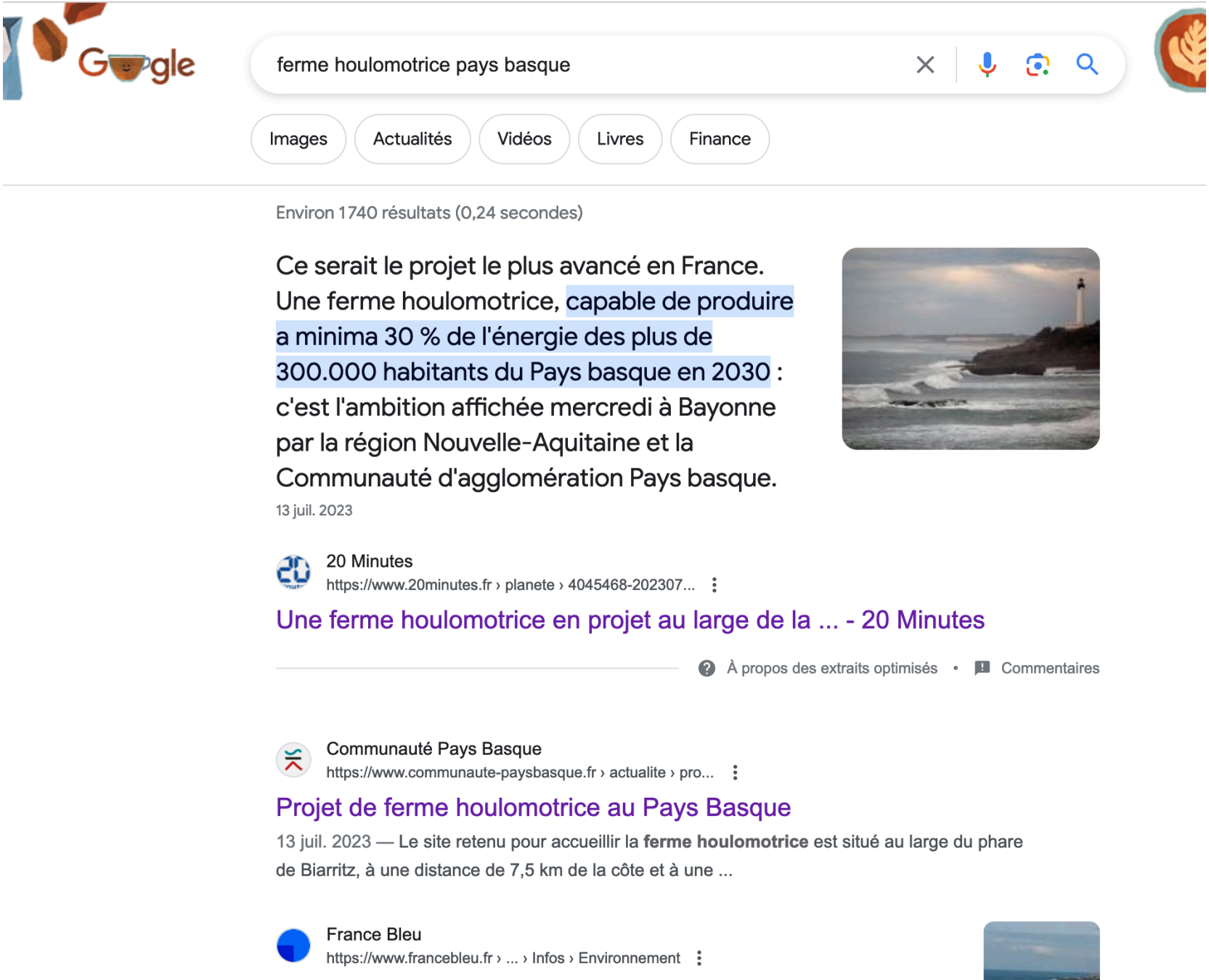
# Quelques enjeux autour de la modélisation des vagues

D. Lannes
Institut de Mathématiques de Bordeaux

Rencontres chercheur·euse·s et ingénieur·e·s

Novembre 2026

# Don’t worry...


Google
ferme houlomotrice pays basque
Images
Actualités
Vidéos
Livres
Finance
Environ 1740 résultats (0,24 secondes)
Ce serait le projet le plus avancé en France. Une ferme houlomotrice, capable de produire a minima 30 % de l'énergie des plus de 300.000 habitants du Pays basque en 2030 : c'est l'ambition affichée mercredi à Bayonne par la région Nouvelle-Aquitaine et la Communauté d'agglomération Pays basque.
13 juil. 2023
20 Minutes
https://www.20minutes.fr › planete › 4045468-202307...
Une ferme houlomotrice en projet au large de la ... - 20 Minutes
À propos des extraits optimisés
Commentaires
Communauté Pays Basque
https://www.communaute-paysbasque.fr › actualite › pro...
Projet de ferme houlomotrice au Pays Basque
13 juil. 2023 — Le site retenu pour accueillir la ferme houlomotrice est situé au large du phare de Biarritz, à une distance de 7,5 km de la côte et à une ...
France Bleu
https://www.francebleu.fr › ... › Infos › Environnement

# Wave-Energy Convertors

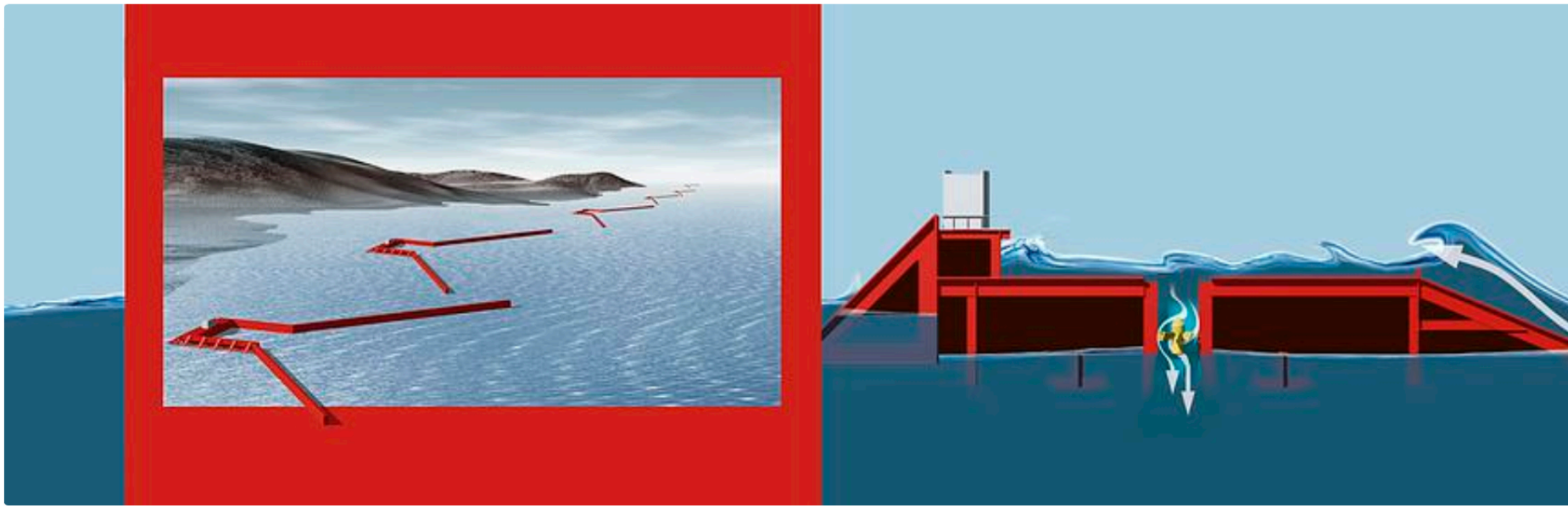
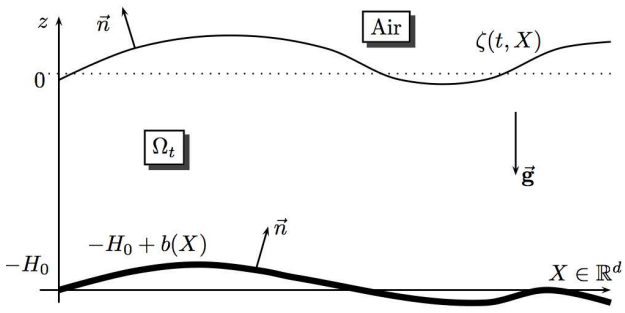
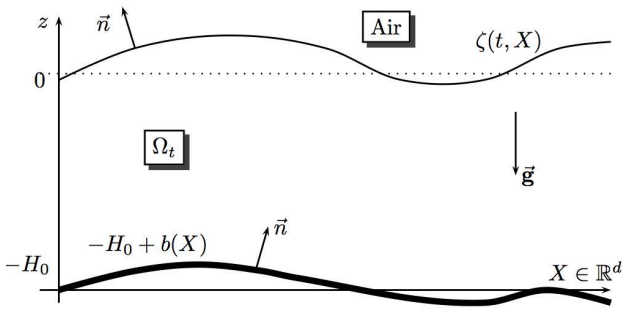


Diagram by Claus Lunau

# Which energy ?

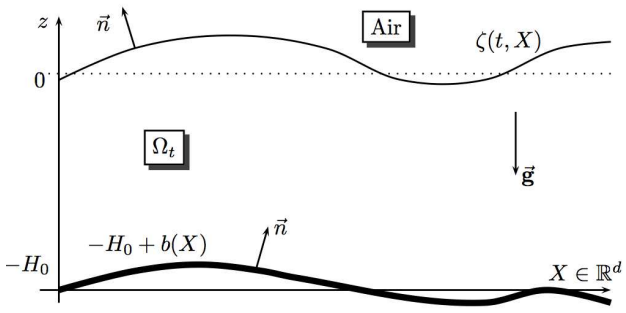


# Which energy ?




$$\begin{cases} \partial_t U + (U\cdot\nabla_{X,z})U = -\frac{1}{\rho}\nabla_{X,z}P + \mathbf{g}, \\ \mathrm{div}\, U = 0, \\ \mathrm{curl}\, U = 0 \end{cases} \qquad \text{in } \Omega_t$$

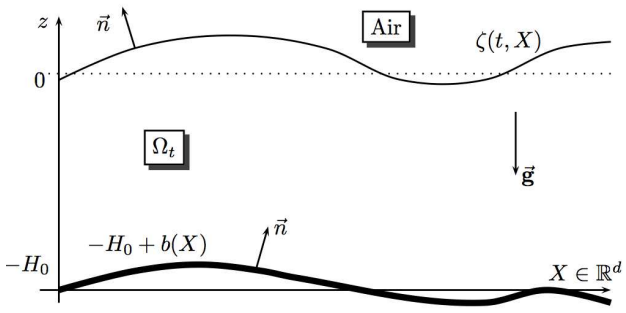
# Which energy ?




$$\begin{cases} \partial_t U + (U\cdot\nabla_{X,z})U = -\frac{1}{\rho}\nabla_{X,z}P + \mathbf{g}, \\ \mathrm{div}\, U = 0, & \text{in } \Omega_t \\ \mathrm{curl}\, U = 0 \end{cases}$$

$$U\cdot N = 0 \qquad \text{at the bottom}$$

# Which energy ?

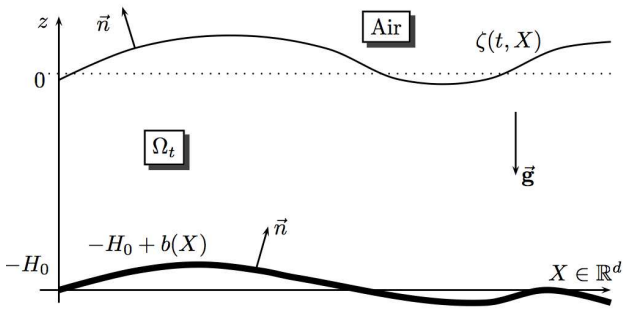



$$\begin{cases} \partial_t U + (U\cdot\nabla_{X,z})U = -\frac{1}{\rho}\nabla_{X,z}P + \mathbf{g}, \\ \mathrm{div}\, U = 0, \\ \mathrm{curl}\, U = 0 \end{cases} \qquad \text{in } \Omega_t$$

$$U\cdot N = 0 \qquad \text{at the bottom}$$

$$\begin{cases} \partial_t \zeta - U\cdot N = 0, \\ P = P_{\mathrm{atm}} \end{cases} \qquad \text{at the surface}$$

# Which energy ?




$$\begin{cases} \partial_t U + (U\cdot\nabla_{X,z})U = -\frac{1}{\rho}\nabla_{X,z}P + \mathbf{g}, \\ \mathrm{div}\, U = 0, \\ \mathrm{curl}\, U = 0 \end{cases} \qquad \text{in } \Omega_t$$

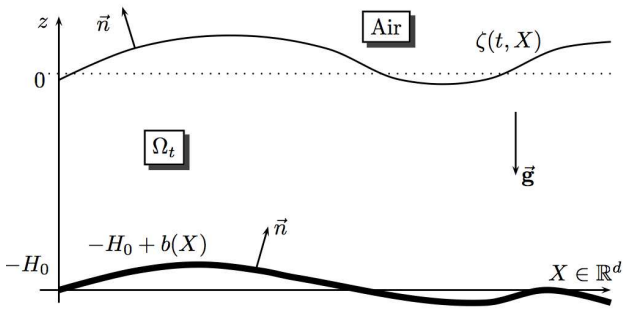
$$U\cdot N = 0 \qquad \text{at the bottom}$$

$$\begin{cases} \partial_t \zeta - U\cdot N = 0, \\ P = P_{\mathrm{atm}} \end{cases} \qquad \text{at the surface}$$

**Energy conservation**

$$e = \frac{1}{2}\rho g\zeta^2 + \rho\int_{b(X)}^{\zeta(t,X)} \frac{1}{2}(|V|^2 + w^2),$$

# Which energy ?




$$\begin{cases} \partial_t U + (U\cdot\nabla_{X,z})U = -\frac{1}{\rho}\nabla_{X,z}P + \mathbf{g}, \\ \mathrm{div}\, U = 0, \\ \mathrm{curl} U = 0 \end{cases} \quad \text{in } \Omega_t$$

$$U\cdot N = 0 \qquad \text{at the bottom}$$

$$\begin{cases} \partial_t \zeta - U\cdot N = 0, \\ P = P_{\mathrm{atm}} \end{cases} \quad \text{at the surface}$$

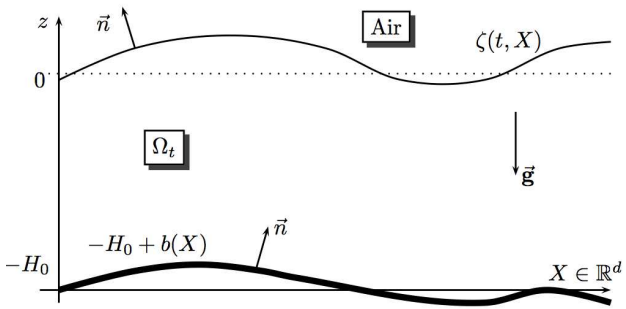
**Energy conservation**

$$e = \frac{1}{2}\rho g\zeta^2 + \rho\int_{b(X)}^{\zeta(t,X)} \frac{1}{2}(|V|^2 + w^2),$$

$$\partial_t e + \nabla\cdot\mathcal{F} = 0$$

with

# Which energy ?




$$\begin{cases} \partial_t U + (U\cdot\nabla_{X,z})U = -\frac{1}{\rho}\nabla_{X,z}P + \mathbf{g}, \\ \mathrm{div}\, U = 0, \\ \mathrm{curl} U = 0 \end{cases} \quad \text{in } \Omega_t$$

$$U\cdot N = 0 \qquad \text{at the bottom}$$

$$\begin{cases} \partial_t \zeta - U\cdot N = 0, \\ P = P_{\mathrm{atm}} \end{cases} \quad \text{at the surface}$$

**Energy conservation**

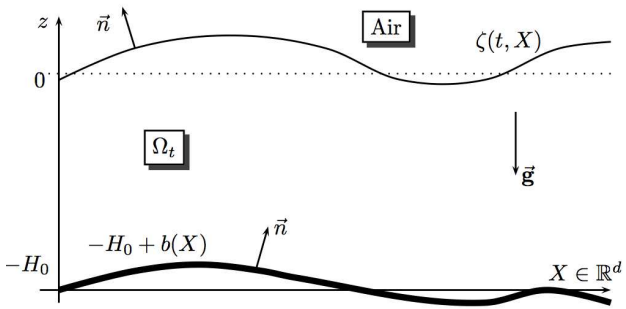
$$e = \frac{1}{2}\rho g\zeta^2 + \rho\int_{b(X)}^{\zeta(t,X)} \frac{1}{2}(|V|^2 + w^2),$$

$$\partial_t e + \nabla\cdot\mathcal{F} = 0$$

with

$$\mathcal{F} = \rho\int_{b(X)}^{\zeta(t,X)} V(g\zeta + \frac{1}{\rho}(P-P_{\mathrm{atm}}) + \frac{1}{2}|U|^2) \qquad (kW\cdot m^{-1})$$

# Which energy ?




$$\begin{cases} \partial_t U + (U \cdot \nabla_{X,z}) U = -\frac{1}{\rho} \nabla_{X,z} P + \mathbf{g}, \\ \operatorname{div} U = 0, \\ U = \nabla_{X,z} \phi \end{cases} \quad \text{in } \Omega_t$$

$$U \cdot N = 0 \qquad \text{at the bottom}$$

$$\begin{cases} \partial_t \zeta - U \cdot N = 0, \\ P = P_{\text{atm}} \end{cases} \quad \text{at the surface}$$

**Energy conservation**

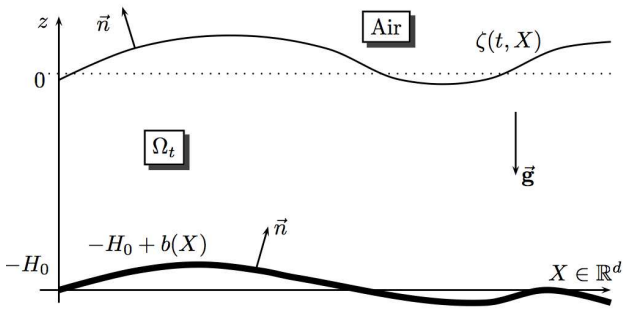
$$e = \frac{1}{2} \rho g \zeta^2 + \rho \int_{b(X)}^{\zeta(t,X)} \frac{1}{2} (|V|^2 + w^2),$$

$$\partial_t e + \nabla \cdot \mathcal{F} = 0$$

with

$$\mathcal{F} = \rho \int_{b(X)}^{\zeta(t,X)} V \left( g\zeta + \frac{1}{\rho}(P - P_{\text{atm}}) + \frac{1}{2}|U|^2 \right) \qquad (kW \cdot m^{-1})$$

# Which energy ?




$$\begin{cases} \partial_t U + (U \cdot \nabla_{X,z}) U = -\frac{1}{\rho} \nabla_{X,z} P + \mathbf{g}, \\ \Delta_{X,z} \phi = 0 \\ U = \nabla_{X,z} \phi \end{cases} \quad \text{in } \Omega_t$$

$$U \cdot N = 0 \qquad \text{at the bottom}$$

$$\begin{cases} \partial_t \zeta - U \cdot N = 0, \\ P = P_{\text{atm}} \end{cases} \qquad \text{at the surface}$$

**Energy conservation**

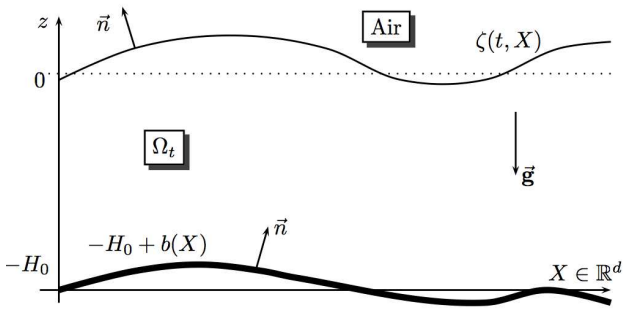
$$e = \frac{1}{2} \rho g \zeta^2 + \rho \int_{b(X)}^{\zeta(t,X)} \frac{1}{2} (|V|^2 + w^2),$$

$$\partial_t e + \nabla \cdot \mathcal{F} = 0$$

with

$$\mathcal{F} = \rho \int_{b(X)}^{\zeta(t,X)} V \left( g\zeta + \frac{1}{\rho}(P - P_{\text{atm}}) + \frac{1}{2}|U|^2 \right) \qquad (kW \cdot m^{-1})$$

# Which energy ?




$$\begin{cases} \partial_t\phi + g\zeta + \frac{1}{2}|U|^2 + \frac{1}{\rho}(P - P_{\text{atm}}) = 0 \\ \Delta_{X,z}\phi = 0 \\ U = \nabla_{X,z}\phi \end{cases} \quad \text{in } \Omega_t$$

$$U \cdot N = 0 \qquad \text{at the bottom}$$

$$\begin{cases} \partial_t\zeta - U \cdot N = 0, \\ P = P_{\text{atm}} \end{cases} \qquad \text{at the surface}$$

**Energy conservation**

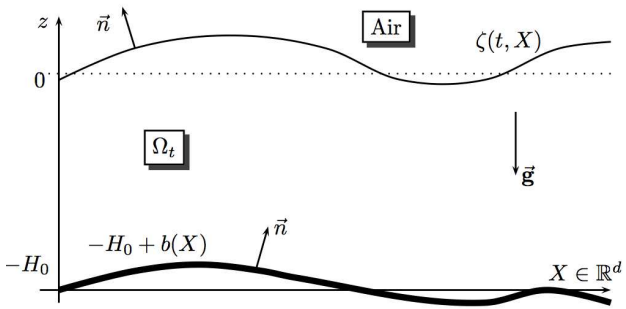
$$e = \frac{1}{2}\rho g\zeta^2 + \rho\int_{b(X)}^{\zeta(t,X)} \frac{1}{2}(|V|^2 + w^2),$$

$$\partial_t e + \nabla \cdot \mathcal{F} = 0$$

with

$$\mathcal{F} = \rho\int_{b(X)}^{\zeta(t,X)} V(g\zeta + \frac{1}{\rho}(P - P_{\text{atm}}) + \frac{1}{2}|U|^2) \qquad (kW \cdot m^{-1})$$

# Which energy ?




$$\begin{cases} \partial_t\phi + g\zeta + \frac{1}{2}|U|^2 + \frac{1}{\rho}(P - P_{\text{atm}}) = 0 \\ \Delta_{X,z}\phi = 0 \\ U = \nabla_{X,z}\phi \end{cases} \quad \text{in } \Omega_t$$

$$U \cdot N = 0 \qquad \text{at the bottom}$$

$$\begin{cases} \partial_t\zeta - U \cdot N = 0, \\ P = P_{\text{atm}} \end{cases} \qquad \text{at the surface}$$

**Energy conservation**

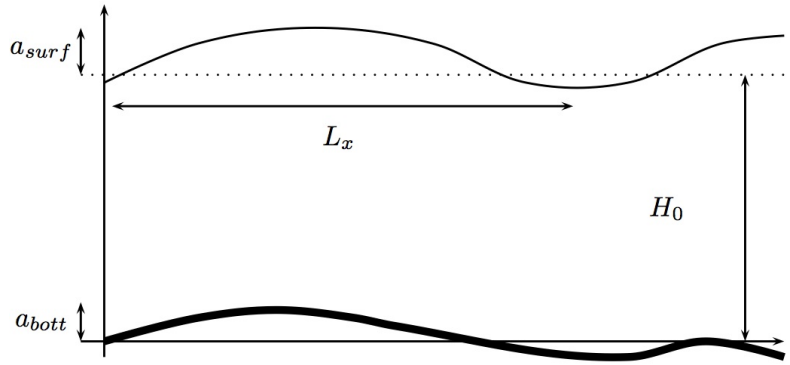
$$e = \frac{1}{2}\rho g\zeta^2 + \rho\int_{b(X)}^{\zeta(t,X)} \frac{1}{2}(|V|^2 + w^2),$$

$$\partial_t e + \nabla \cdot \mathcal{F} = 0$$

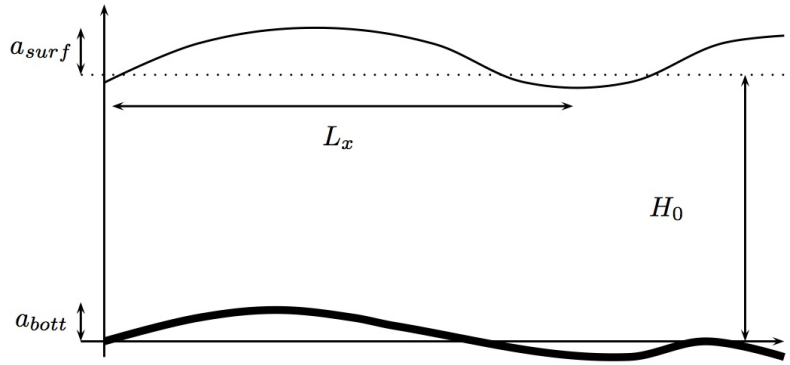
with

$$\mathcal{F} = \rho\int_{b(X)}^{\zeta(t,X)} \nabla\phi\partial_t\phi \qquad (kW \cdot m^{-1})$$

# Asymptotic approximations



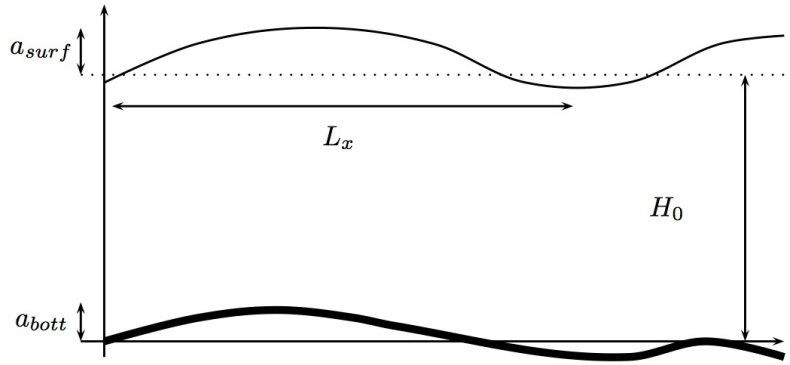
# Asymptotic approximations




**Dimensionless parameters**

$$\varepsilon = \frac{a}{H_0} \quad \text{(nonlinearity parameter)} \qquad \mu = \frac{H_0^2}{L^2} \quad \text{(shallowness parameter)}$$

# Asymptotic approximations




## Dimensionless parameters

$$\varepsilon = \frac{a}{H_0} \quad \text{(nonlinearity parameter)} \qquad \mu = \frac{H_0^2}{L^2} \quad \text{(shallowness parameter)}$$

## Asymptotic regimes

- $\varepsilon \ll 1 \rightsquigarrow$ Linear approximation
- $\mu \ll 1 \rightsquigarrow$ Shallow water approximations

# Linear approximation

$$\mathcal{F}(t,X) = \rho \int_{-H_0}^{0} \nabla\phi(t,X,z)\partial_t\phi(t,X,z)\mathrm{d}z$$

# Linear approximation

$$\boxed{\mathcal{F}(t,X) = \rho \int_{-H_0}^{0} \nabla\phi(t,X,z)\partial_t\phi(t,X,z)\mathrm{d}z}$$

Airy theory:

$$\zeta(t,X) = a e^{i(-\omega t + \mathbf{k}\cdot X)}$$

$$\phi(t,X,z) = i\frac{ag}{\omega}\frac{\cosh(k(z+H_0))}{\cosh(kH_0)} e^{i(-\omega t + \mathbf{k}\cdot X)}$$

# Linear approximation

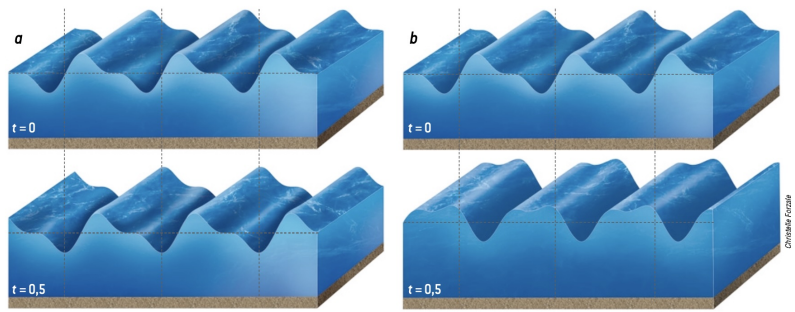
$$\boxed{\mathcal{F}(t,X) = \rho \int_{-H_0}^{0} \nabla\phi(t,X,z)\partial_t\phi(t,X,z)\mathrm{d}z}$$

Airy theory:

$$\zeta(t,X) = a e^{i(-\omega t + \mathbf{k}\cdot X)}$$

$$\phi(t,X,z) = i\frac{ag}{\omega}\frac{\cosh(k(z+H_0))}{\cosh(kH_0)} e^{i(-\omega t + \mathbf{k}\cdot X)}$$

$$\omega^2 = gk\tanh(H_0 k) \qquad \text{(dispersion relation)}$$



# Linear approximation

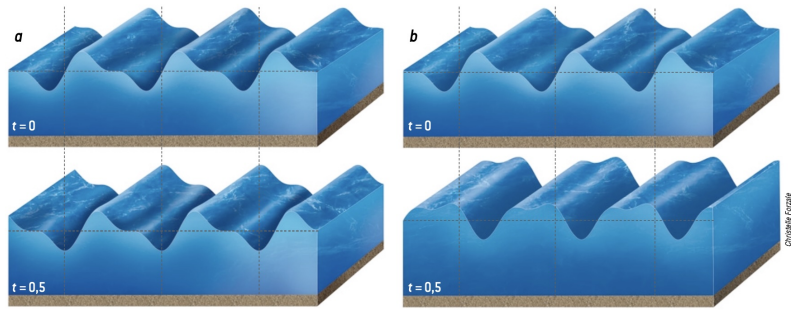
$$\mathcal{F}(t,X) = \rho \int_{-H_0}^{0} \nabla\phi(t,X,z)\partial_t\phi(t,X,z)\mathrm{d}z$$

Airy theory:

$$\zeta(t,X) = a e^{i(-\omega t + \mathbf{k}\cdot X)}$$

$$\phi(t,X,z) = i\frac{ag}{\omega}\frac{\cosh(k(z+H_0))}{\cosh(kH_0)} e^{i(-\omega t + \mathbf{k}\cdot X)}$$

$$\omega^2 = gk\tanh(H_0 k) \qquad \text{(dispersion relation)}$$




Average energy flux on a period $T = 2\pi/\omega$:

$$\langle \mathcal{F}(a,\omega)\rangle_T = \frac{1}{8\pi}\rho g^2 a^2 T\left(1 + \frac{2k(\omega)H_0}{\sinh(2k(\omega)H_0)}\right)$$

# Wave energy : how much ?

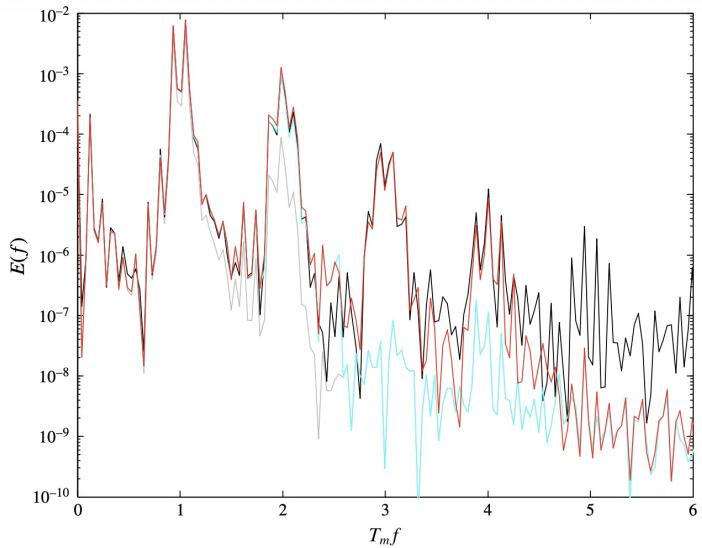



FIGURE 9. (Colour online) Surface elevation energy density spectra, $E(f)$, as a function of the dimensionless frequency $T_m f$, for bichromatic waves, $f_1 = 0.5515$ Hz, $f_2 = 0.6250$ Hz ($T_m = ((f_1 + f_2)/2)^{-1}$), $h_0 = 0.326$ m and $\delta_m = 0.5$ cm. Cutoff frequency $f_c = 1.5$ Hz. Black line: direct measurement of $\zeta$; grey line: hydrostatic reconstruction $\zeta_H$, equation (C 1); cyan line: $\zeta_L$, equation (C 10); red line: $\zeta_{NL}$, equation (C 4).

# Wave energy : how much ?

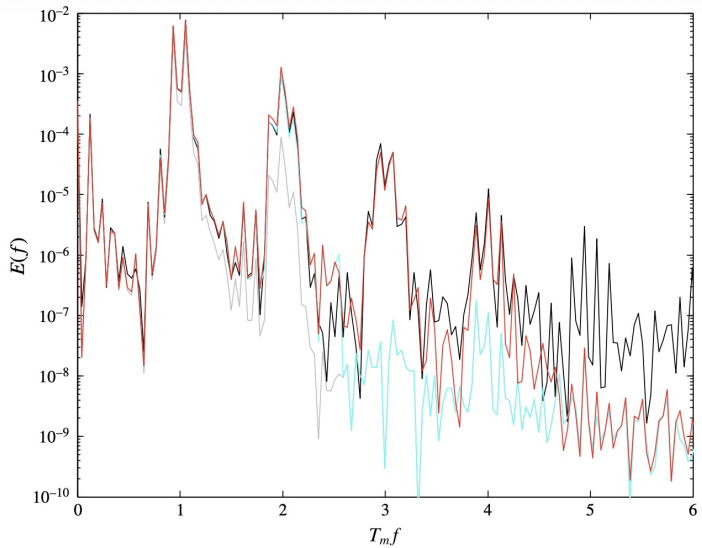
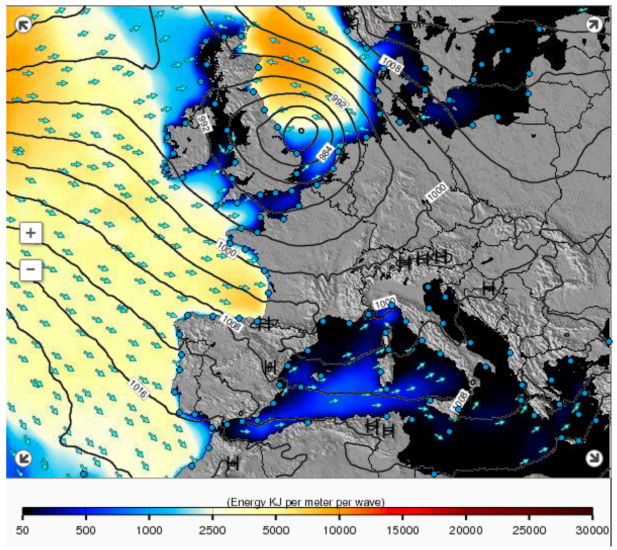



FIGURE 9. (Colour online) Surface elevation energy density spectra, $E(f)$, as a function of the dimensionless frequency $T_m f$, for bichromatic waves, $f_1 = 0.5515$ Hz, $f_2 = 0.6250$ Hz ($T_m = ((f_1 + f_2)/2)^{-1}$), $h_0 = 0.326$ m and $\delta_m = 0.5$ cm. Cutoff frequency $f_c = 1.5$ Hz. Black line: direct measurement of $\zeta$; grey line: hydrostatic reconstruction $\zeta_H$, equation (C 1); cyan line: $\zeta_L$, equation (C 10); red line: $\zeta_{NL}$, equation (C 4).

The annual average of wave energy in France, kilowatt/meter.

**12.5 KW/m**



# Shallow water approximation

Recall the linear approximation:

$$V(t, X, z) = -\frac{ag}{\omega}\mathbf{k}\frac{\cosh(k(z + H_0))}{\cosh(kH_0)}e^{i(-\omega t+\mathbf{k}\cdot X)}$$

# Shallow water approximation

Recall the linear approximation:

$$V(t, X, z) = -\frac{ag}{\omega}\mathbf{k}\frac{\cosh(k(z + H_0))}{\cosh(kH_0)}e^{i(-\omega t+\mathbf{k}\cdot X)}$$

If $kH_0 \ll 1$, that is if $\mu \ll 1$ (shallow water)

$$V(t, X, z) \sim -\frac{ag}{\omega}\mathbf{k}e^{i(-\omega t+\mathbf{k}\cdot X)}$$

# Shallow water approximation

Recall the linear approximation:

$$V(t,X,z) = -\frac{ag}{\omega}\mathbf{k}\frac{\cosh(k(z+H_0))}{\cosh(kH_0)}e^{i(-\omega t+\mathbf{k}\cdot X)}$$

If $kH_0 \ll 1$, that is if $\mu \ll 1$ (shallow water)

$$V(t,X,z) \sim -\frac{ag}{\omega}\mathbf{k}e^{i(-\omega t+\mathbf{k}\cdot X)}$$

Can be justfified at the nonlinear level:

$$V(t,x,z) \sim \overline{V}(t,X) := \frac{1}{H_0+\zeta}\int_{-H_0}^{\zeta(t,X)} V(t,X,z)\mathrm{d}z$$

# Shallow water approximation

Recall the linear approximation:

$$V(t,X,z) = -\frac{ag}{\omega}\mathbf{k}\frac{\cosh(k(z+H_0))}{\cosh(kH_0)}e^{i(-\omega t+\mathbf{k}\cdot X)}$$

If $kH_0 \ll 1$, that is if $\mu \ll 1$ (shallow water)

$$V(t,X,z) \sim -\frac{ag}{\omega}\mathbf{k}e^{i(-\omega t+\mathbf{k}\cdot X)}$$

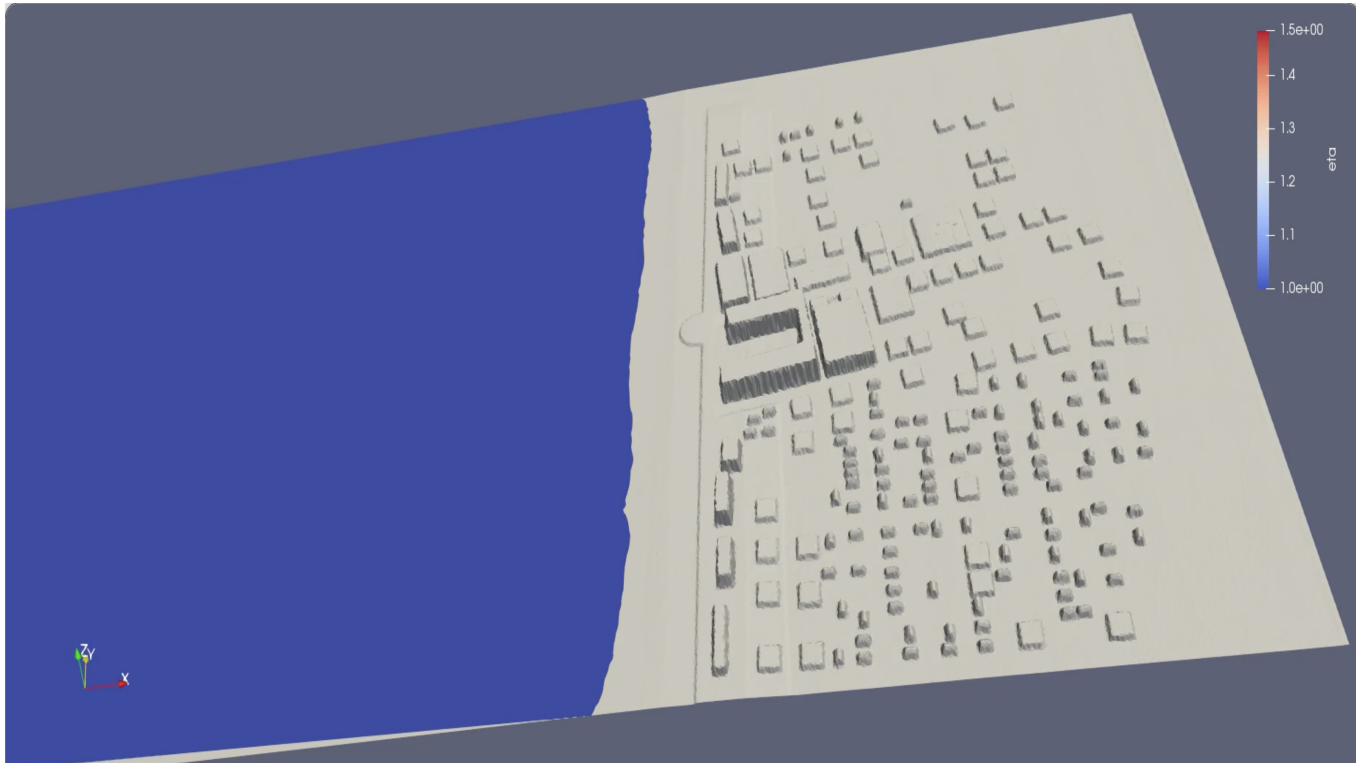
Can be justfified at the nonlinear level:

$$V(t,x,z) \sim \overline{V}(t,X) := \frac{1}{H_0+\zeta}\int_{-H_0}^{\zeta(t,X)} V(t,X,z)\mathrm{d}z$$

and

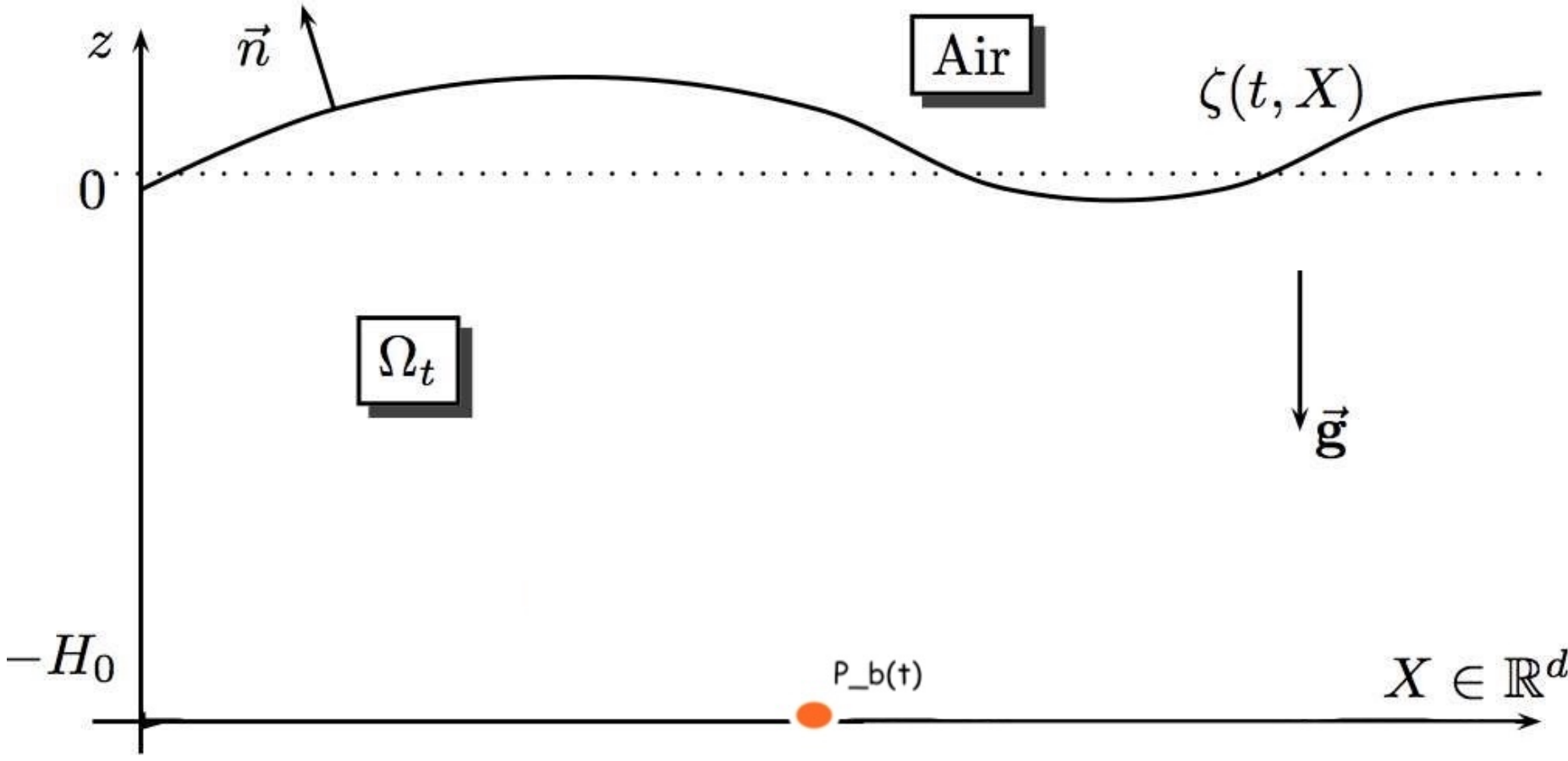
$$\begin{cases} \partial_t\zeta + \nabla\cdot((H_0+\zeta)\overline{V}) = 0, \\ \partial_t\overline{V} + g\nabla\zeta + \overline{V}\cdot\nabla\overline{V} = 0. \end{cases} \quad \text{for } t>0, \quad x\in\mathbb{R}^d$$

# Shallow water models for submersion risks

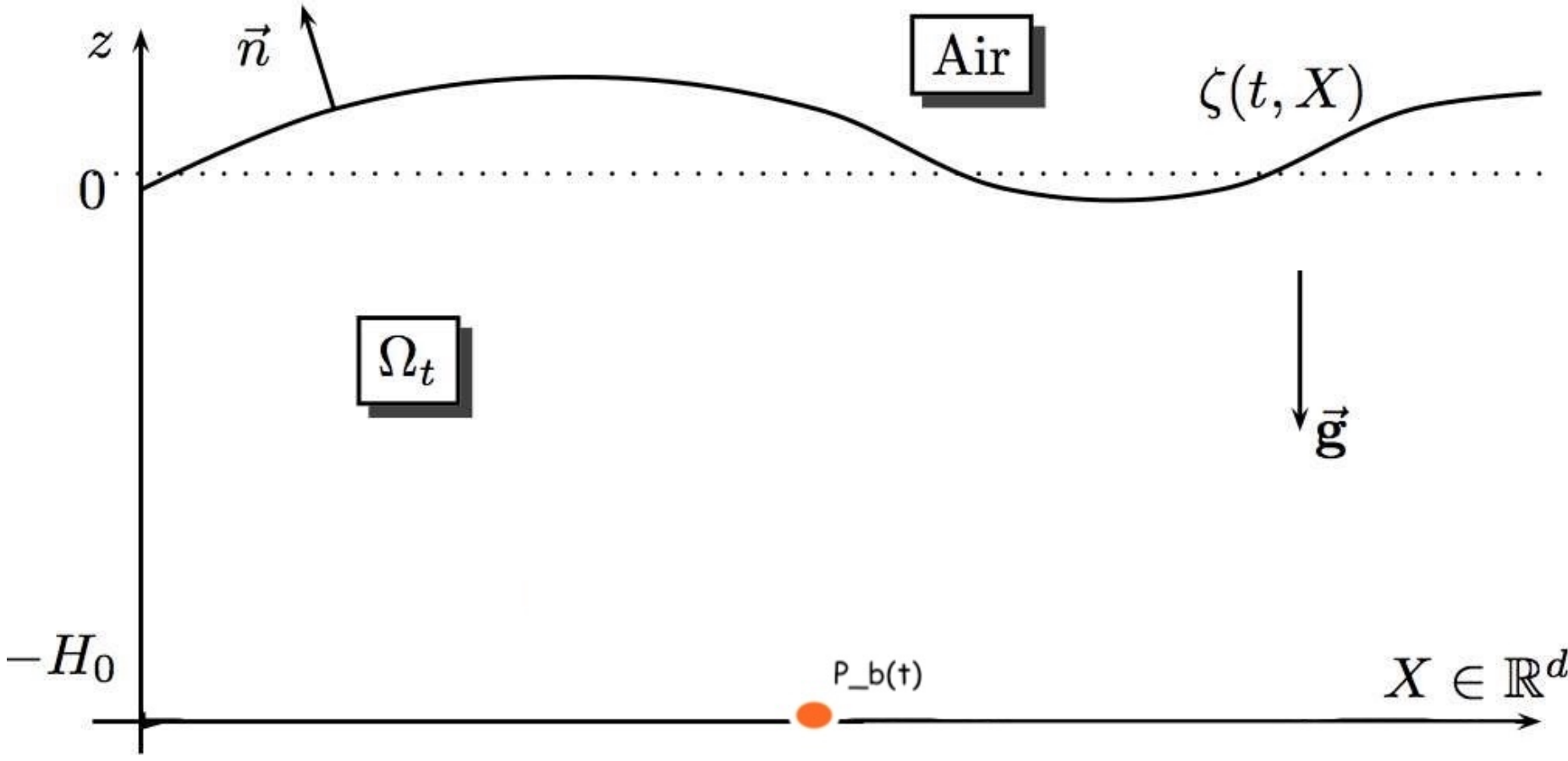



[Uhaina]

# Surface reconstruction



# Surface reconstruction




Reconstruction formula [Oliveras, Vasan, Deconinck, Henderson (2012)]:

> *This formula is obtained somewhat heuristically, and its justification rests on the fact that it agrees extremely well with both numerical and experimental data.*

# Hydrostatic reconstruction

Pressure from vertical component of the Euler equation

$$\int_z^\zeta \left(\partial_t w + \mathbf{U} \cdot \nabla_{x,z} w + g + \frac{1}{\rho}\partial_z P\right) = 0$$

# Hydrostatic reconstruction

Pressure from vertical component of the Euler equation

$$\int_z^\zeta \left(\partial_t w + \mathbf{U} \cdot \nabla_{X,z} w + g + \frac{1}{\rho}\partial_z P\right) = 0$$

so that we can decompose

$$P(t, X, z) = \underbrace{P_{\text{atm}} + \rho g(\zeta(t, X) - z)}_{:=P_{\text{H}}} + P_{\text{NH}}$$

# Hydrostatic reconstruction

Pressure from vertical component of the Euler equation

$$\int_z^\zeta \left(\partial_t w + \mathbf{U} \cdot \nabla_{X,z} w + g + \frac{1}{\rho}\partial_z P\right) = 0$$

so that we can decompose

$$P(t, X, z) = \underbrace{P_{\mathrm{atm}} + \rho g(\zeta(t, X) - z)}_{:=P_{\mathrm{H}}} + P_{\mathrm{NH}}$$

When we can neglect $P_{\mathrm{NH}}$, and if the sensor is located at $X = 0$,

$$P_b(t) \sim P_{\mathrm{atm}} + \rho g(\zeta(t, 0) + H_0)$$

# Hydrostatic reconstruction

Pressure from vertical component of the Euler equation

$$\int_z^\zeta \left(\partial_t w + \mathbf{U}\cdot\nabla_{X,z} w + g + \frac{1}{\rho}\partial_z P\right) = 0$$

so that we can decompose

$$P(t,X,z) = \underbrace{P_{\mathrm{atm}} + \rho g(\zeta(t,X) - z)}_{:=P_{\mathrm{H}}} + P_{\mathrm{NH}}$$

When we can neglect $P_{\mathrm{NH}}$, and if the sensor is located at $X = 0$,

$$P_b(t) \sim P_{\mathrm{atm}} + \rho g(\zeta(t,0) + H_0)$$

The hydrostatic reconstruction of the surface elevation is

$$\zeta_{\mathrm{H}}(t,0) = \frac{1}{\rho g}(P_b(t) - P_{\mathrm{atm}}) - H_0$$

# Other reconstructions

- Hydrostatic reconstruction

$$\zeta_H(t,x) = \frac{1}{\rho g}\left(P_b(t,x) - P_{\mathrm{atm}}\right) - H_0\Big)$$

# Other reconstructions

- Hydrostatic reconstruction

$$\zeta_H(t,x) = \frac{1}{\rho g}(P_b(t,x) - P_{\text{atm}}) - H_0)$$

- Linear reconstruction

$$\zeta_L(t,x) = \cosh(H_0 D)\zeta_H(t,x)$$

# Other reconstructions

- Hydrostatic reconstruction

$$\zeta_H(t,x) = \frac{1}{\rho g}(P_b(t,x) - P_{\text{atm}}) - H_0)$$

- Linear reconstruction

$$\zeta_L(t,x) = \cosh(H_0 D)\zeta_H(t,x)$$

- Heuristic nonlinear reconstruction of Oliveras etal (JFM 2012)

$$\zeta_{HE}(t,x) = \frac{\zeta_L}{1 - D\sinh(H_0 D)\zeta_H}$$

# Other reconstructions

- Hydrostatic reconstruction

$$\zeta_H(t,x) = \frac{1}{\rho g}(P_b(t,x) - P_{\text{atm}}) - H_0)$$

- Linear reconstruction

$$\zeta_L(t,x) = \cosh(H_0 D)\zeta_H(t,x)$$

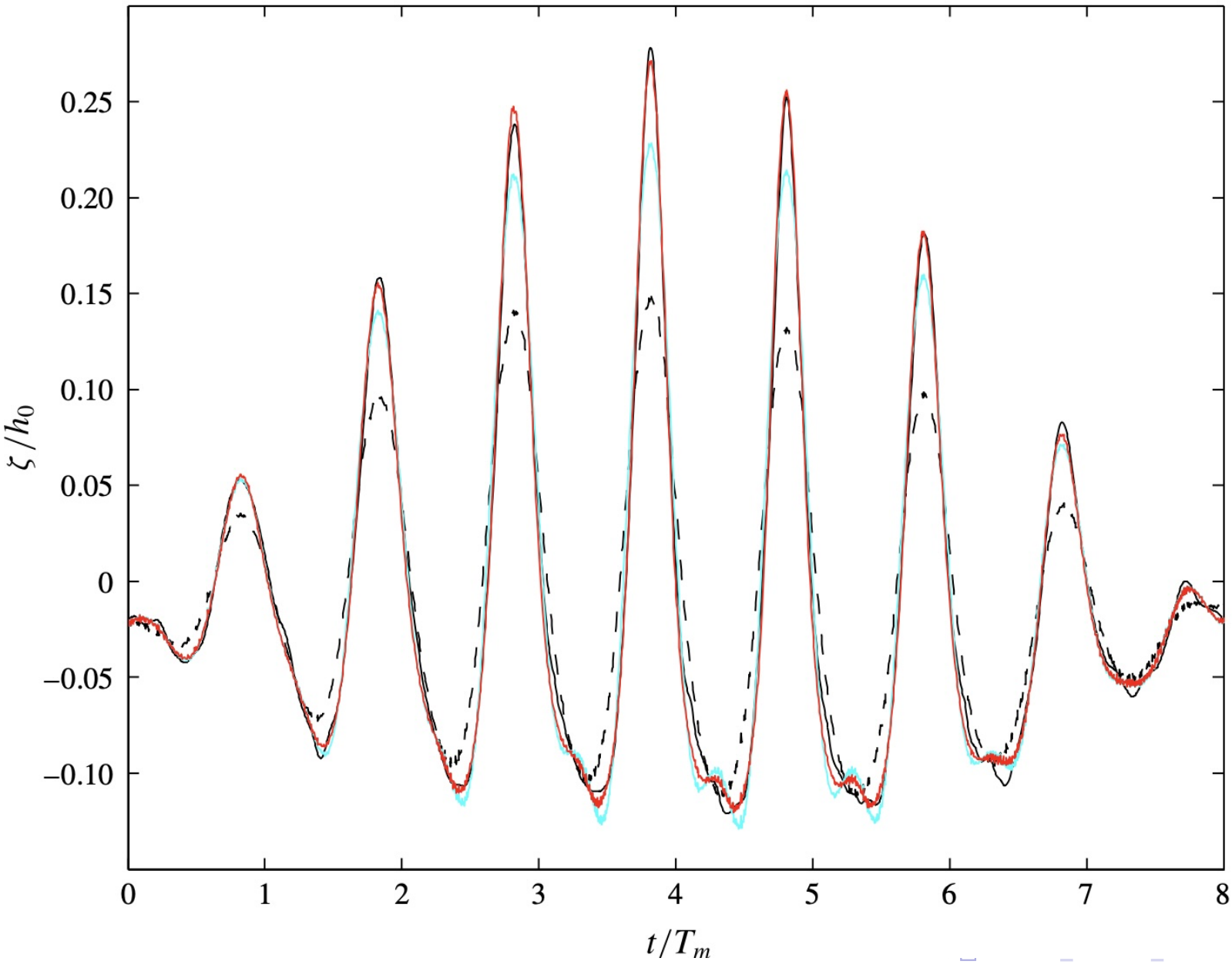
- Heuristic nonlinear reconstruction of Oliveras etal (JFM 2012)

$$\zeta_{HE}(t,x) = \frac{\zeta_L}{1 - D\sinh(H_0 D)\zeta_H}$$

- Asymptotic nonlinear reconstruction of Bonneton and L. (JFM 2017)

$$\zeta_{NL} = \zeta_L - \partial_t(\zeta_L \partial_t \zeta_L)$$

# Example



# Practical applications (1)

Issue: Linear reconstruction

$$\zeta_L(t, x) = \cosh(H_0 D)\zeta_H(t, x)$$

but we only know $\zeta_H(t, x_0)$.

# Practical applications (1)

Issue: Linear reconstruction

$$\zeta_L(t, x) = \cosh(H_0 D)\zeta_H(t, x)$$

but we only know $\zeta_H(t, x_0)$.

- Waves of constant speed $c$ [Clamond, Constantin, Labarbe, Oliveras et al. ...]

$$\zeta_L^{\mathrm{p}}(t, x_0) = [\cosh(H_0 \frac{D_t}{c})\zeta_H(\cdot, x_0)](t)$$

# Practical applications (1)

Issue: Linear reconstruction

$$\zeta_L(t, x) = \cosh(H_0 D)\zeta_H(t, x)$$

but we only know $\zeta_H(t, x_0)$.

- Waves of constant speed $c$ [Clamond, Constantin, Labarbe, Oliveras et al. ...]

$$\zeta_L^{\mathrm{p}}(t, x_0) = [\cosh(H_0 \frac{D_t}{c})\zeta_H(\cdot, x_0)](t)$$

- Irregular linear waves

$$\zeta(t, x) = \int_{\mathbb{R}} e^{ikx-\omega(k)t} A_+(k)\mathrm{d}k + \int_{\mathbb{R}} e^{ikx+\omega(k)t} A_-(k)\mathrm{d}k,$$

with $\omega(k)^2 = gk \tanh(H_0 k)$.

# Practical applications (1)

Issue: Linear reconstruction

$$\zeta_L(t,x) = \cosh(H_0 D)\zeta_H(t,x)$$

but we only know $\zeta_H(t,x_0)$.

- Waves of constant speed $c$ [Clamond, Constantin, Labarbe, Oliveras et al. ...]

$$\zeta_L^{\mathrm{p}}(t,x_0) = [\cosh(H_0 \frac{D_t}{c})\zeta_H(\cdot,x_0)](t)$$

- Irregular linear waves

$$\zeta(t,x) = \int_{\mathbb{R}} e^{ikx-\omega(k)t} A_+(k)\mathrm{d}k + \int_{\mathbb{R}} e^{ikx+\omega(k)t} A_-(k)\mathrm{d}k,$$

with $\omega(k)^2 = gk\tanh(H_0 k)$.

Transfer function $\quad \zeta_L^{\mathrm{p}}(t,x_0) = [\cosh(H_0 k(D_t))\zeta_H(\cdot,x_0)](t)$

# Practical applications (2)

Transfer function $\zeta_L^{\mathrm{p}}(t, x_0) = [\cosh(H_0 k(D_t))\zeta_H(\cdot, x_0)](t)$

# Practical applications (2)

Transfer function $\zeta_L^{\mathrm{p}}(t, x_0) = [\cosh(H_0 k(D_t))\zeta_H(\cdot, x_0)](t)$

$\rightsquigarrow$ Needs a cutoff frequency

# Practical applications (2)

Transfer function $\zeta_L^{\mathrm{p}}(t, x_0) = [\cosh(H_0 k(D_t))\zeta_H(\cdot, x_0)](t)$

$\rightsquigarrow$ Needs a cutoff frequency

$$\widetilde{\zeta}_L^{\mathrm{p}}(\omega, 0) = \begin{cases} [\cosh(H_0 k(\omega))\widetilde{\zeta}_H(\omega, x_0)] & \text{if } \frac{\omega}{2\pi} < f_c, \\ \widetilde{\zeta}_H(\omega, x_0)] & \text{if } \frac{\omega}{2\pi} \geq f_c \end{cases}$$

# Practical applications (2)

Transfer function $\zeta_L^{\mathrm{p}}(t, x_0) = [\cosh(H_0 k(D_t))\zeta_H(\cdot, x_0)](t)$

$\rightsquigarrow$ Needs a cutoff frequency

$$\widetilde{\zeta}_L^{\mathrm{p}}(\omega, 0) = \begin{cases} [\cosh(H_0 k(\omega))\widetilde{\zeta}_H(\omega, x_0)] & \text{if } \frac{\omega}{2\pi} < f_c, \\ \widetilde{\zeta}_H(\omega, x_0)] & \text{if } \frac{\omega}{2\pi} \geq f_c \end{cases}$$

**Question.** Due to noise?

# Practical applications (2)

Transfer function $\zeta_L^{\mathrm{p}}(t, x_0) = [\cosh(H_0 k(D_t))\zeta_H(\cdot, x_0)](t)$

$\rightsquigarrow$ Needs a cutoff frequency

$$\widetilde{\zeta}_L^{\mathrm{p}}(\omega, 0) = \begin{cases} [\cosh(H_0 k(\omega))\widetilde{\zeta}_H(\omega, x_0)] & \text{if } \frac{\omega}{2\pi} < f_c, \\ \widetilde{\zeta}_H(\omega, x_0)] & \text{if } \frac{\omega}{2\pi} \geq f_c \end{cases}$$

**Question.** Due to noise? No!

# Practical applications (2)

Transfer function $\zeta_L^{\mathrm{p}}(t, x_0) = [\cosh(H_0 k(D_t))\zeta_H(\cdot, x_0)](t)$

$\rightsquigarrow$ Needs a cutoff frequency

$$\widetilde{\zeta}_L^{\mathrm{p}}(\omega, 0) = \begin{cases} [\cosh(H_0 k(\omega))\widetilde{\zeta}_H(\omega, x_0)] & \text{if } \frac{\omega}{2\pi} < f_c, \\ \widetilde{\zeta}_H(\omega, x_0)] & \text{if } \frac{\omega}{2\pi} \geq f_c \end{cases}$$

**Question.** Due to noise? No!

Asymptotic nonlinear reconstruction

$$\zeta_{NL}^{\mathrm{p}} = \zeta_L^{\mathrm{p}} - \partial_t(\zeta_L^{\mathrm{p}} \partial_t \zeta_L^{\mathrm{p}})$$

# Example (P. Bonneton, D. L. (JMF 2017))

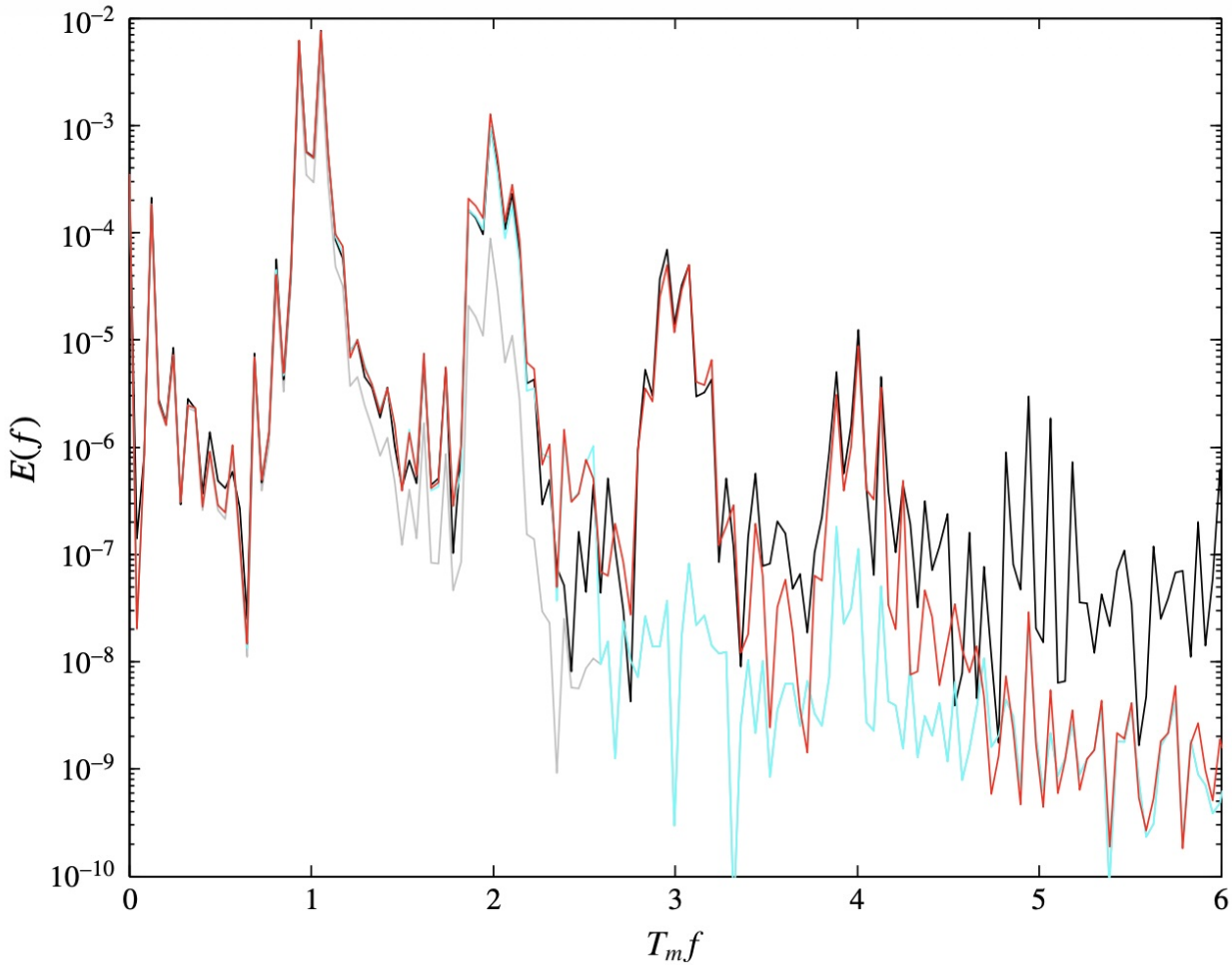



FIGURE 9. (Colour online) Surface elevation energy density spectra, $E(f)$, as a function of the dimensionless frequency $T_m f$, for bichromatic waves, $f_1 = 0.5515$ Hz, $f_2 = 0.6250$ Hz ($T_m = ((f_1 + f_2)/2)^{-1}$), $h_0 = 0.326$ m and $\delta_m = 0.5$ cm. Cutoff frequency $f_c = 1.5$ Hz. Black line: direct measurement of $\zeta$; grey line: hydrostatic reconstruction $\zeta_H$, equation (C 1); cyan line: $\zeta_L$, equation (C 10); red line: $\zeta_{NL}$, equation (C 4).

# Extreme waves (Mouragues et al, Coastal Engineering 2019)

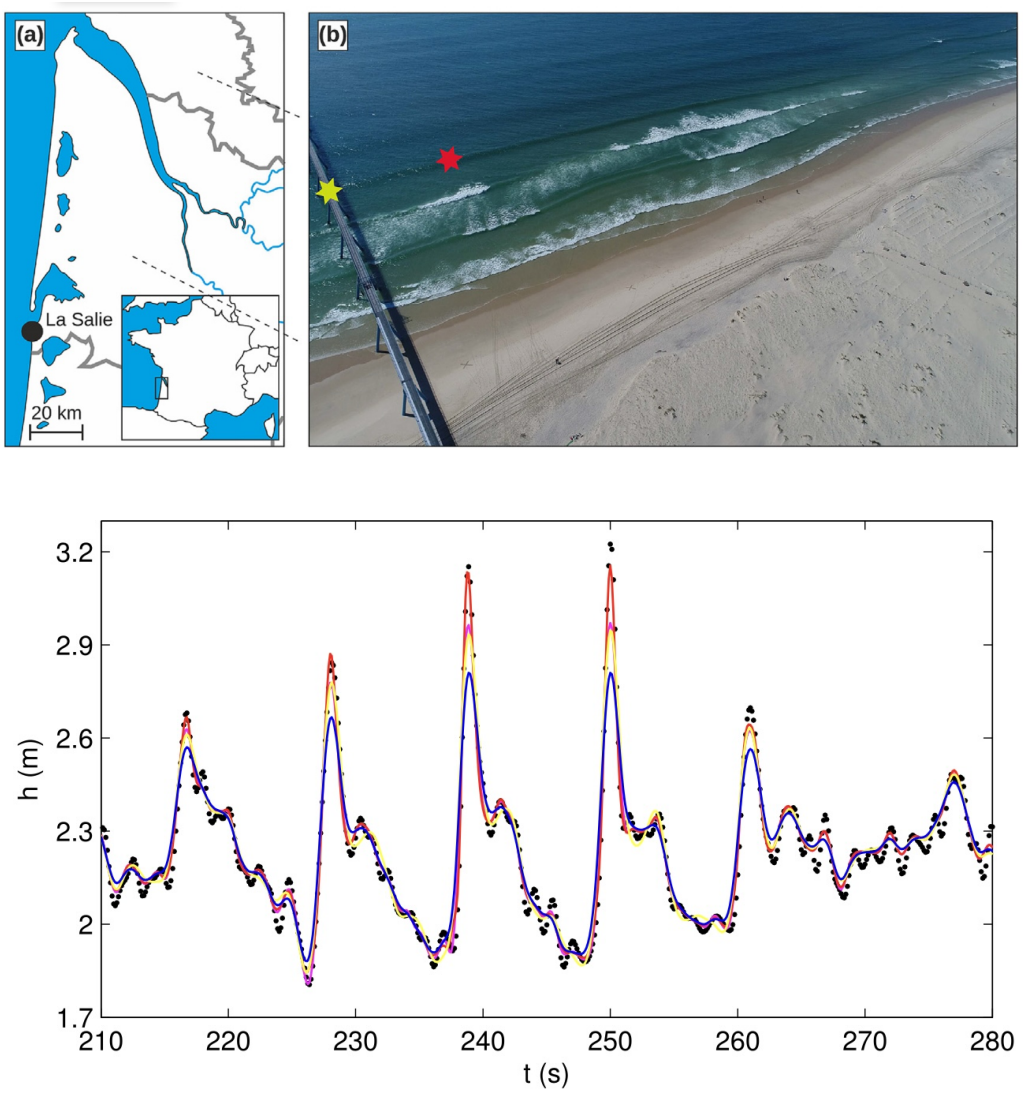



**Fig. 12.** Water depth time series of a group of waves. AST measurements: $\zeta_m$ (black points); hydrostatic reconstruction: $\zeta_H^{\delta_m}$ (blue line); weakly-dispersive linear reconstruction: $\zeta_{SL}$ (magenta line); weakly-dispersive nonlinear reconstruction: $\zeta_{SNL}$ (red line); heuristic reconstruction: $\zeta_{HE}$ (yellow line); see the associated equations in Table 2. $f_c = 0.32$ Hz, $f_{c,noise} = 1$ Hz and $h_0 = 2.25$ m. (For interpretation of the references to color in this figure legend, the reader is

# Two mathematical issues

Remark 1: the transition

$$\zeta_{NL} = \zeta_L - \partial_t(\zeta_L \partial_t \zeta_L) \quad \rightsquigarrow \quad \zeta^{\mathrm{p}}_{NL} = \zeta^{\mathrm{p}}_L - \partial_t(\zeta^{\mathrm{p}}_L \partial_t \zeta^{\mathrm{p}}_L)$$

# Two mathematical issues

Remark 1: the transition

$$\zeta_{NL} = \zeta_L - \partial_t(\zeta_L \partial_t \zeta_L) \quad \rightsquigarrow \quad \zeta^{\mathrm{p}}_{NL} = \zeta^{\mathrm{p}}_L - \partial_t(\zeta^{\mathrm{p}}_L \partial_t \zeta^{\mathrm{p}}_L)$$

is (formally) asymptotically FALSE

# Two mathematical issues

Remark 1: the transition

$$\zeta_{NL} = \zeta_L - \partial_t(\zeta_L \partial_t \zeta_L) \quad \rightsquigarrow \quad \zeta^{\mathrm{p}}_{NL} = \zeta^{\mathrm{p}}_L - \partial_t(\zeta^{\mathrm{p}}_L \partial_t \zeta^{\mathrm{p}}_L)$$

is (formally) asymptotically FALSE
(because everything is nonsense from the beginning)

# Two mathematical issues

Remark 1: the transition

$$\zeta_{NL} = \zeta_L - \partial_t(\zeta_L \partial_t \zeta_L) \quad \rightsquigarrow \quad \zeta^{\mathrm{p}}_{NL} = \zeta^{\mathrm{p}}_L - \partial_t(\zeta^{\mathrm{p}}_L \partial_t \zeta^{\mathrm{p}}_L)$$

is (formally) asymptotically FALSE
(because everything is nonsense from the beginning)

Remark 2: the BIG issue:

$$e^{-2i\omega t} \rightsquigarrow \begin{cases} e^{i(-2\omega t - k(2\omega)x)} & (\text{linear}) \\ e^{i(-2(\omega t - k(\omega)x))} & (\text{nonlinear}) \end{cases}$$

# Two mathematical issues

Remark 1: the transition

$$\zeta_{NL} = \zeta_L - \partial_t(\zeta_L \partial_t \zeta_L) \quad \rightsquigarrow \quad \zeta^{\mathrm{p}}_{NL} = \zeta^{\mathrm{p}}_L - \partial_t(\zeta^{\mathrm{p}}_L \partial_t \zeta^{\mathrm{p}}_L)$$

is (formally) asymptotically FALSE
(because everything is nonsense from the beginning)

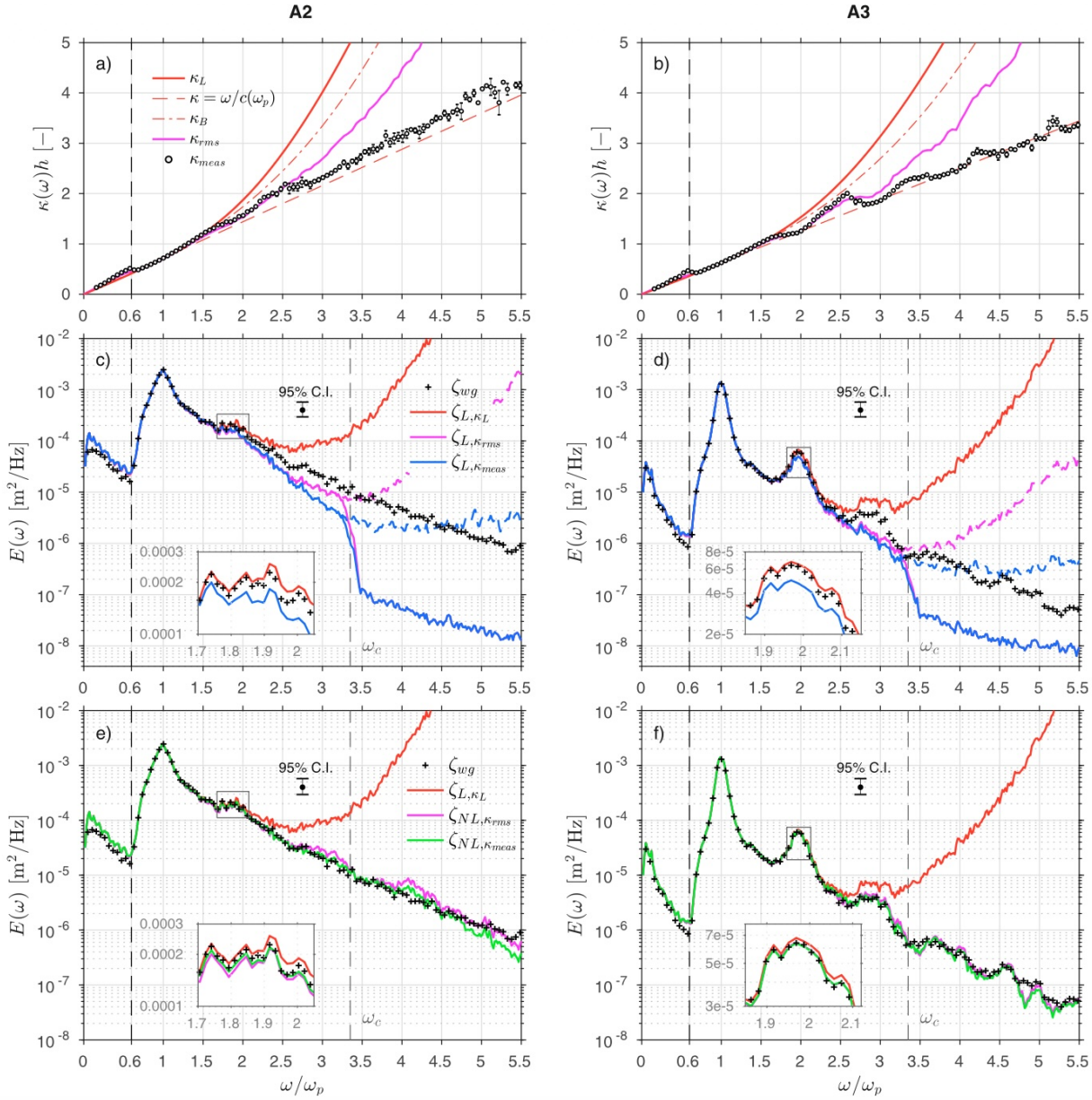
Remark 2: the BIG issue:

$$e^{-2i\omega t} \rightsquigarrow \begin{cases} e^{i(-2\omega t - k(2\omega)x)} & (\mathrm{linear}) \\ e^{i(-2(\omega t - k(\omega)x))} & (\mathrm{nonlinear}) \end{cases}$$

$\rightsquigarrow$ Statistical approach (following Herbers et al)

# "Statistical" wave number

K. Martins, P. Bonneton, D. L. , H. Michallet, J. Phys Oc 2021)



Merci pour votre attention !