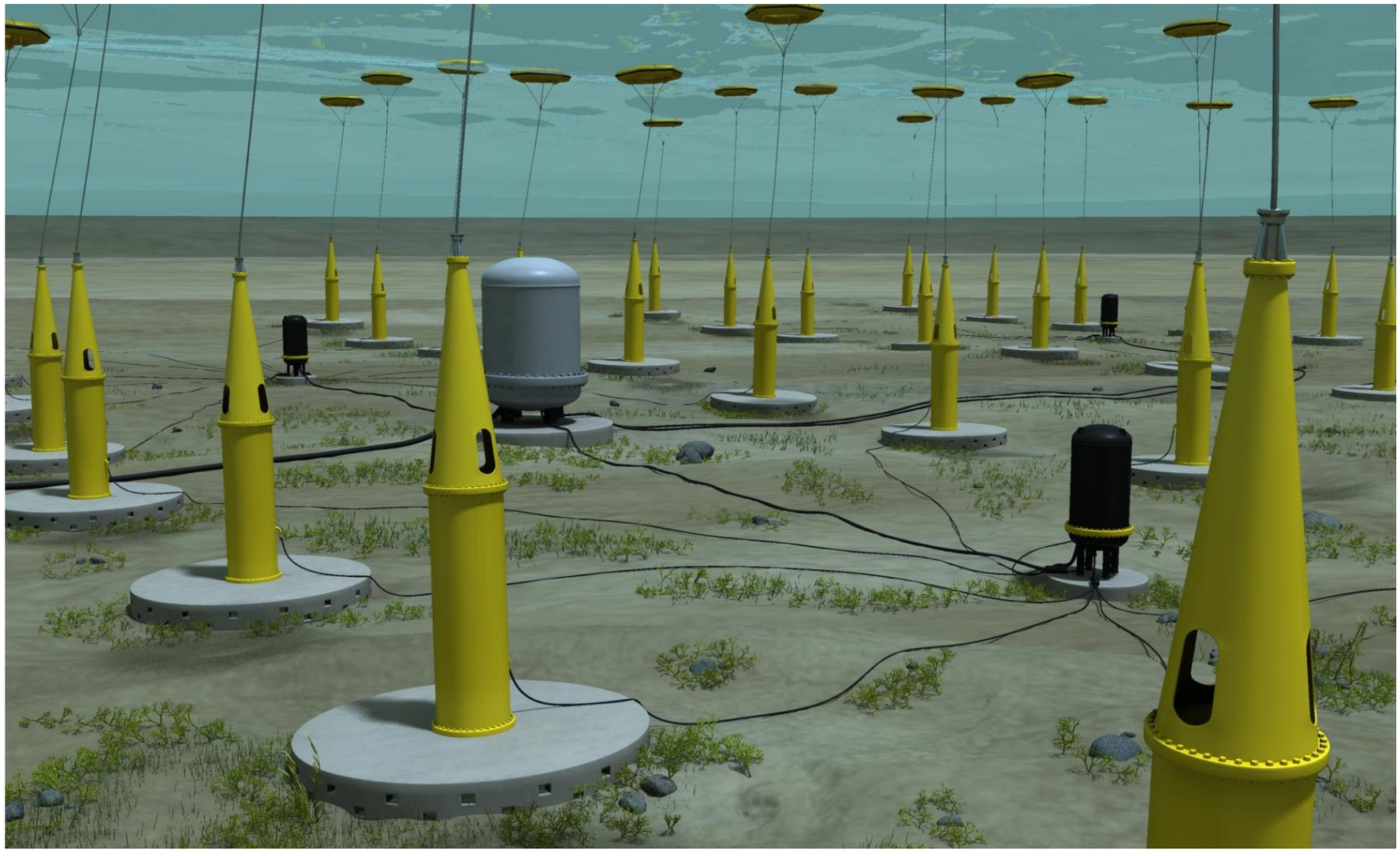
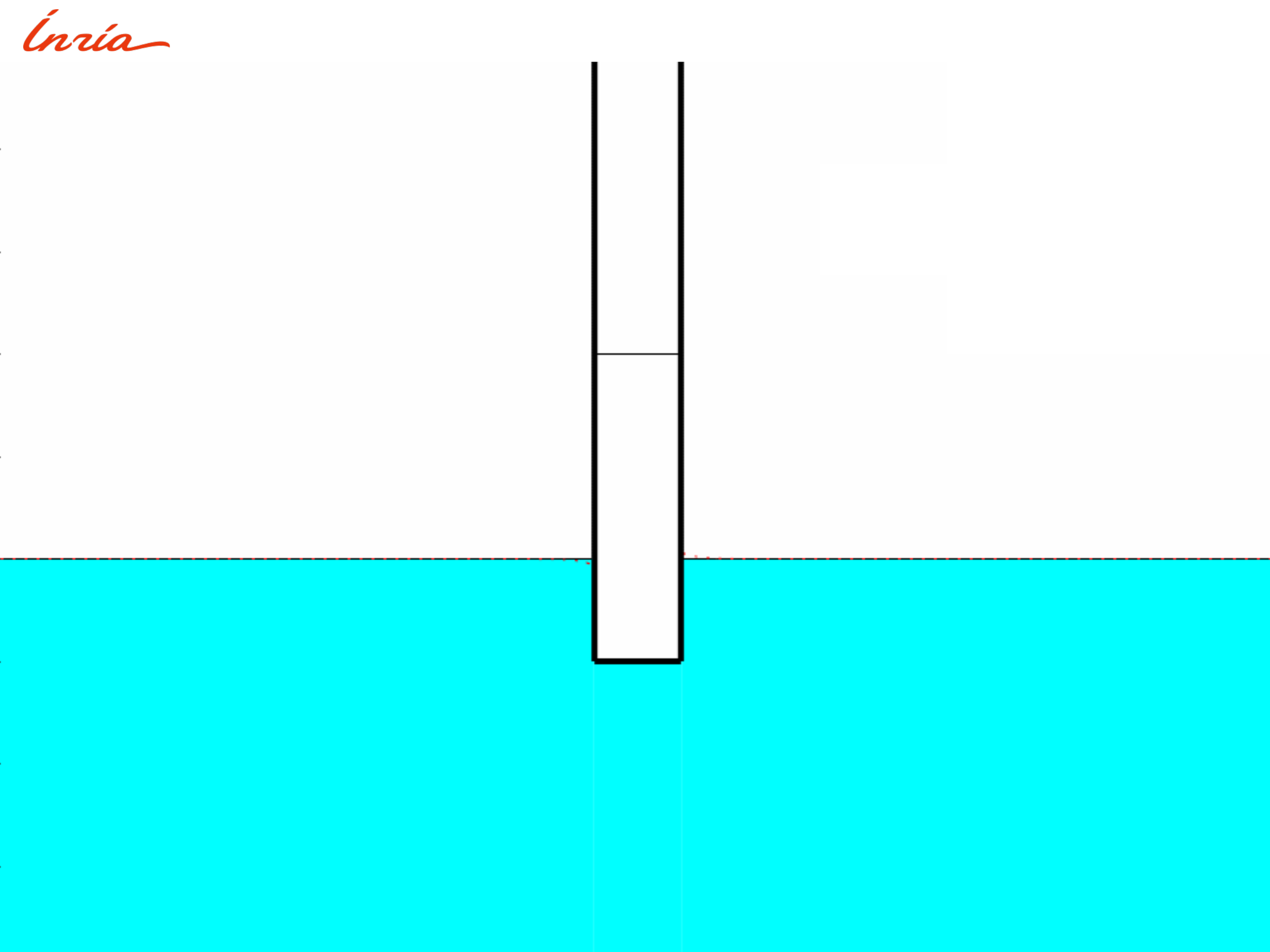


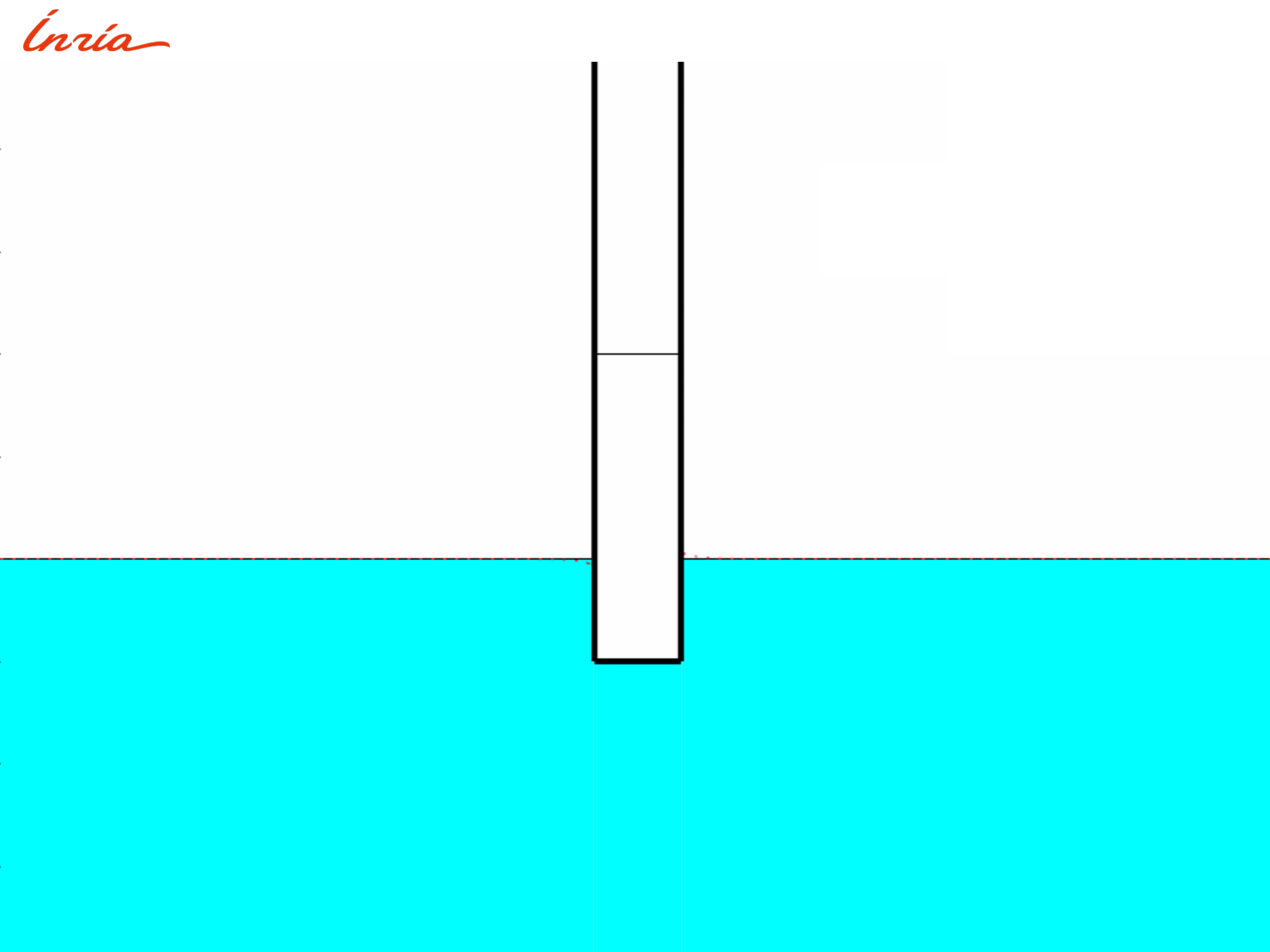
Waves-Floating structures interactions

Applications to wave energy convertors

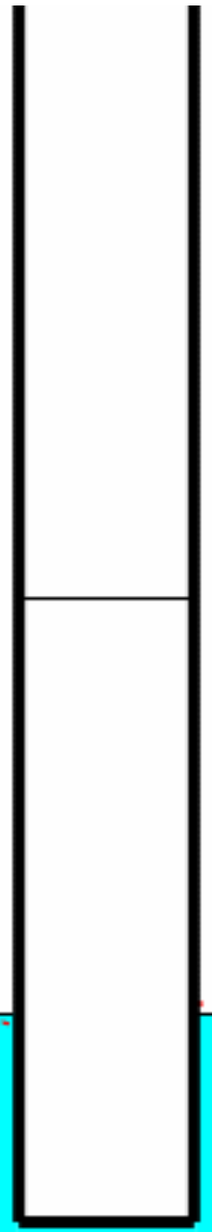
G. Beck, B. Bensiali, **E. Contentin**, N. Crouseilles, R. Grande, D. Lannes, L. Martaud, **A. Riquier**, L. Weynans

C. Berthon, B. Boutin, T. Iguchi, **M. Oen Paulsen**, M. Parisot, C. Perrin





Solid: Newton's equation
with hydrodynamic force
due to pressure



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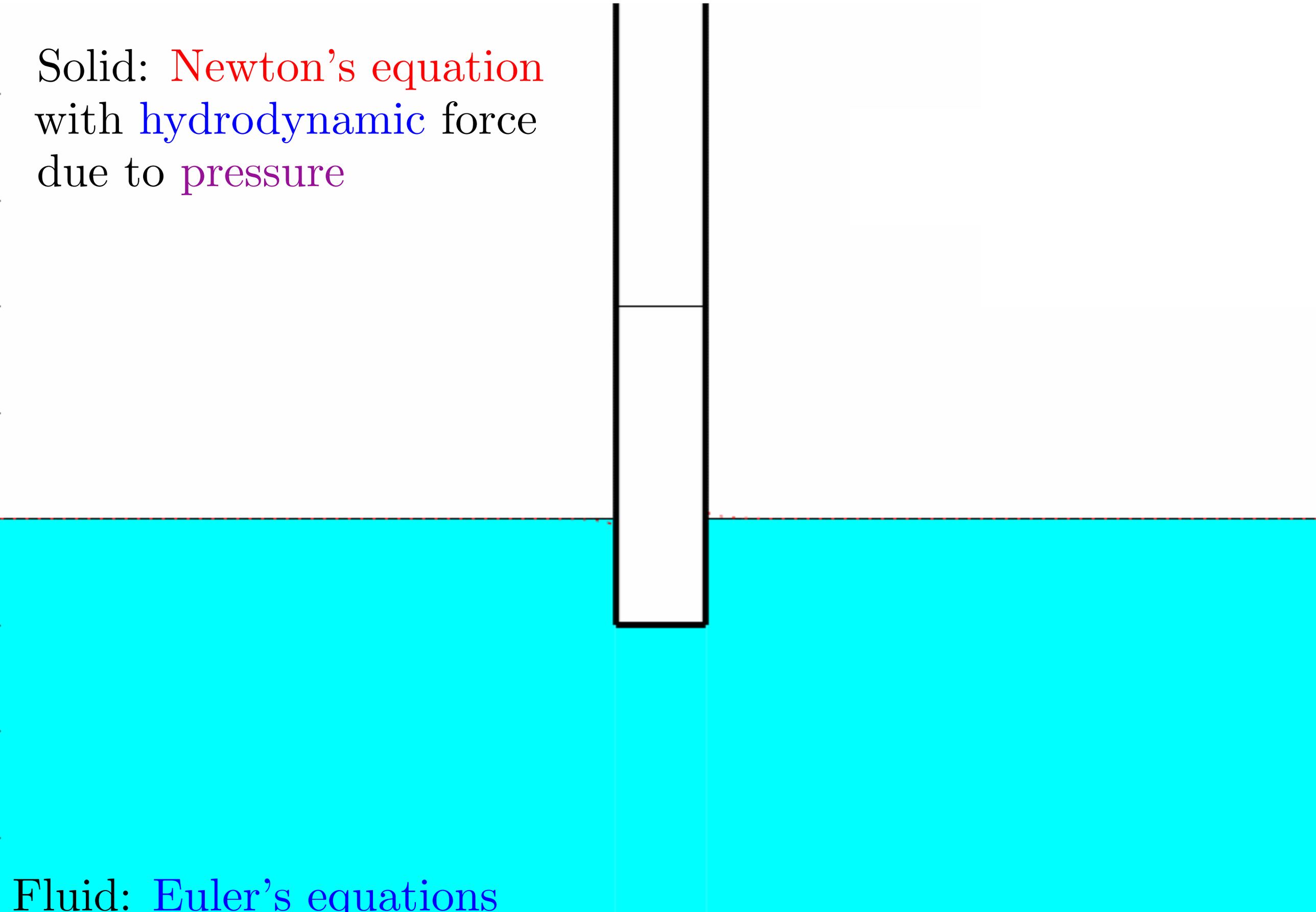

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- water surface
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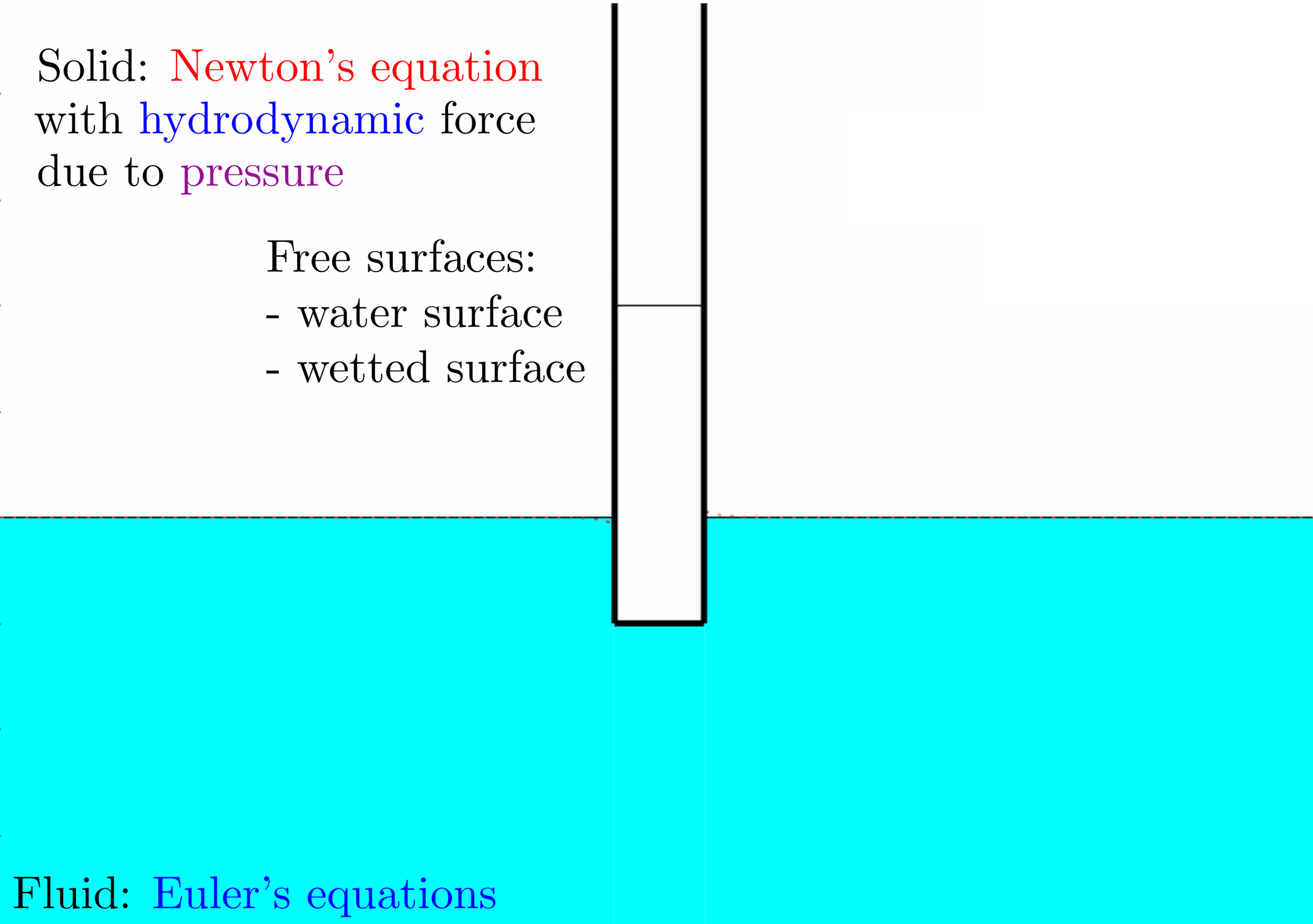

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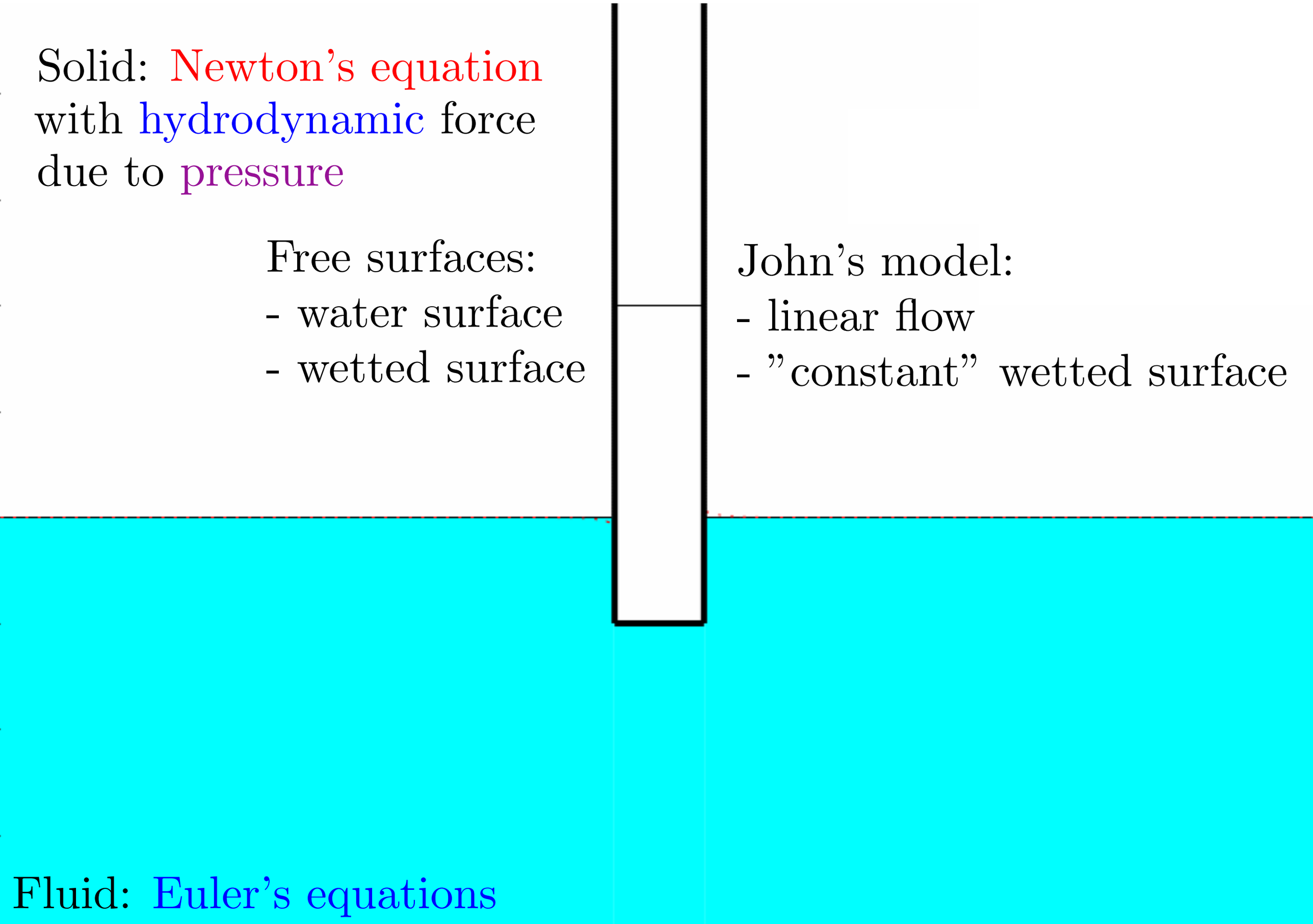
John's model:
- linear flow
- "constant" wetted surface



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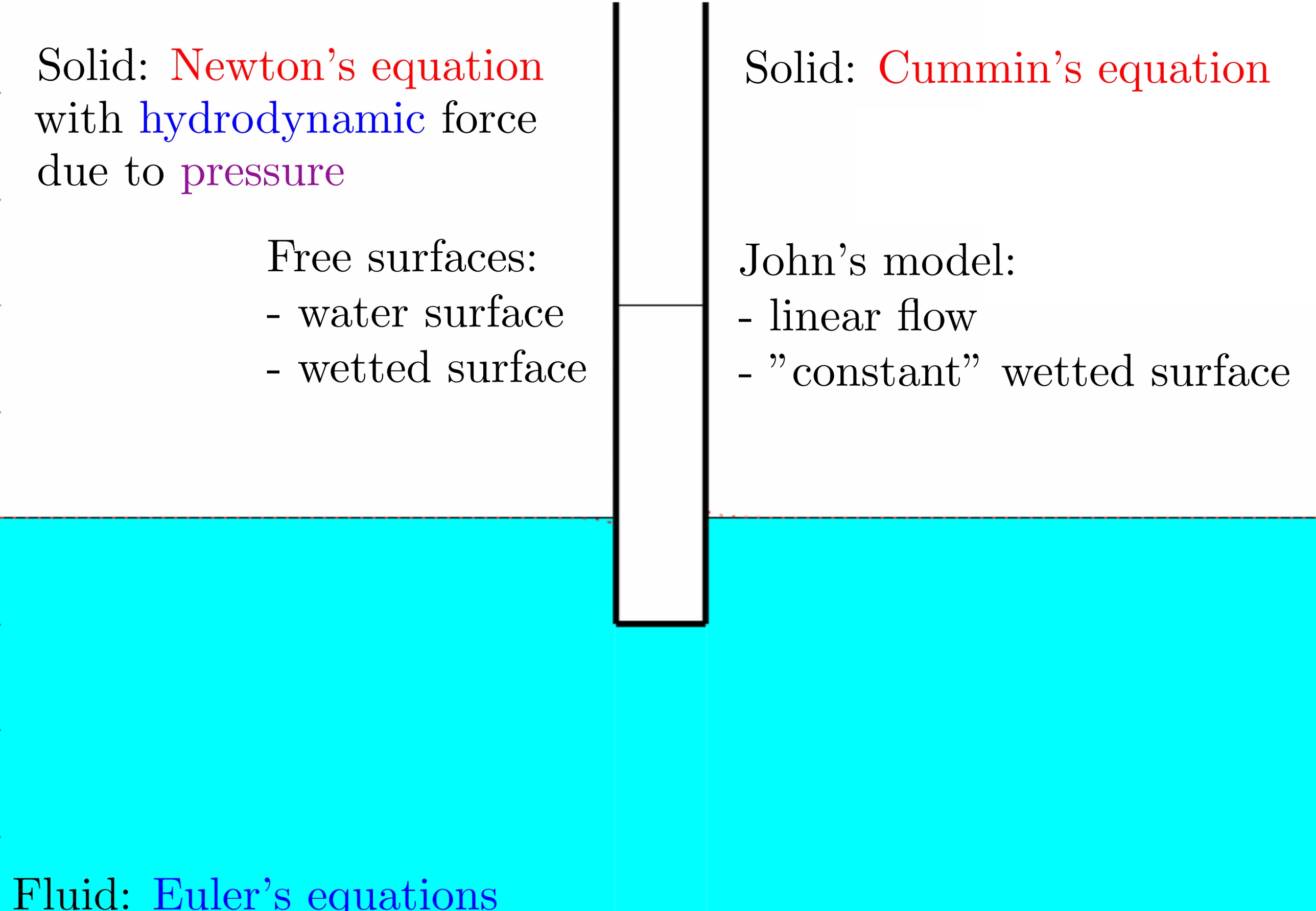

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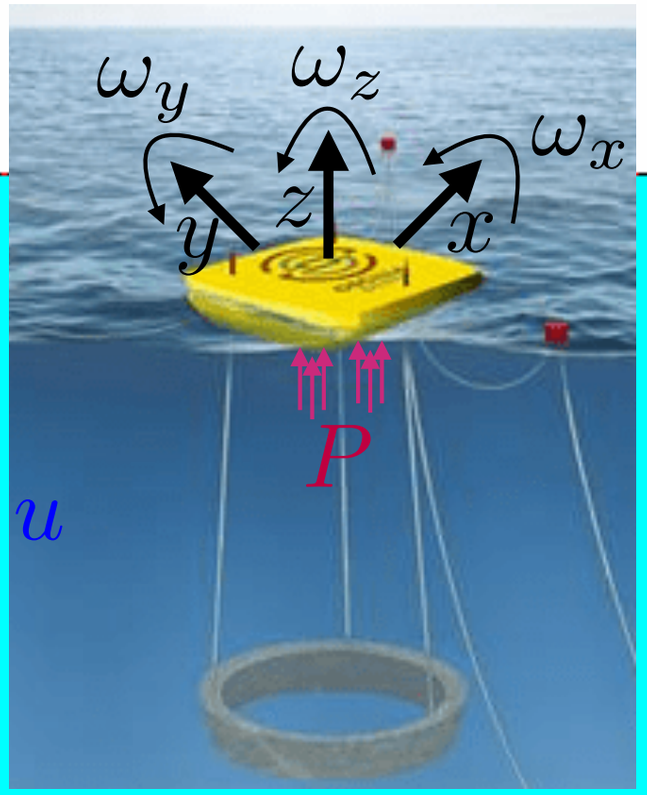

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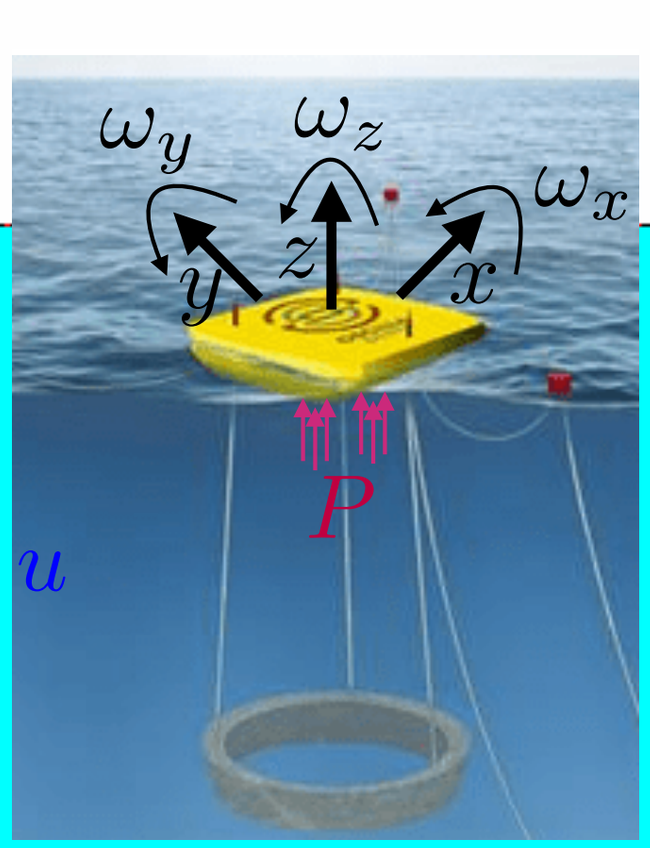

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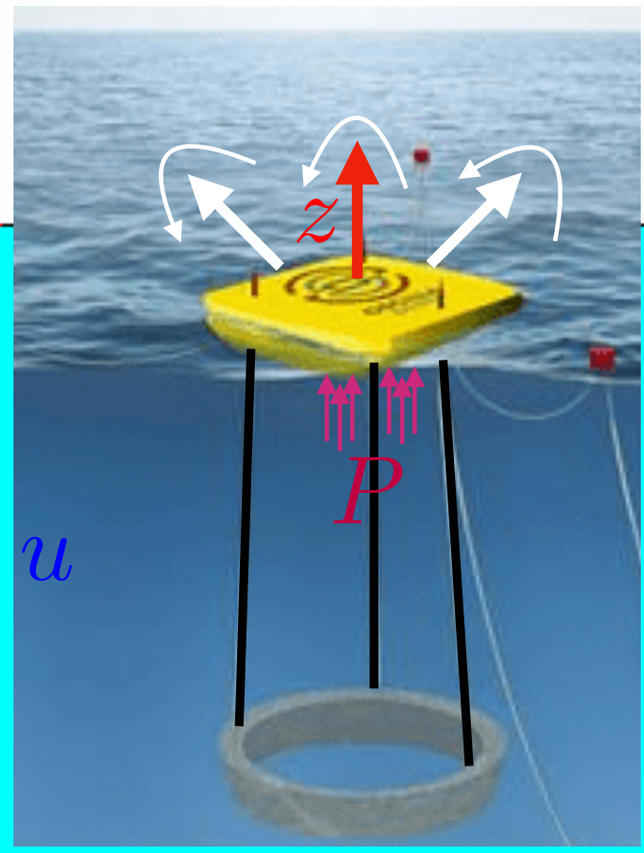

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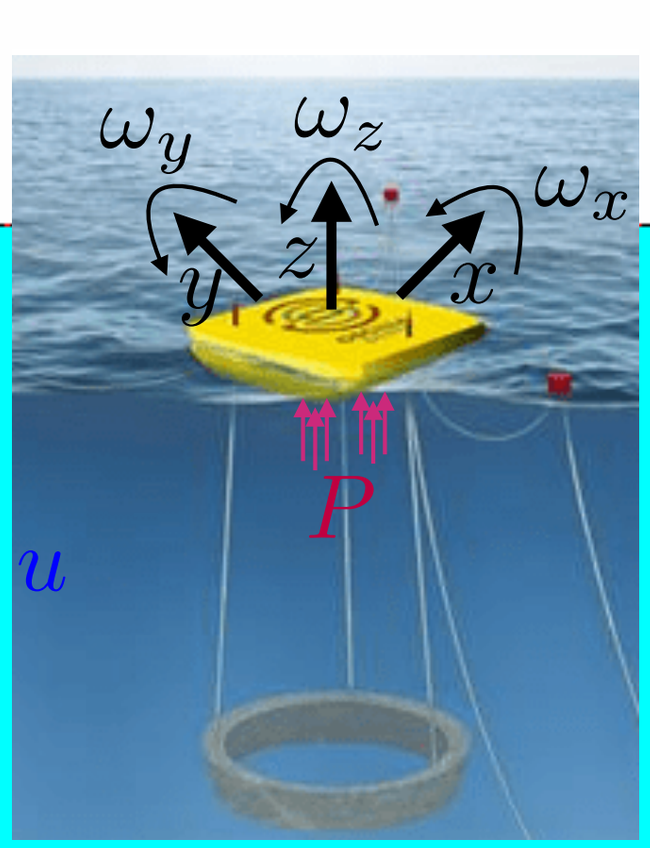

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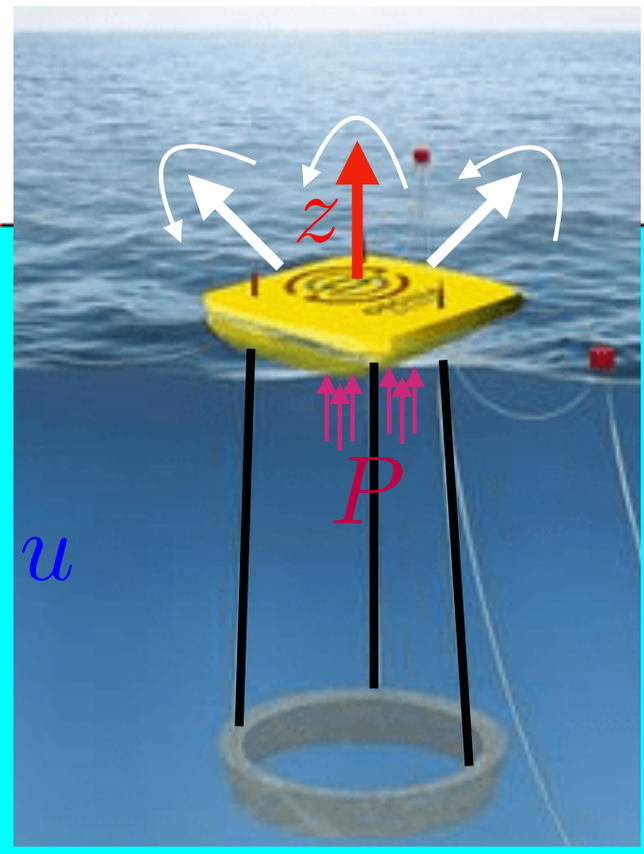

2D fluid $\Rightarrow$ 1D waves

Solid: Cummin's equation

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Solid: Cummin's equation

John's model:
- linear flow
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Heave Motion $\delta = z - z_{\text{eq}}$

Solid: Cummin's equation

$$M\ddot{\delta} + a\delta + k *_t \delta = 0$$

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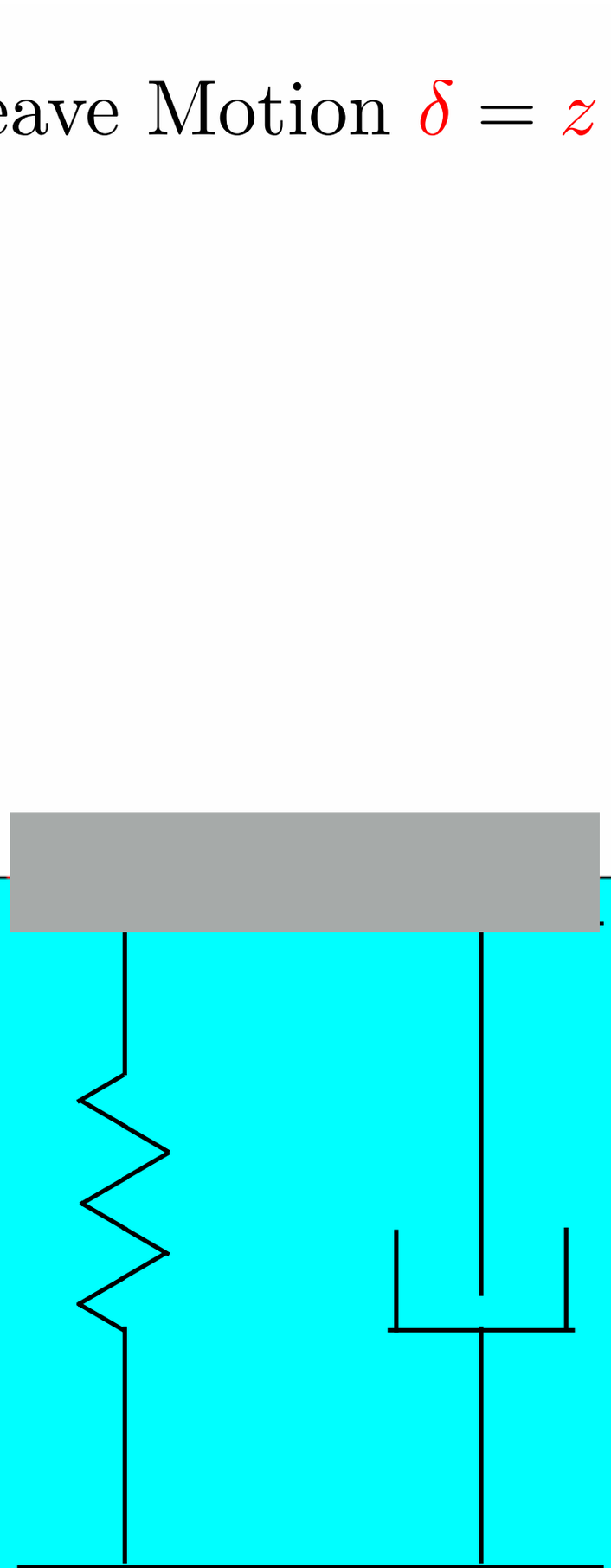

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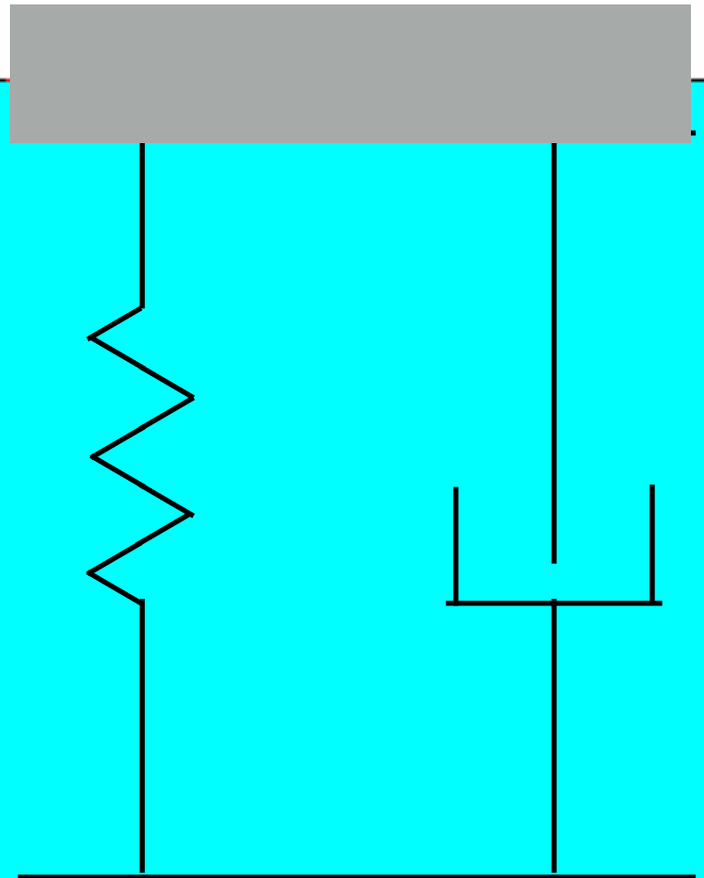

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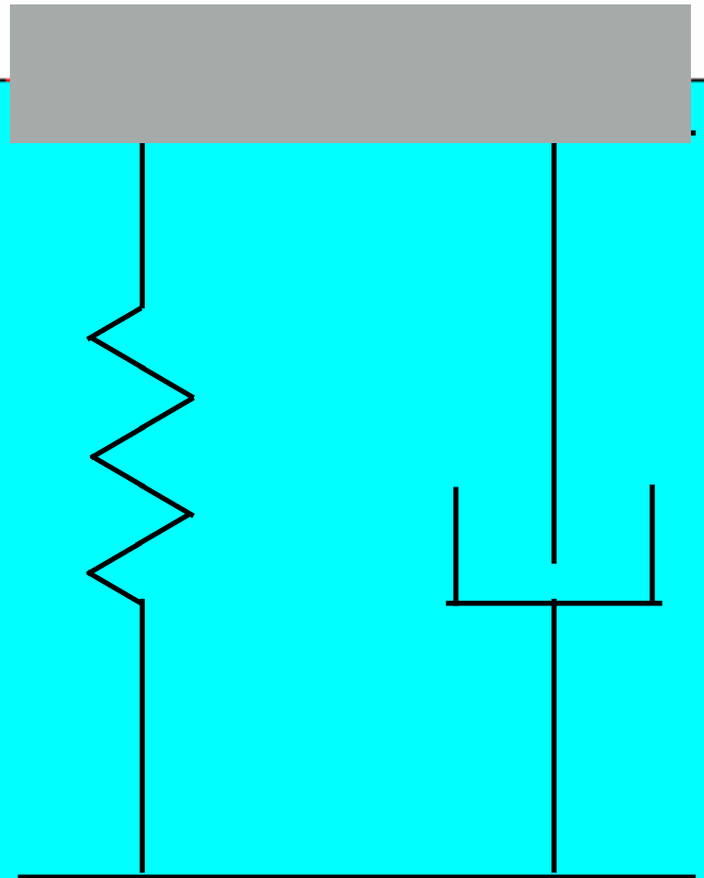

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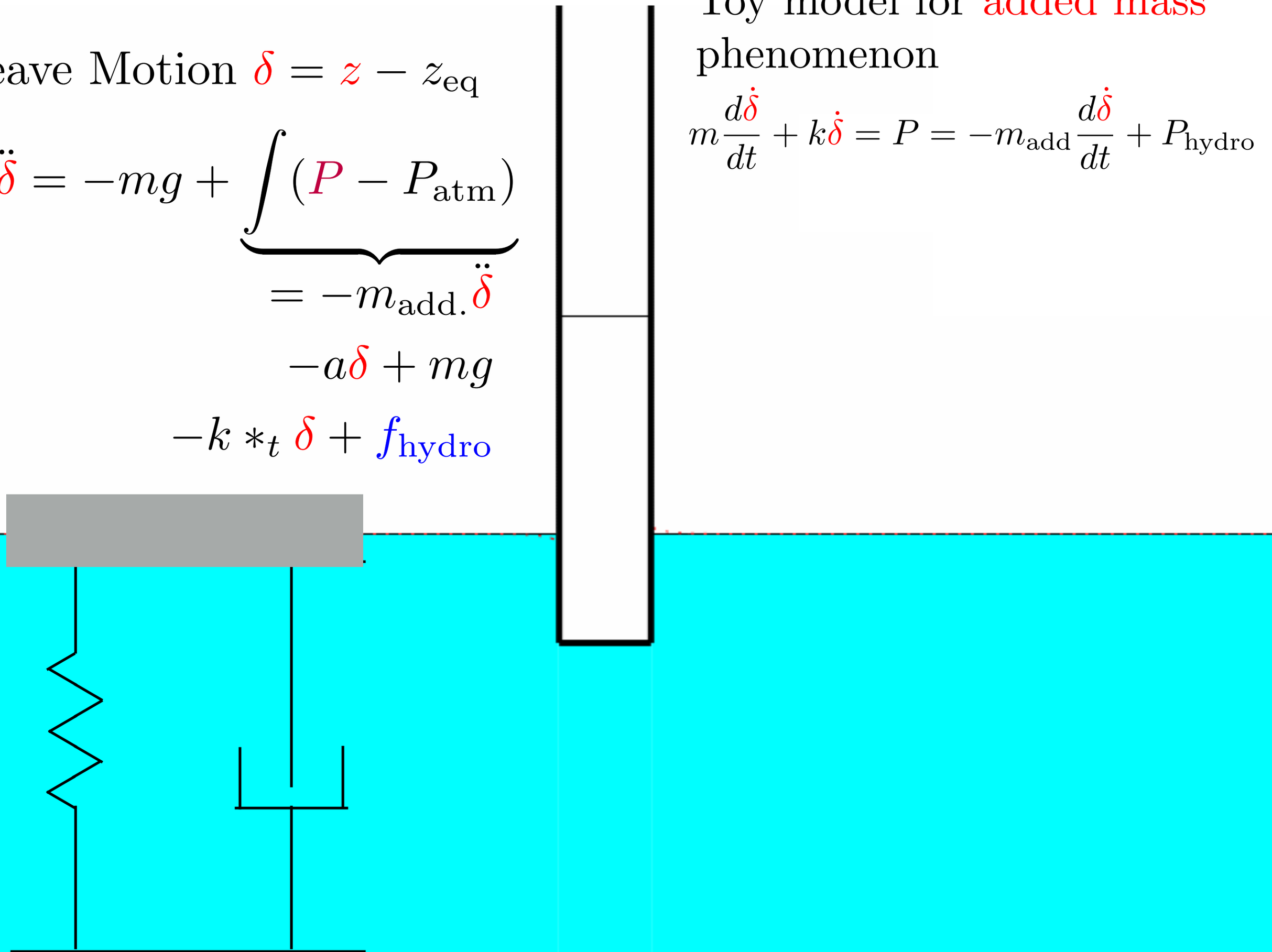
Toy model for added mass phenomenon

$$m\frac{d\dot{\delta}}{dt} + k\dot{\delta} = P = -m_{\text{add}}\frac{d\dot{\delta}}{dt} + P_{\text{hydro}}$$



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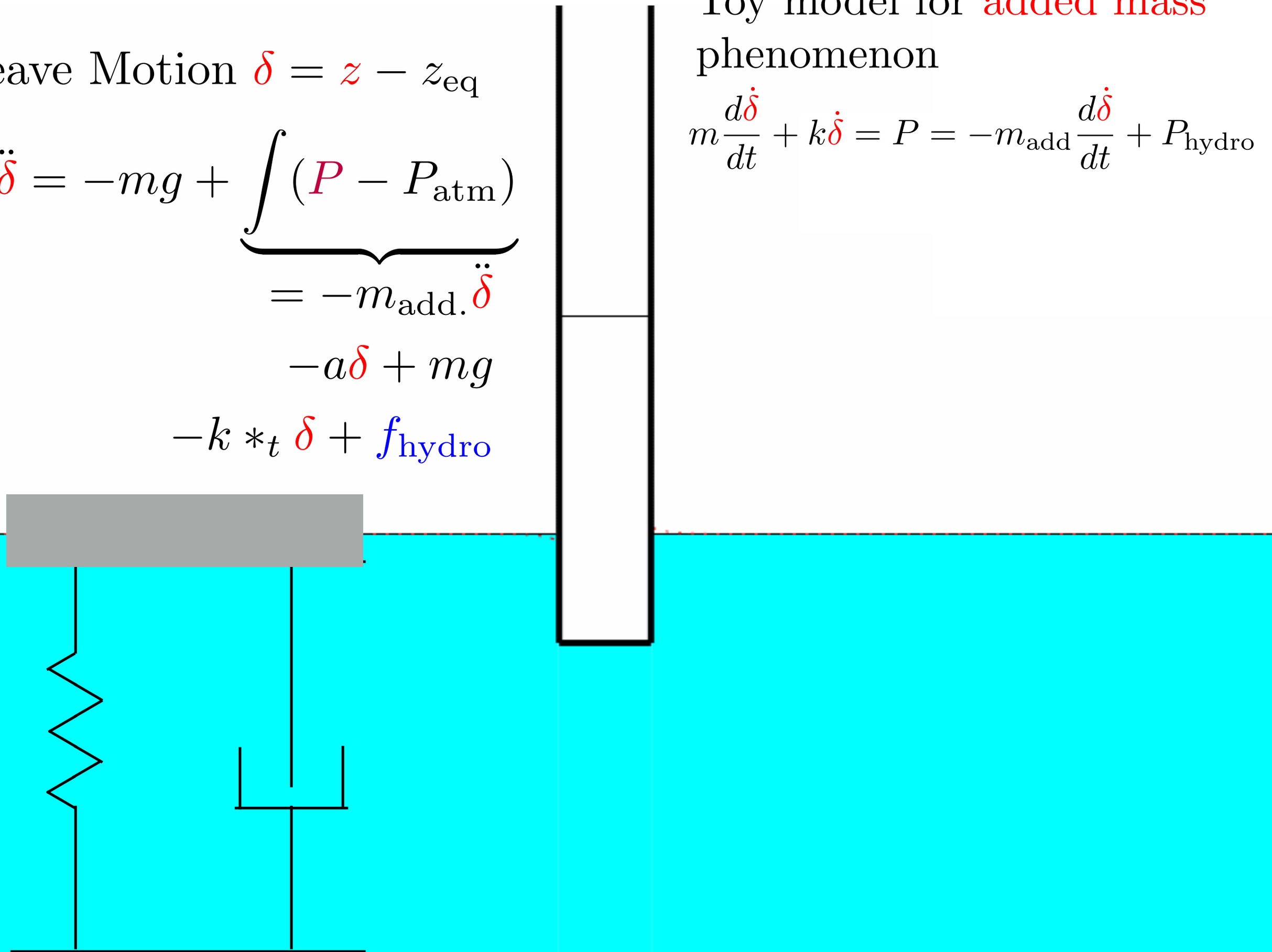

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$$m\frac{\dot{\delta}^{n+1} - \dot{\delta}^{n}}{\Delta t} + k\dot{\delta}^{n} = P^{n}$$

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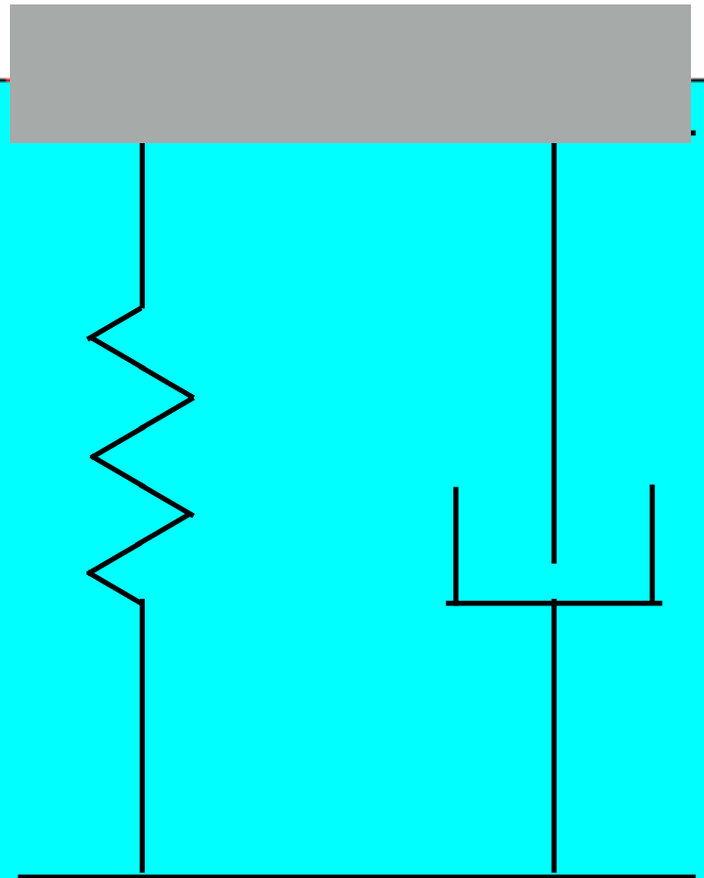

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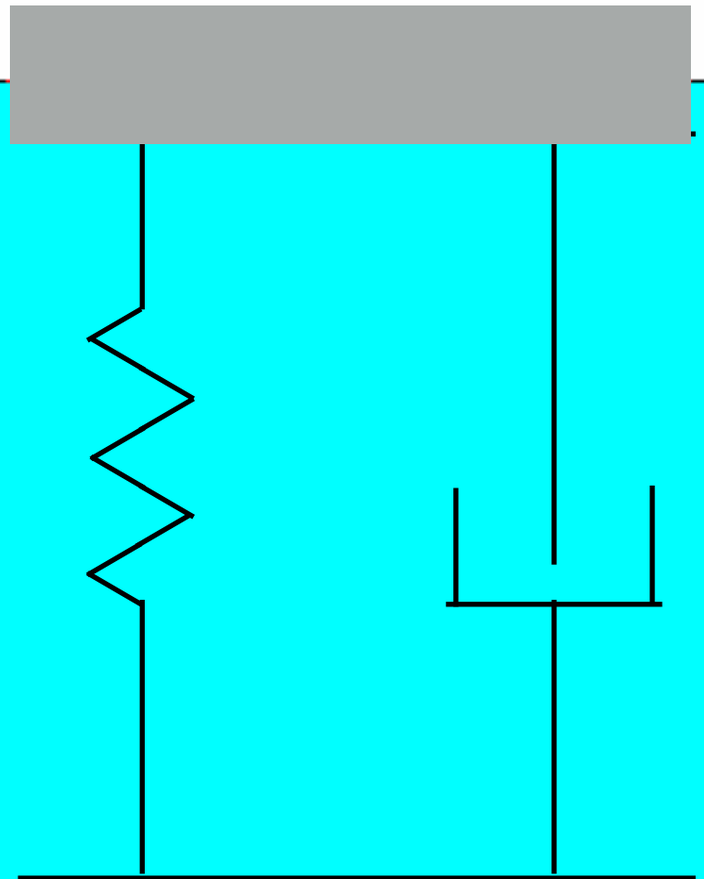

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stability criterium

$$\Delta t \leq \frac{2(m - m_{\text{add}})}{k}$$

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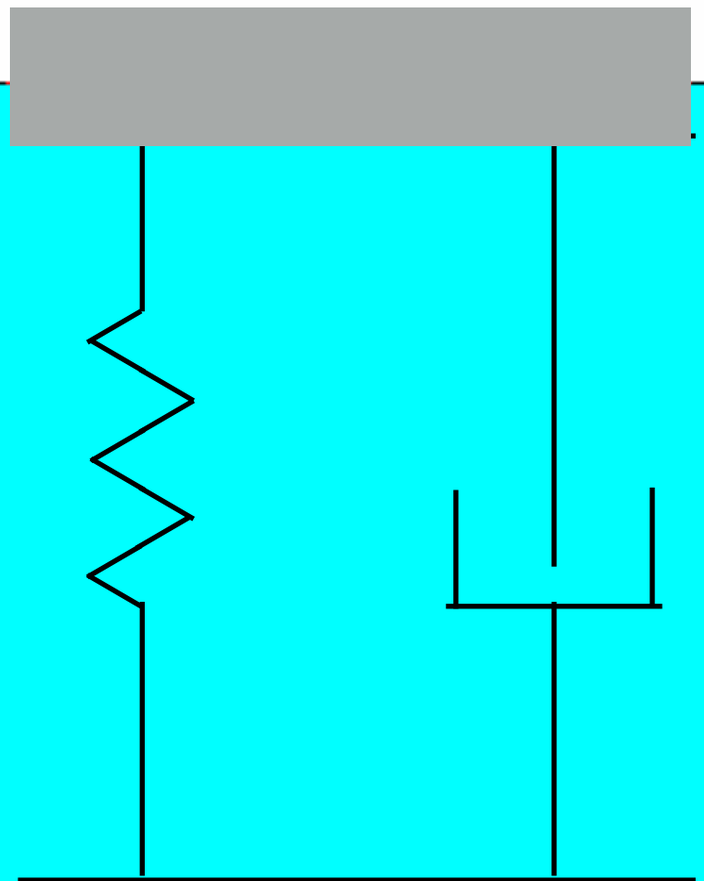

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inertial mass = mass + added mass

$$[m + m_{\text{add}}]\frac{d\dot{\delta}}{dt} + k\dot{\delta} = P_{\text{hydro}}$$

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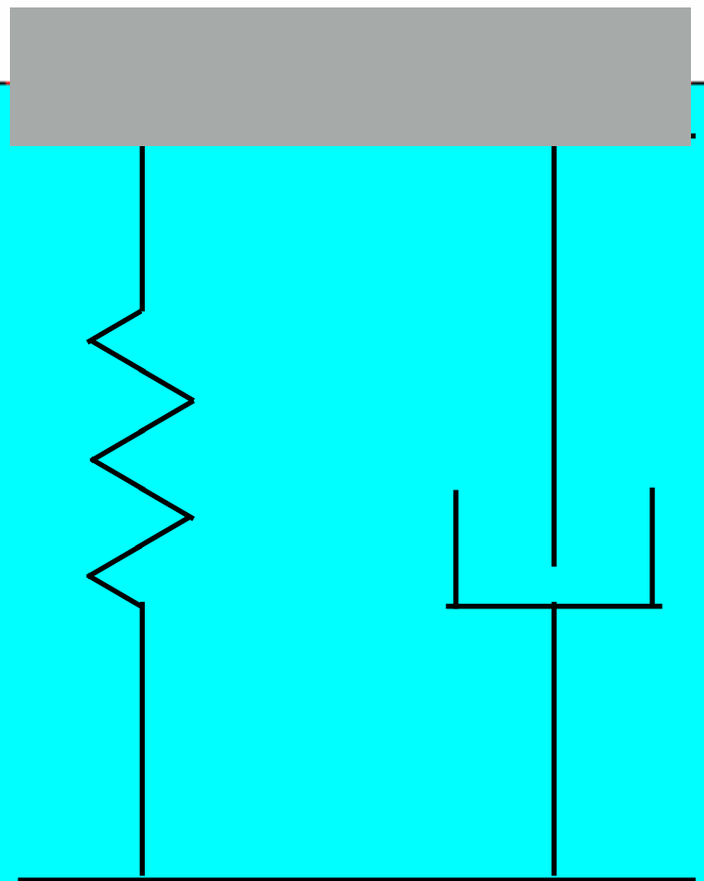

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Solid: Newton's equation
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Free surfaces:
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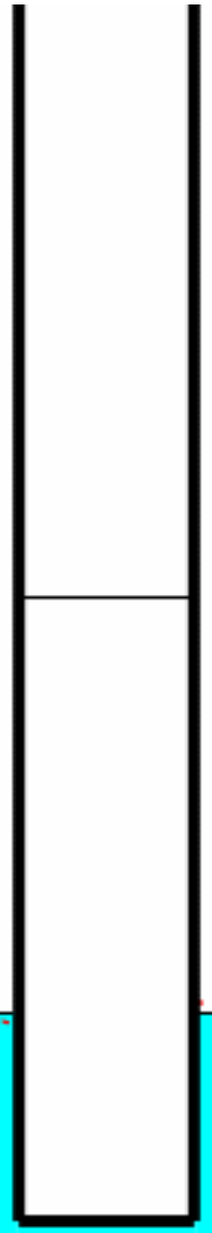

Finite Elements: FreeFem

Fluid: Euler's equations

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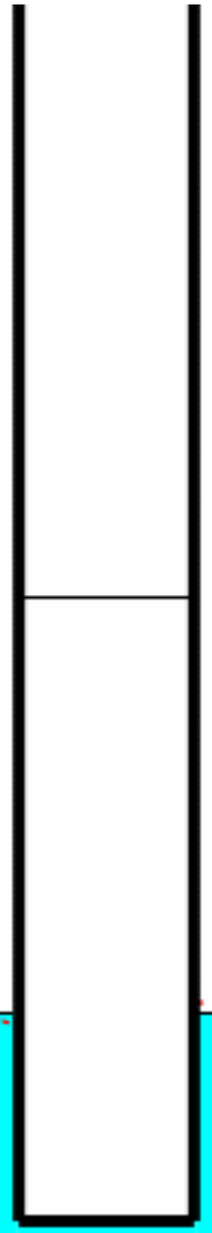

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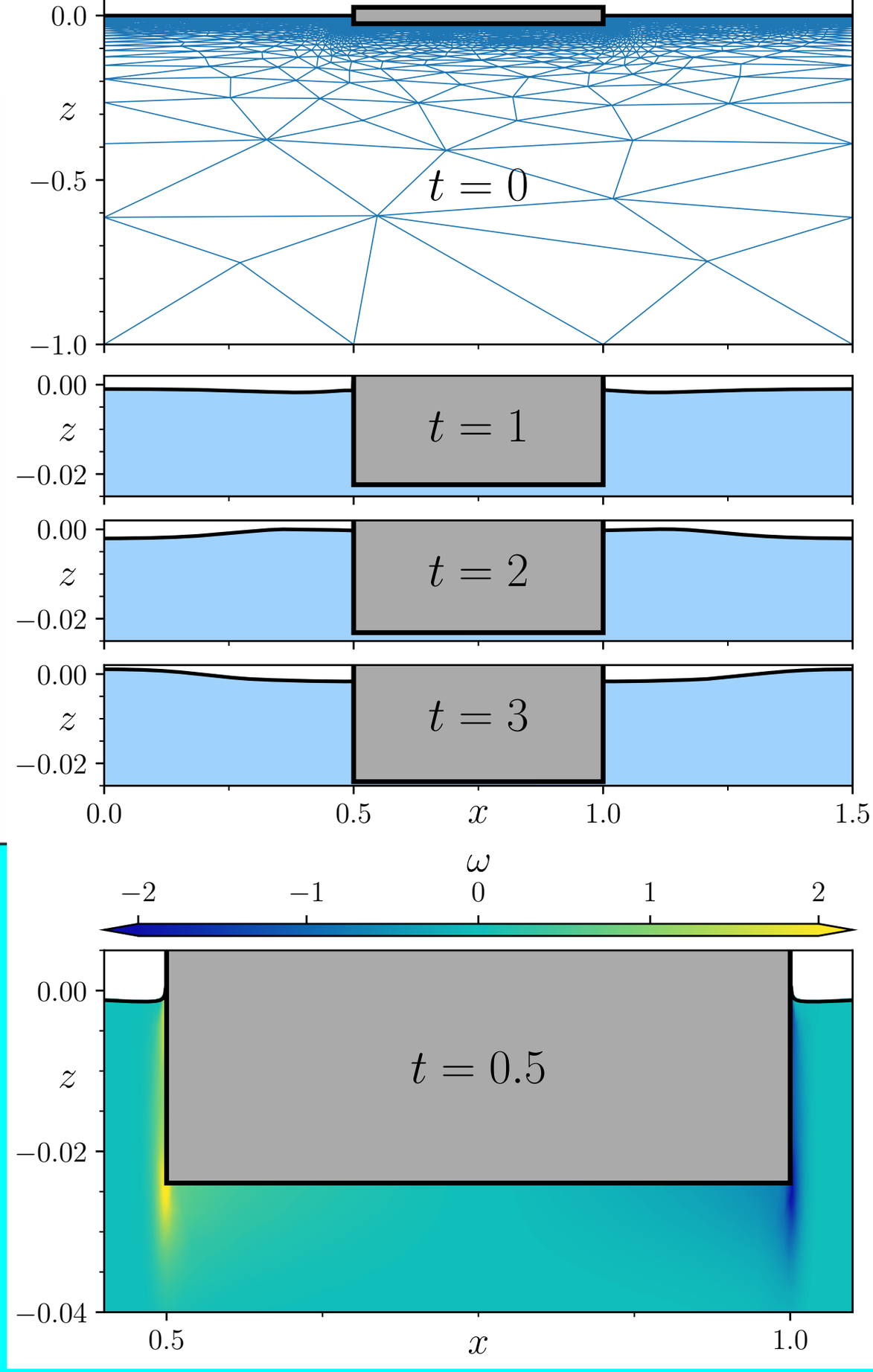

https://gitlab.com/ariquier-research/floating-body

Solid: Newton's equation with hydrodynamic force due to pressure

Free surfaces:
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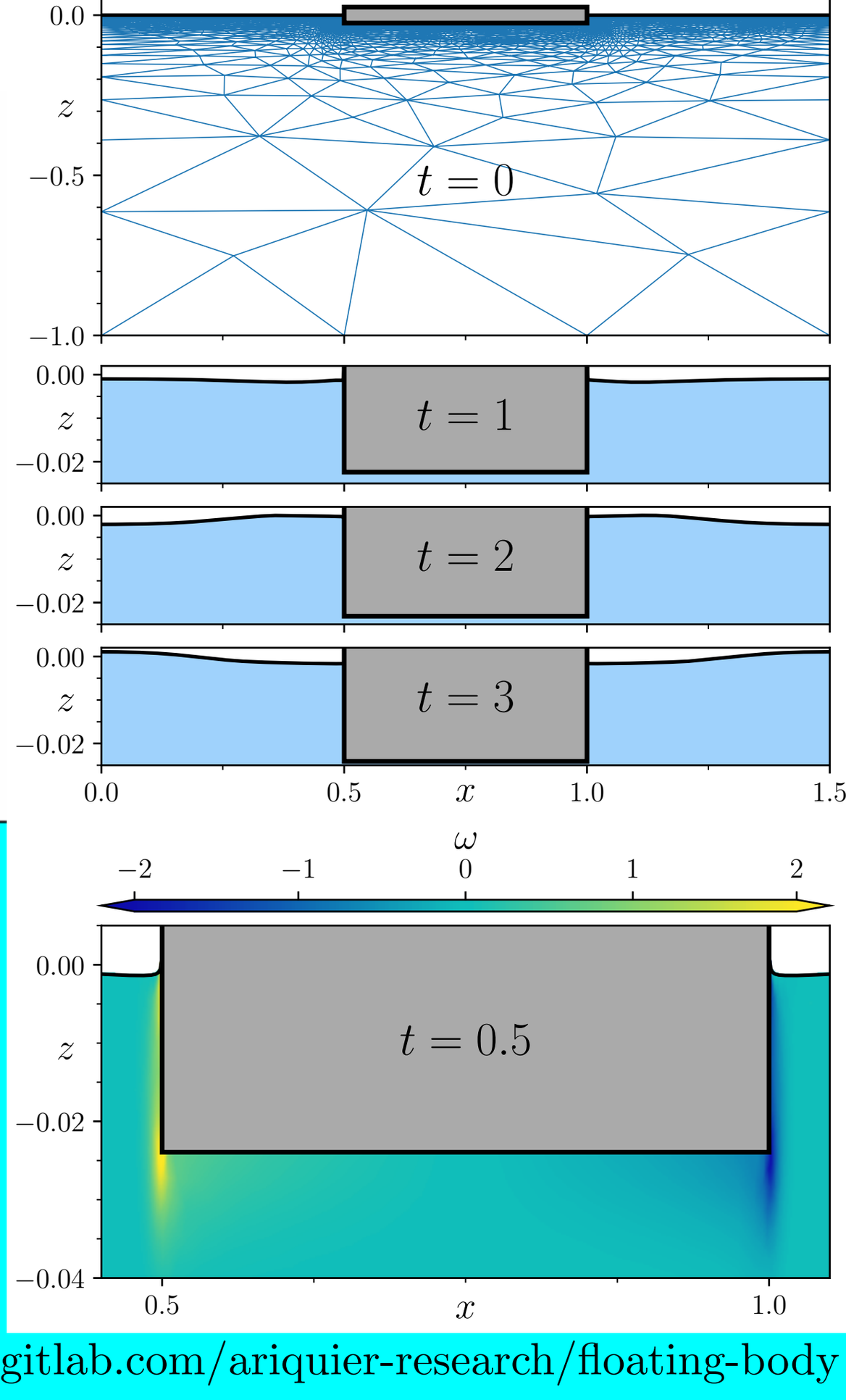
Finite Elements: FreeFem

Advection of the whole triangulation: ALE

Fluid: Euler's equations



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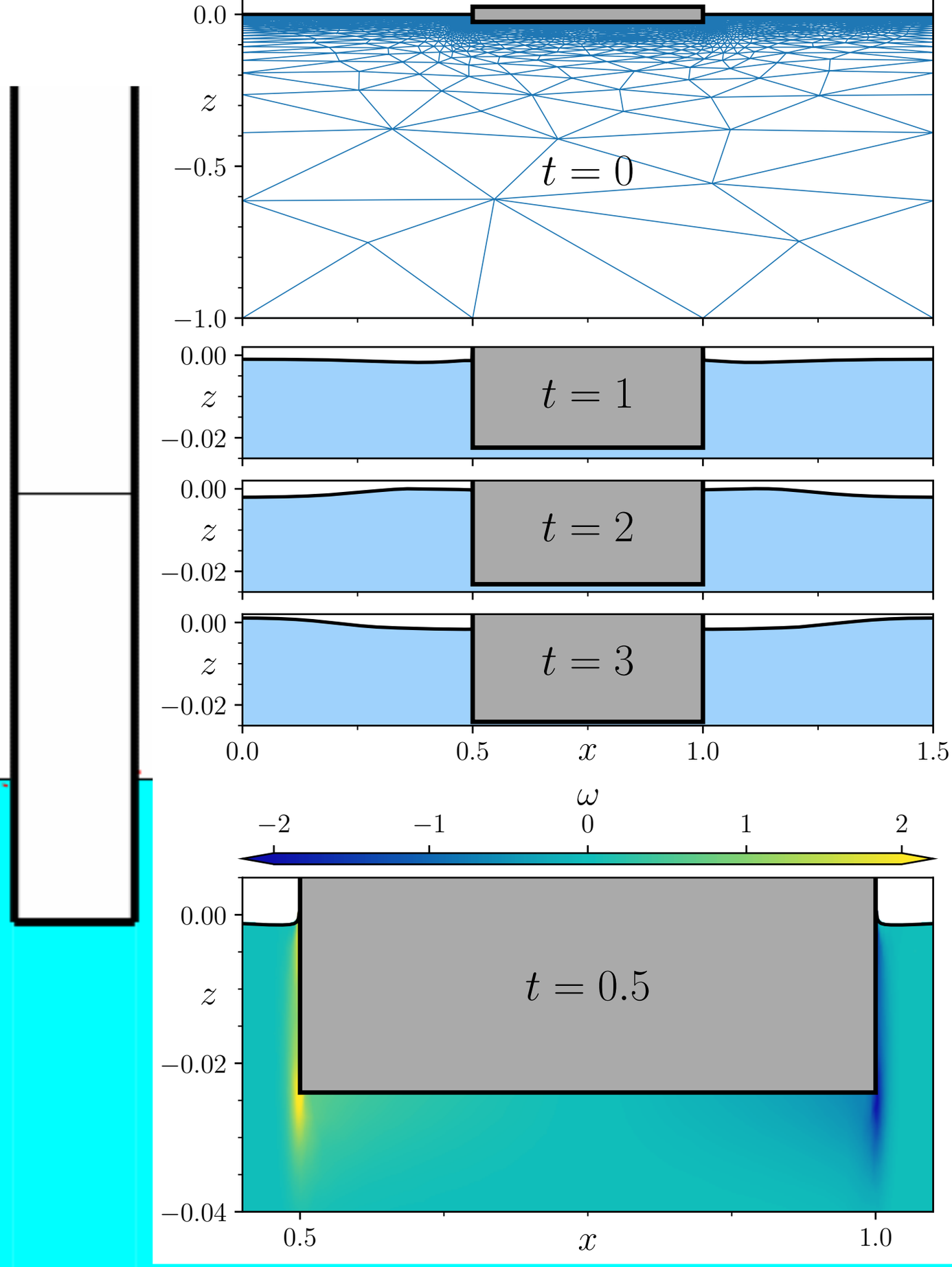

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Stabilization: Navier-Stokes
Fluid: ~~Euler's equations~~

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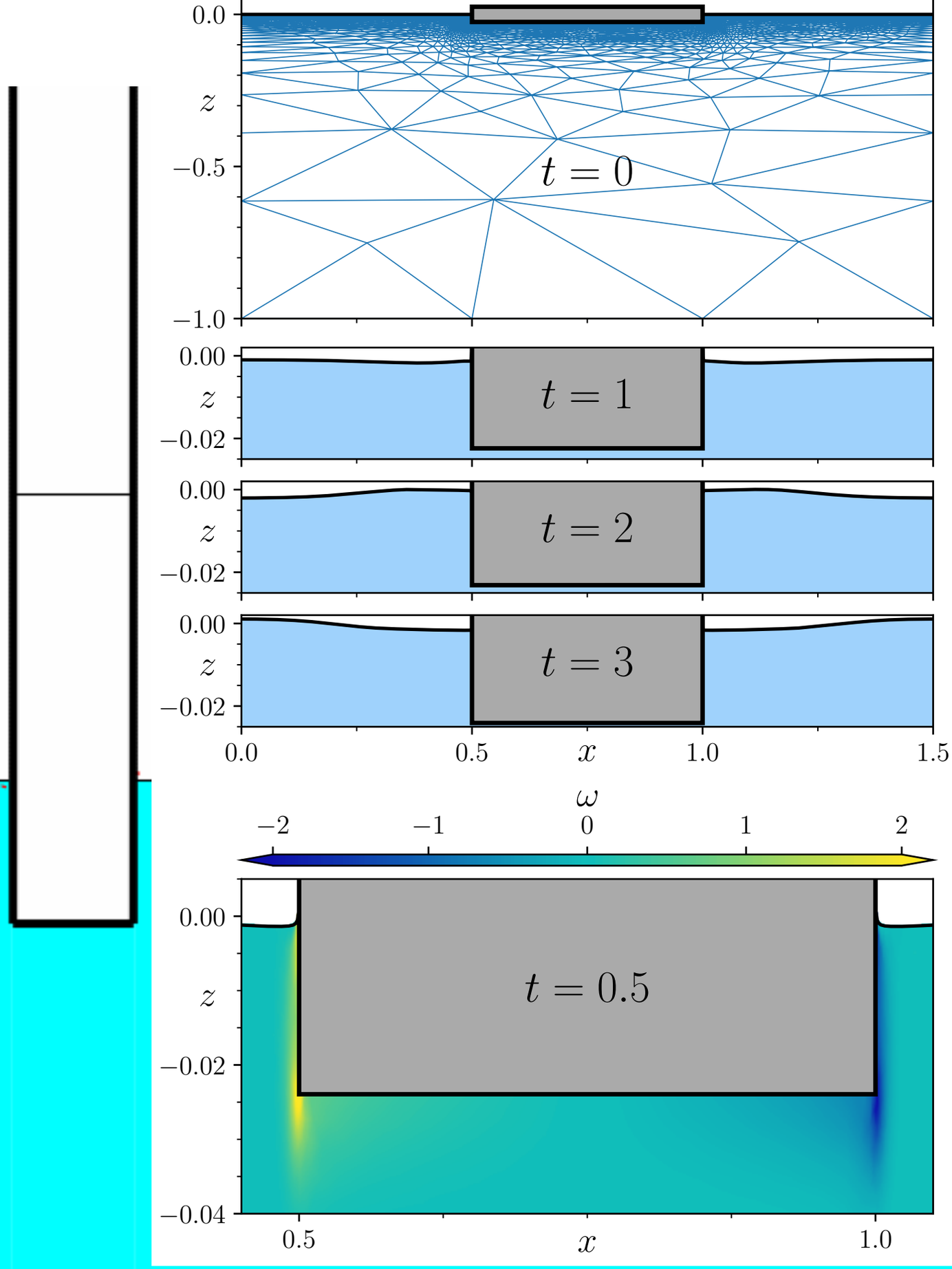

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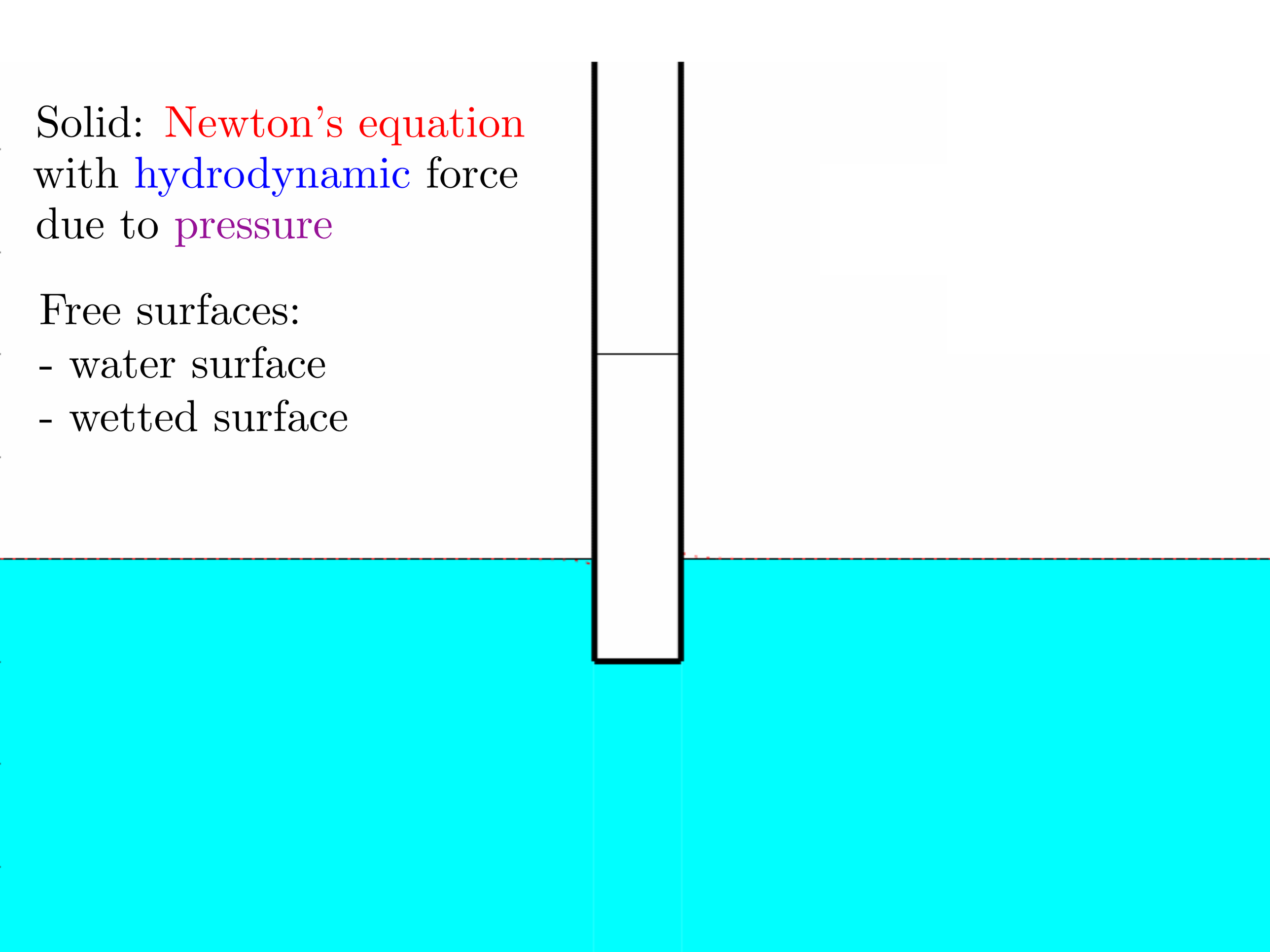

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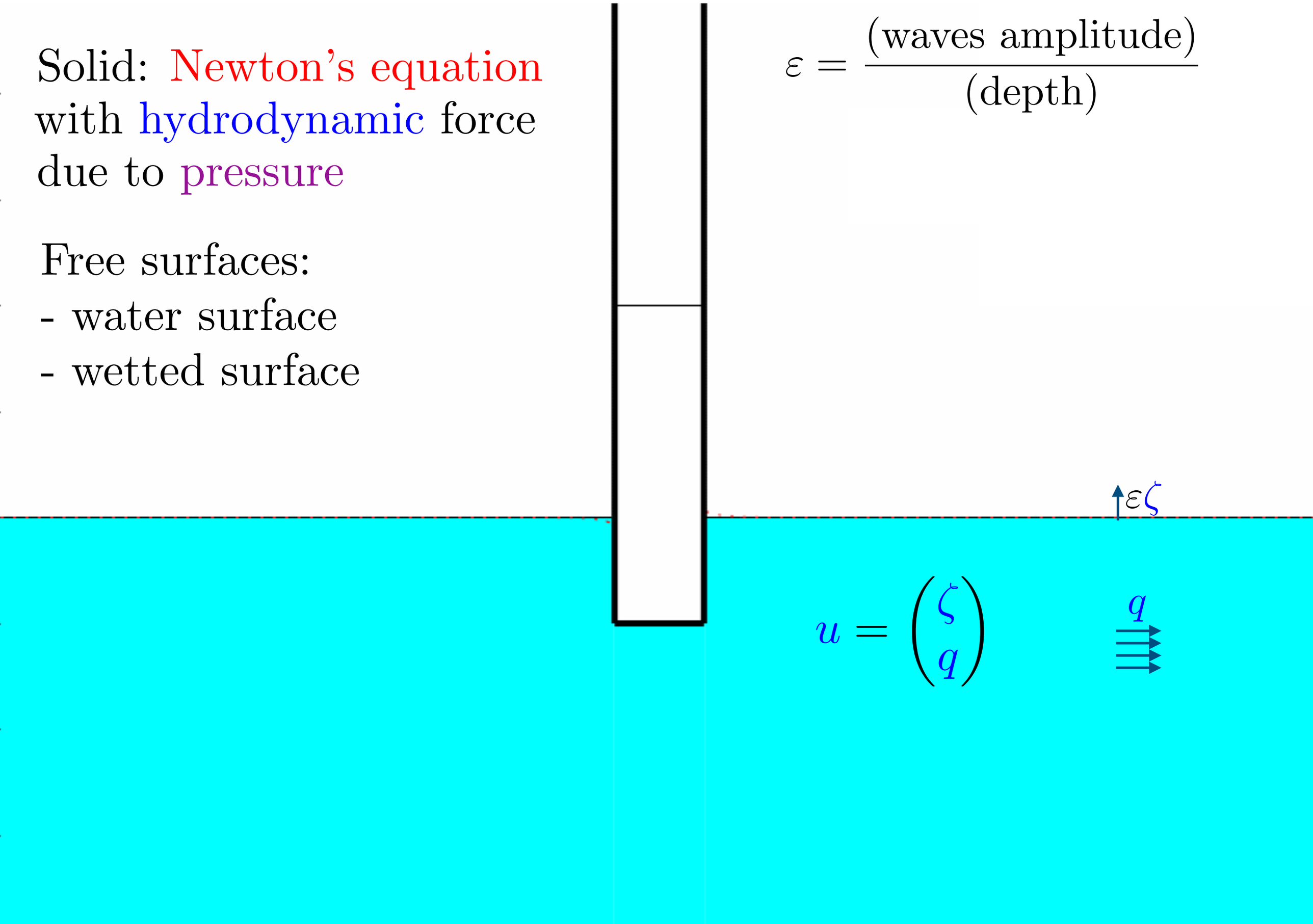

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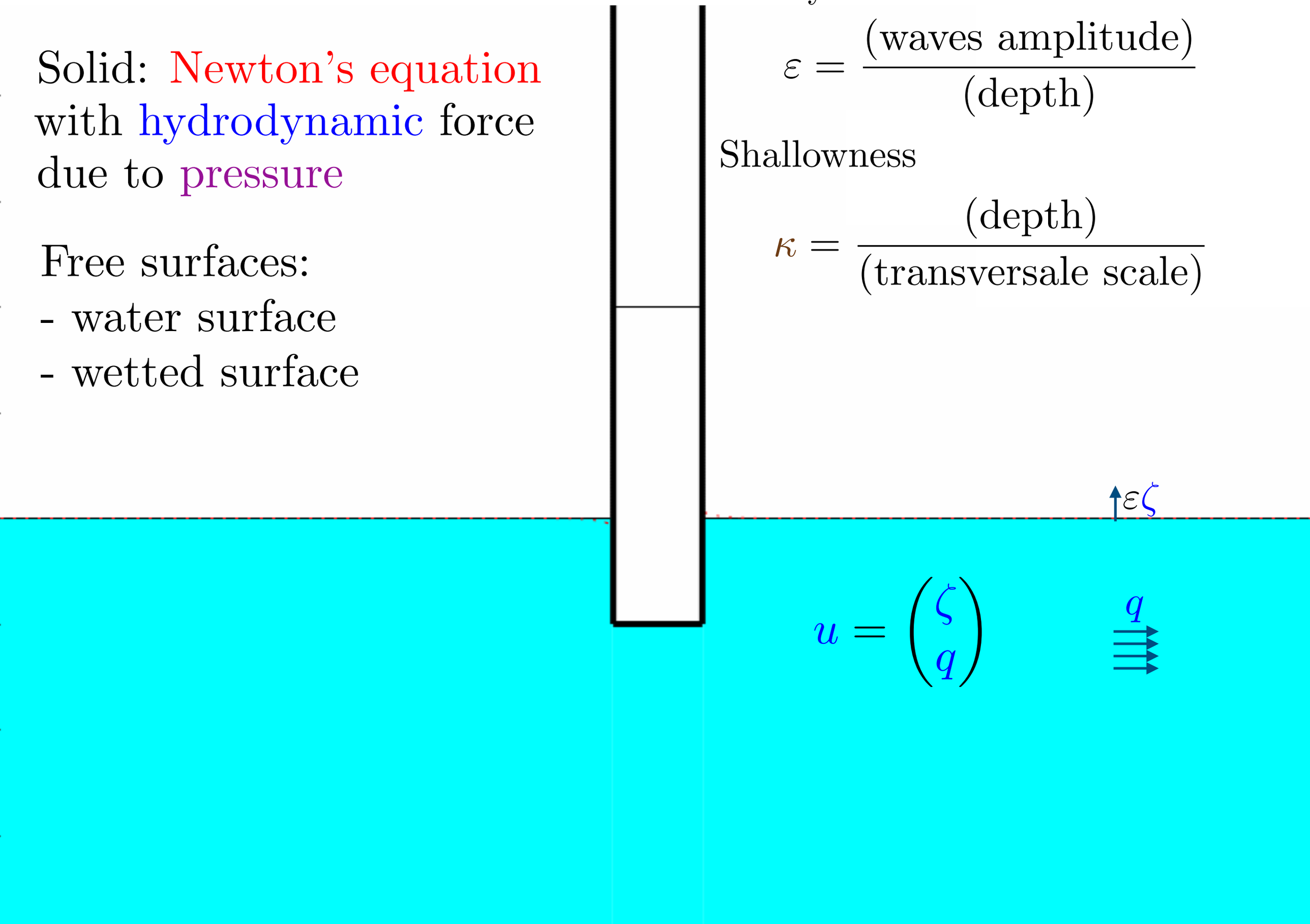
Weakly non-linear

$$\varepsilon = \frac{(\text{waves amplitude})}{(\text{depth})}$$

Shallowness

$$\kappa = \frac{(\text{depth})}{(\text{transversale scale})}$$



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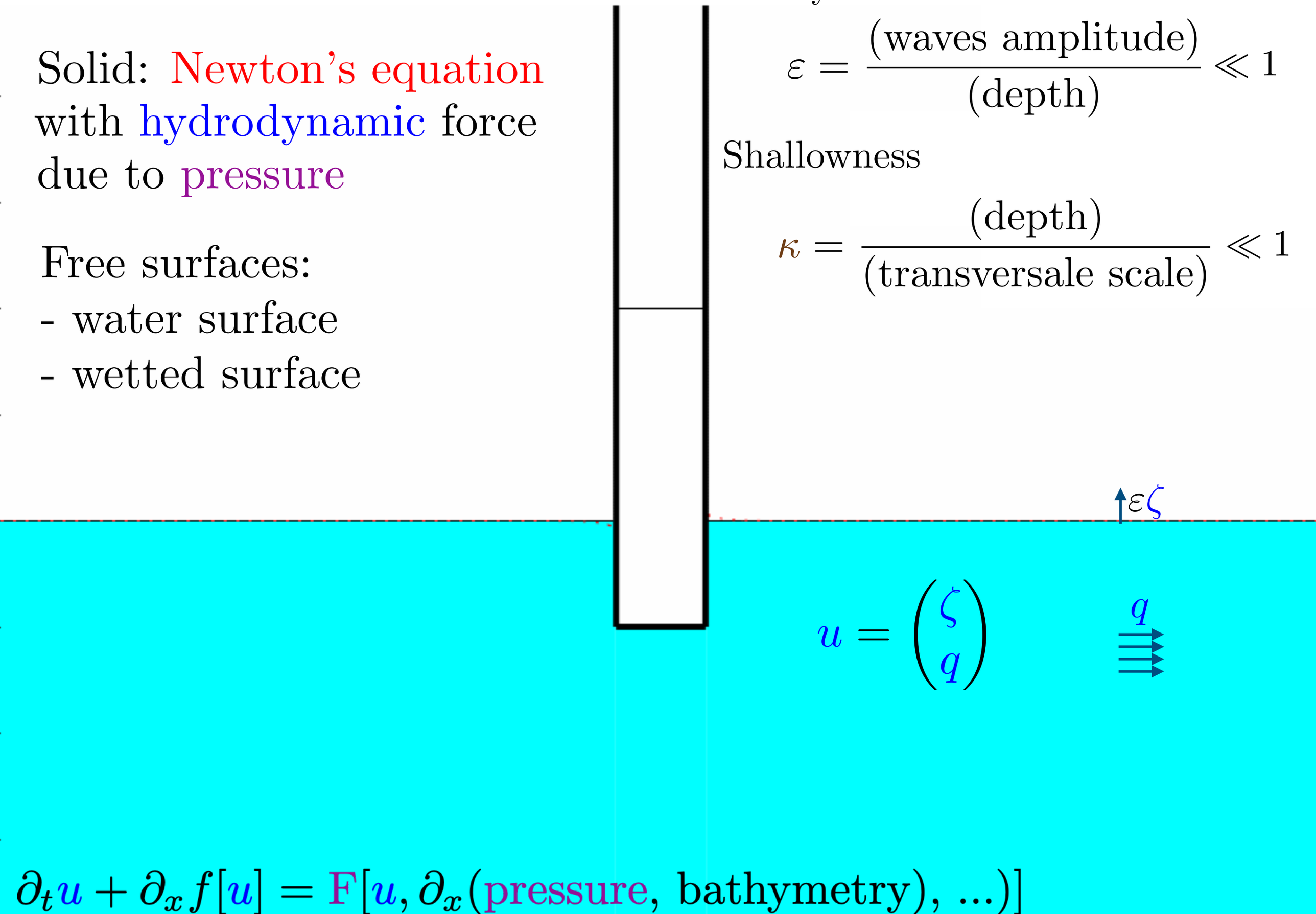
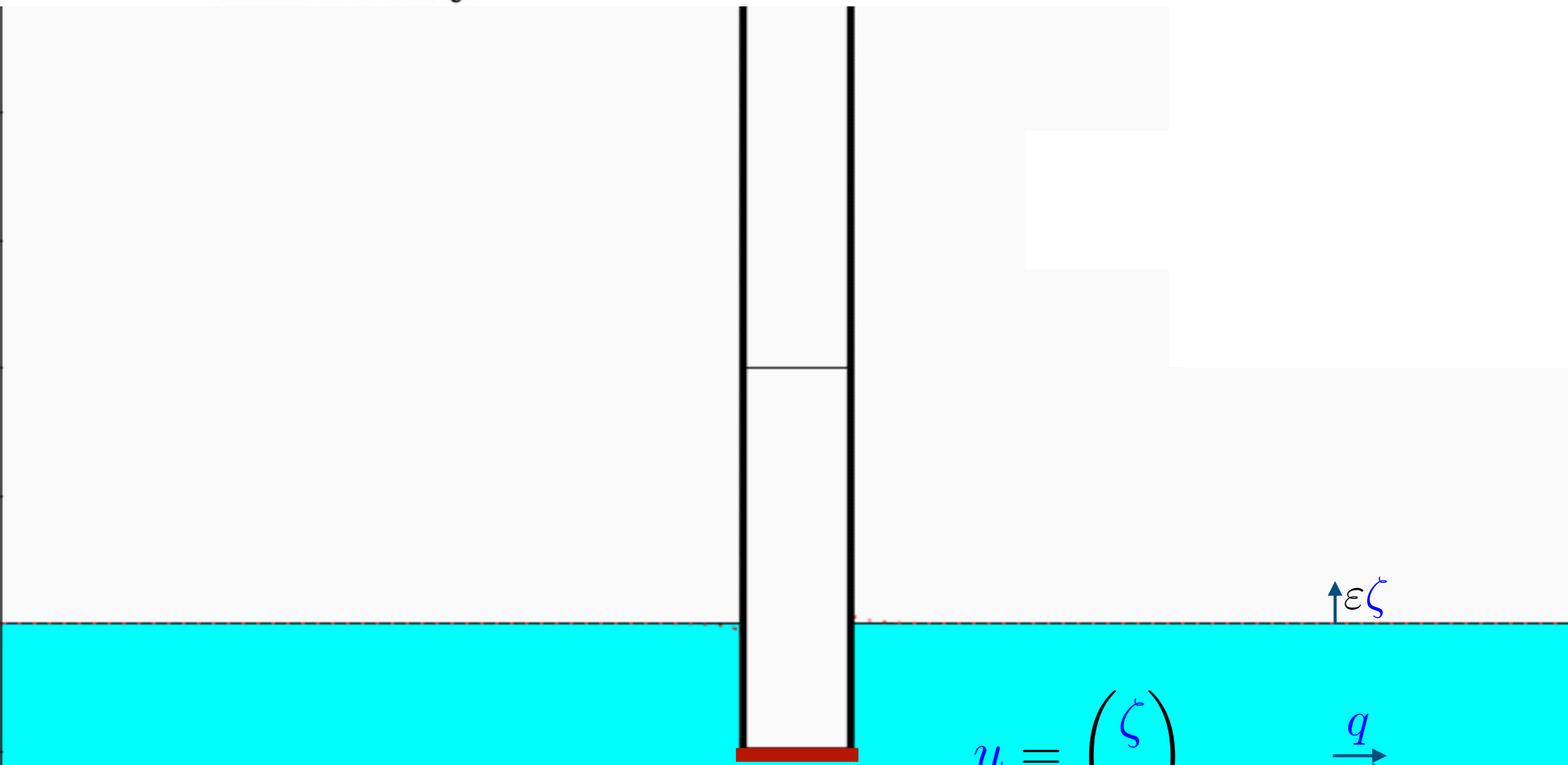


$$\partial_t u + \partial_x f[u] = \mathrm{F}[u, \partial_x(\text{pressure}, \text{ bathymetry}), \ldots)]$$

Fixed body



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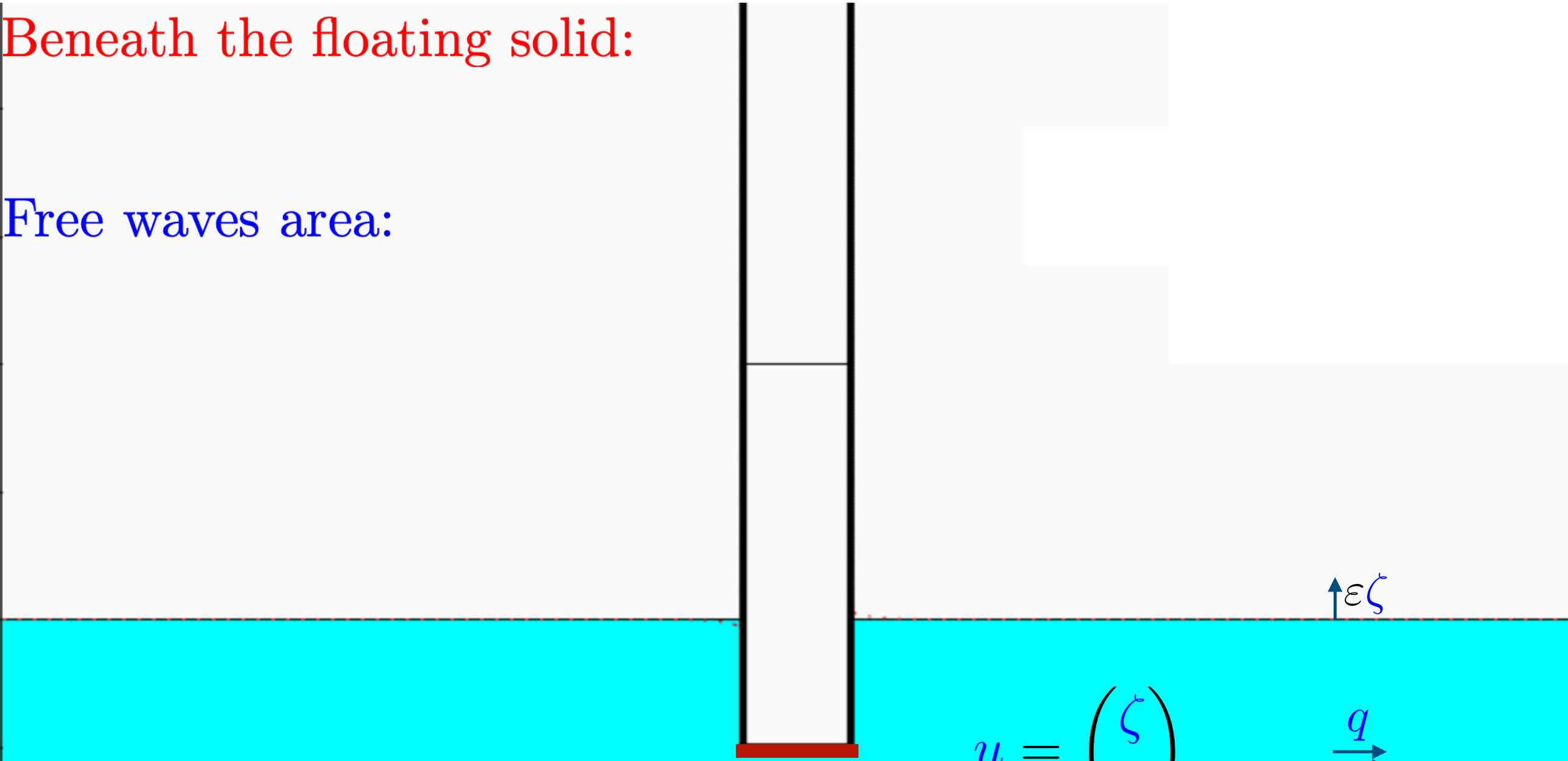
Fixed body

Beneath the floating solid:

Free waves area:



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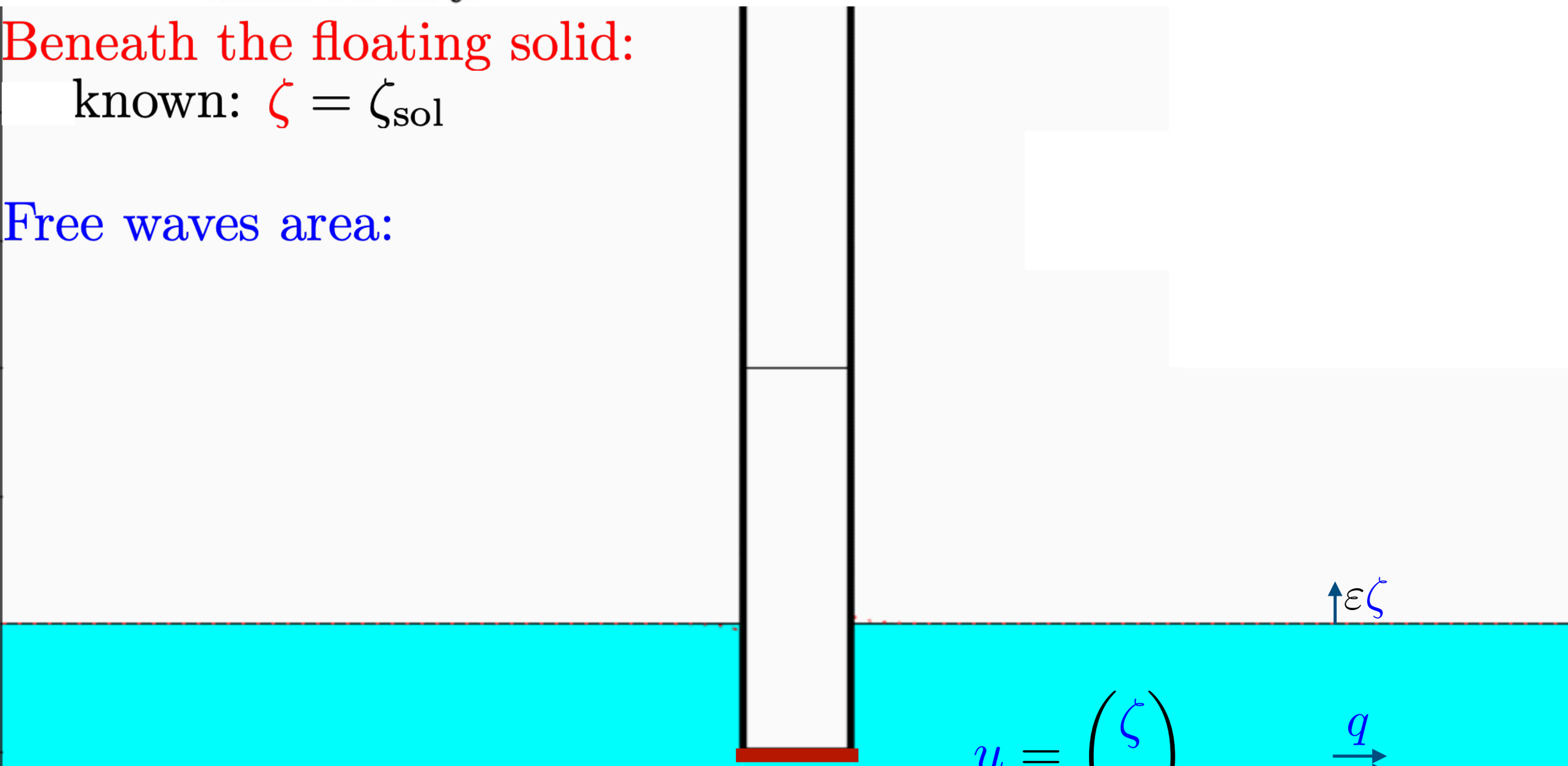
Fixed body

Beneath the floating solid:

known: $\zeta = \zeta_{\text{sol}}$

Free waves area:



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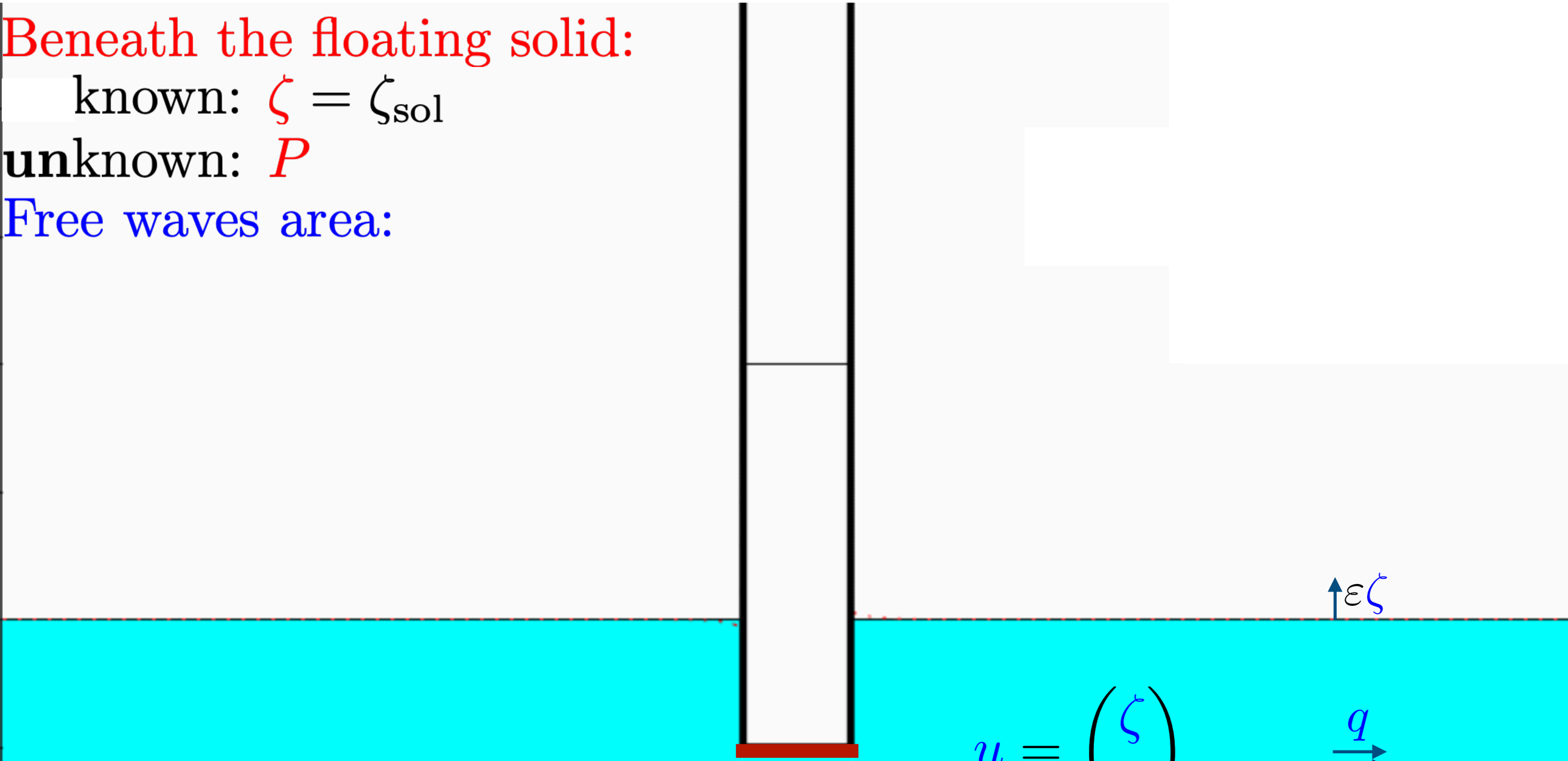
Free waves area:



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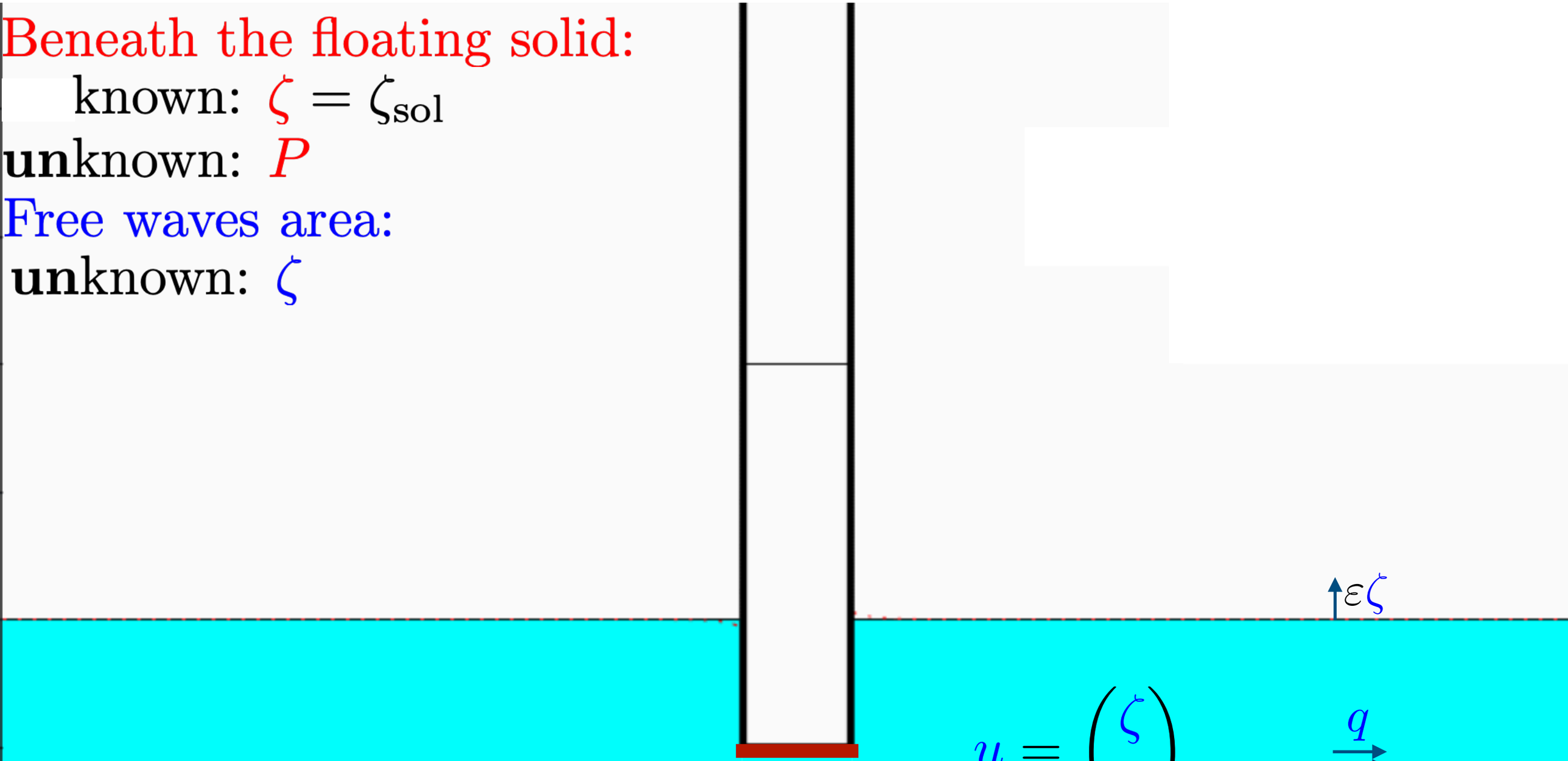
unknown: ζ



$$\partial_t u + \partial_x f[u] = \mathrm{F}[u, \partial_x(\text{pressure}, \text{ bathymetry}), \ldots)]$$

Fixed body

Beneath the floating solid:

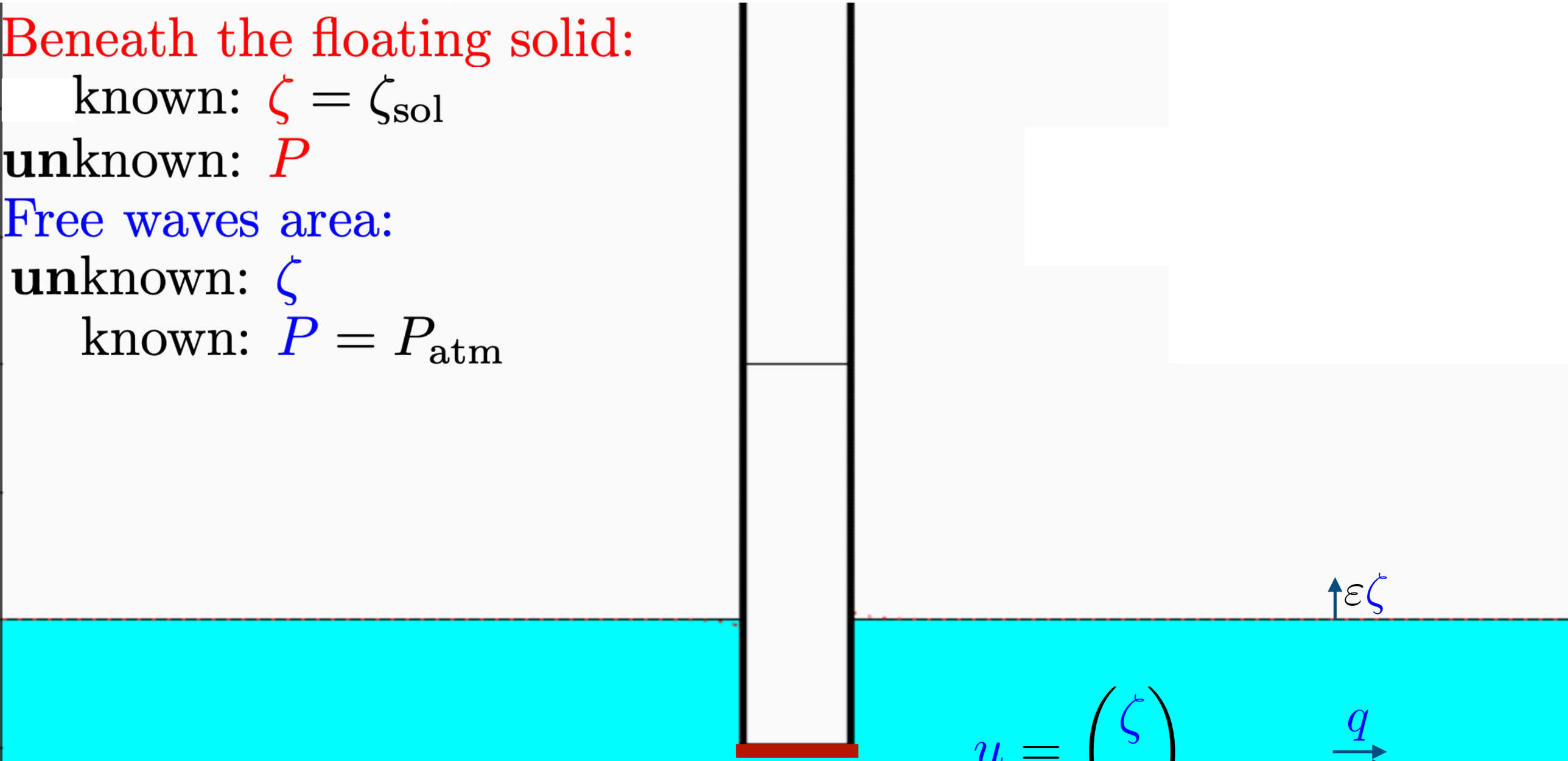
known: $\zeta = \zeta_{\text{sol}}$

unknown: P

Free waves area:

unknown: ζ

known: $P = P_{\text{atm}}$



$$\partial_t u + \partial_x f[u] = \mathrm{F}[u, \partial_x(\text{pressure}, \text{ bathymetry}), \ldots)]$$

Fixed body

Beneath the floating solid:

known: $\zeta = \zeta_{\text{sol}}$

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Free waves area:

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known: $P = P_{\text{atm}}$

Transmission conditions
at contact points:

$\varepsilon\zeta$

$$u = \begin{pmatrix} \zeta \\ q \end{pmatrix}$$

q

$$\partial_t u + \partial_x f[u] = \mathrm{F}[u, \partial_x(\text{pressure}, \text{ bathymetry}), \ldots)]$$

Fixed body

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Transmission conditions
at contact points:

$q = q$

$\varepsilon\zeta$

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Fixed body

Beneath the floating solid:

known: $\zeta = \zeta_{\text{sol}}$

unknown: P

Free waves area:

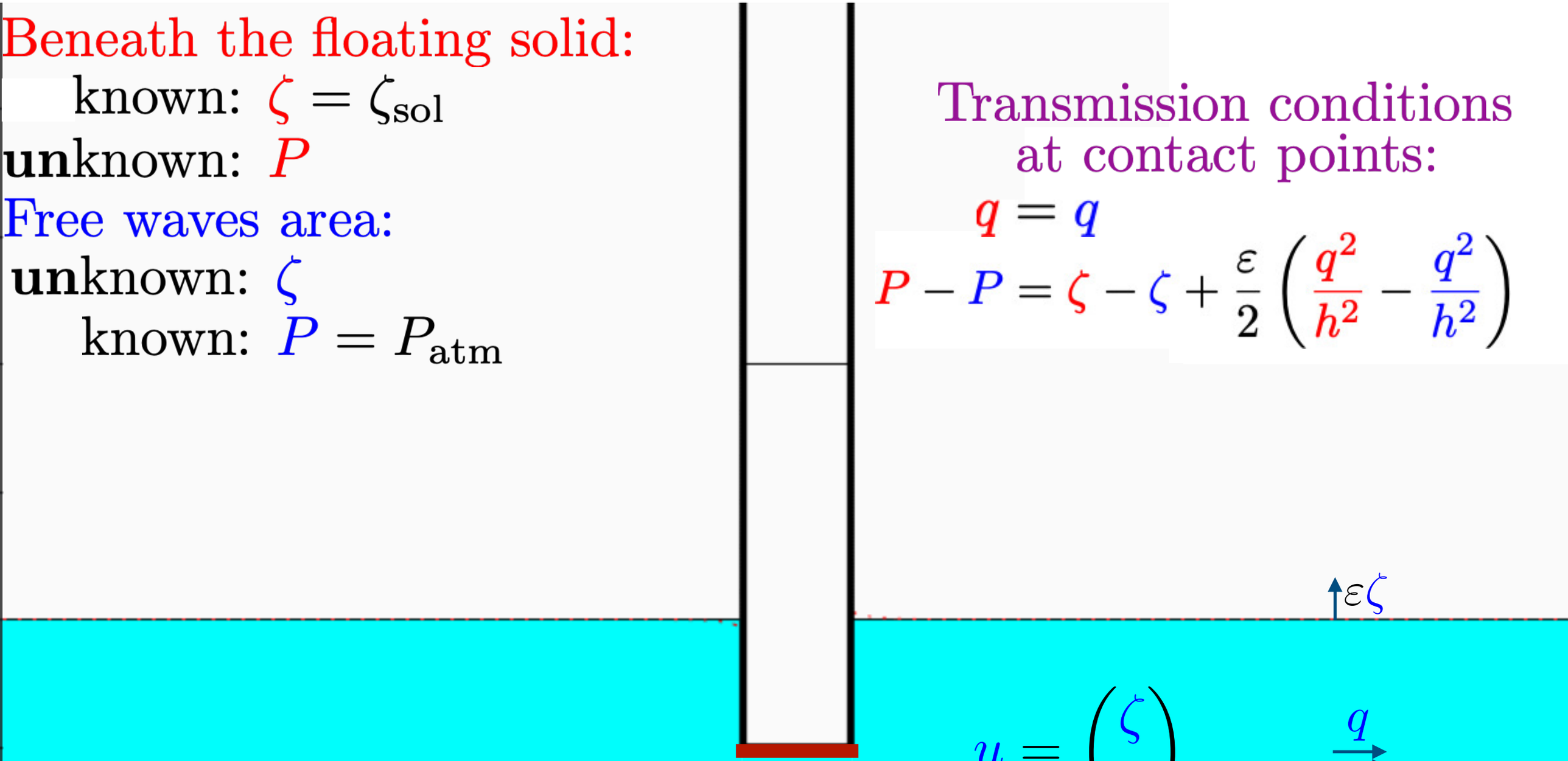
unknown: ζ

known: $P = P_{\text{atm}}$

Transmission conditions at contact points:

$$q = q$$

$$P - P = \zeta - \zeta + \frac{\varepsilon}{2}\left(\frac{q^2}{h^2} - \frac{q^2}{h^2}\right)$$



$$\partial_t u + \partial_x f[u] = \mathrm{F}[u, \partial_x(\text{pressure}, \text{ bathymetry}), \ldots)]$$

Moving body

Beneath the floating solid:

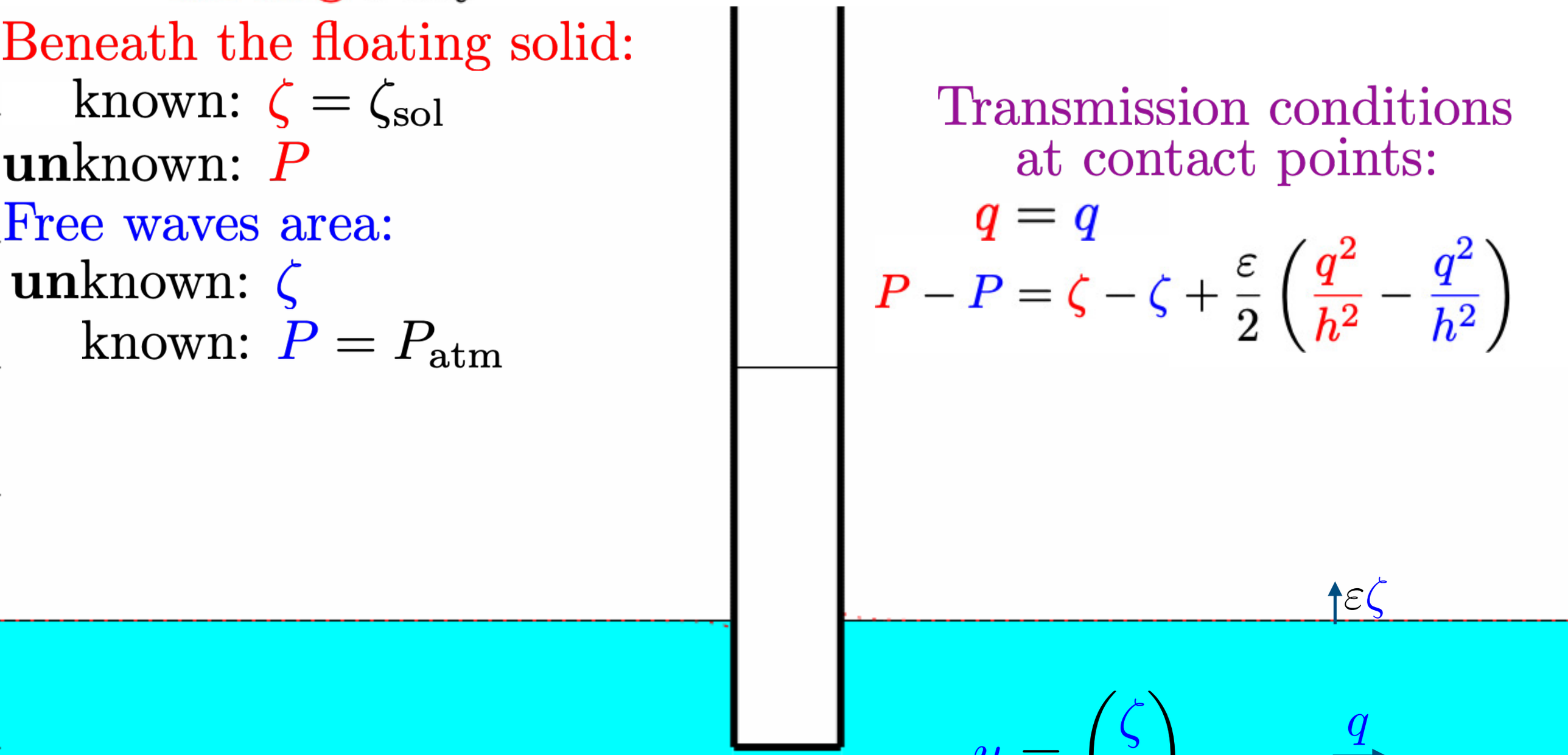
known: $\zeta = \zeta_{\text{sol}}$

unknown: P

Free waves area:

unknown: ζ

known: $P = P_{\text{atm}}$



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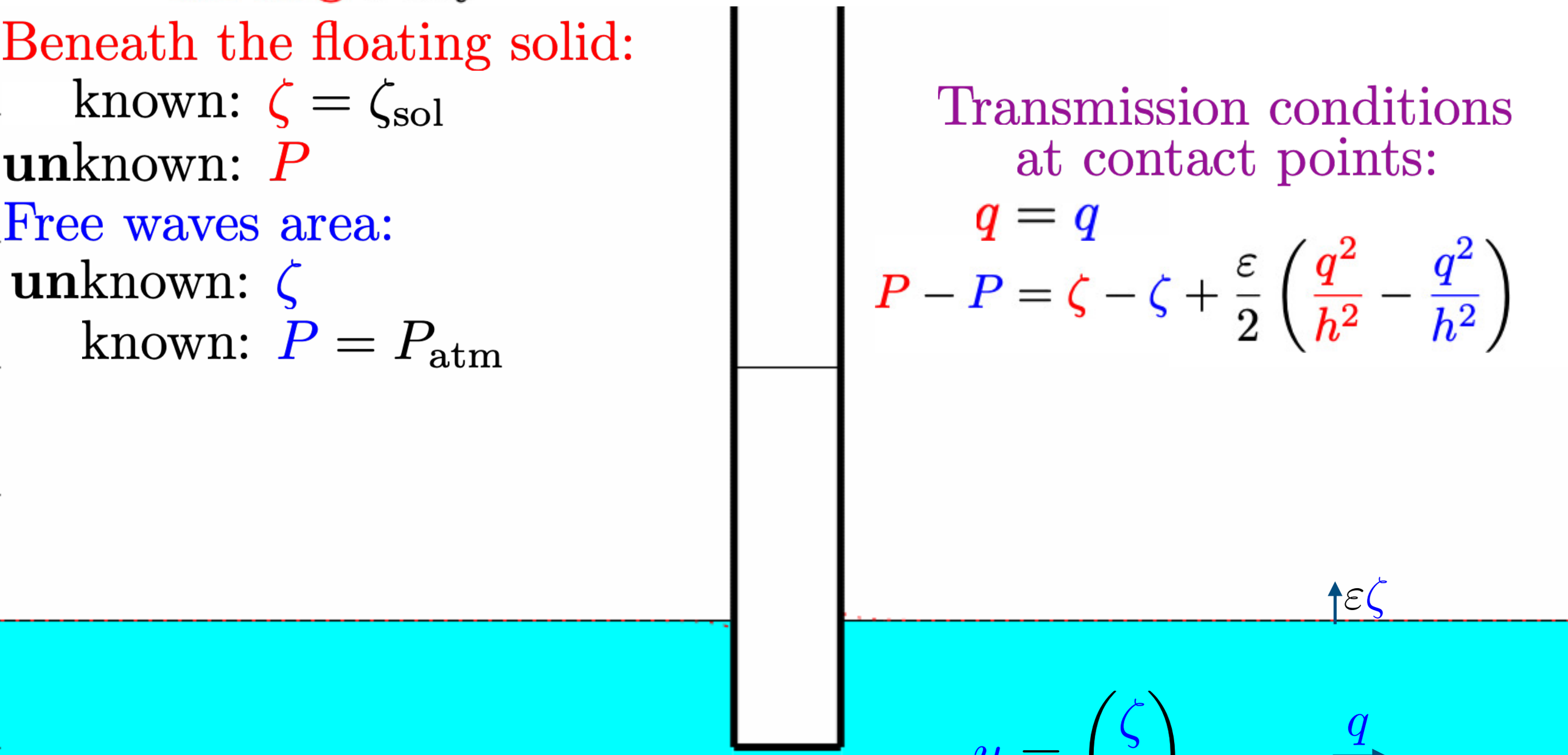

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Moving body

Beneath the floating solid:

unknown: $\zeta = \zeta_{\text{sol}}$+solid motion

unknown: P

Free waves area:

unknown: ζ

known: $P = P_{\text{atm}}$

Transmission conditions at contact points:

$$q = q$$

$$P - P = \zeta - \zeta + \frac{\varepsilon}{2}\left(\frac{q^2}{h^2} - \frac{q^2}{h^2}\right)$$

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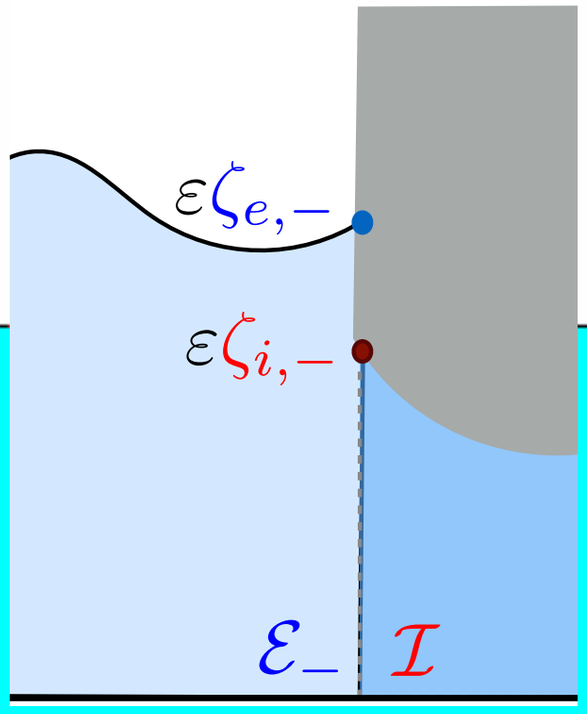

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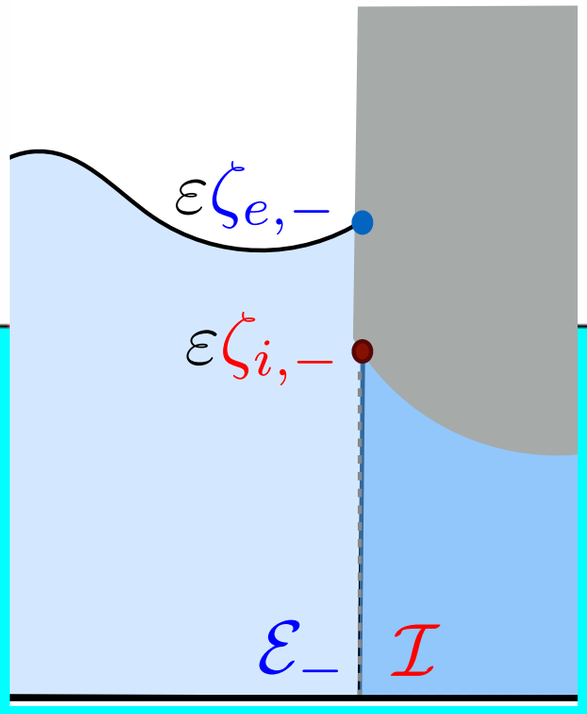

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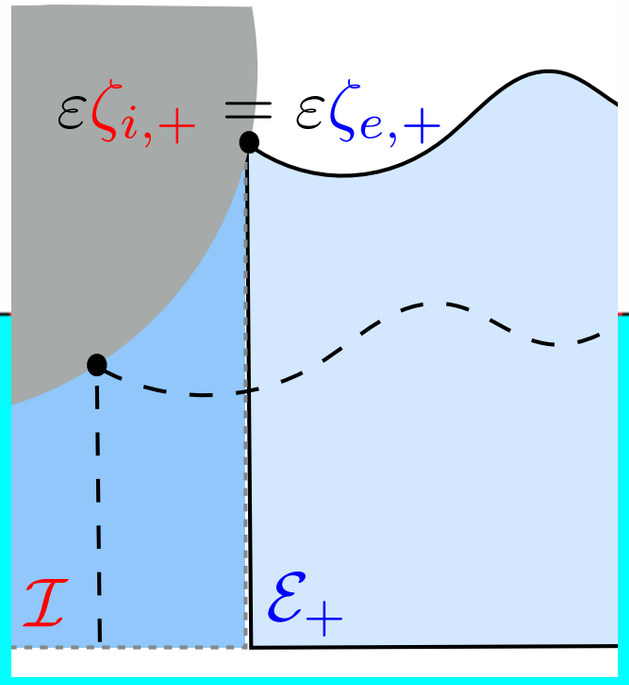

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$\varepsilon\zeta$

$$u = \begin{pmatrix} \zeta \\ q \end{pmatrix}$$

q

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Moving body

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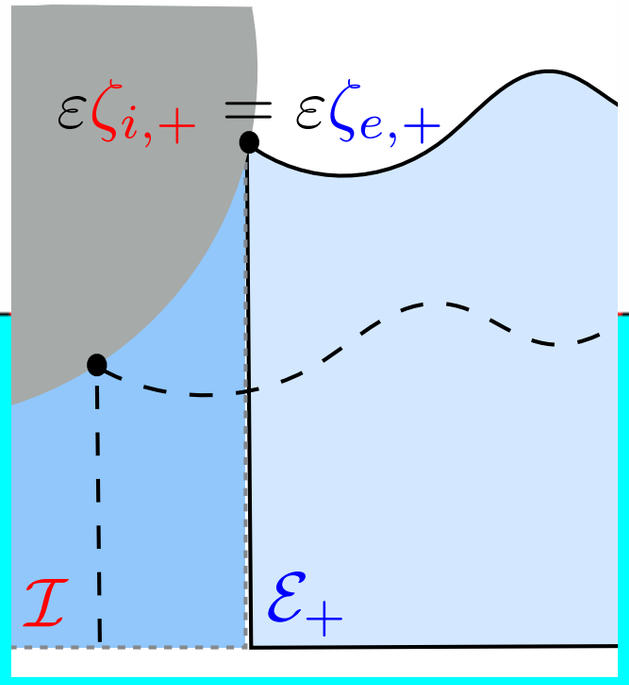

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$$q = q$$

$$P - P = \zeta - \zeta + \frac{\varepsilon}{2}\left(\frac{q^2}{h^2} - \frac{q^2}{h^2}\right)$$

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$$\partial_t u + \partial_x f[u] = \mathrm{F}[u, \partial_x(\text{pressure}, \text{ bathymetry}), \ldots)]$$

Moving body

Beneath the floating solid:

unknown: $\zeta = \zeta_{\text{sol}}$+solid motion

unknown: P

Free waves area:

unknown: ζ

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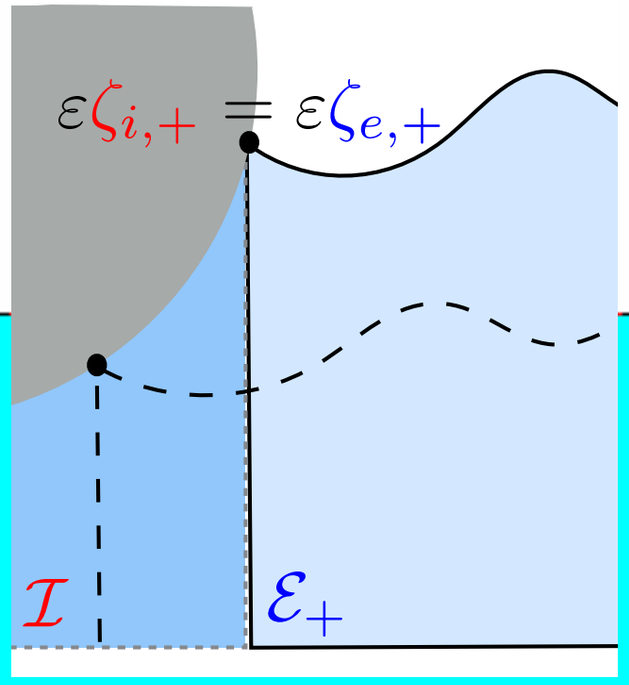


Transmission conditions at contact points:

$q = q$

$P = P, \qquad \zeta = \zeta$

$\varepsilon\zeta$

$$u = \begin{pmatrix} \zeta \\ q \end{pmatrix}$$

q

$$\partial_t u + \partial_x f[u] = \mathrm{F}[u, \partial_x(\text{pressure}, \text{ bathymetry}), \ldots)]$$

Moving body

Beneath the floating solid:

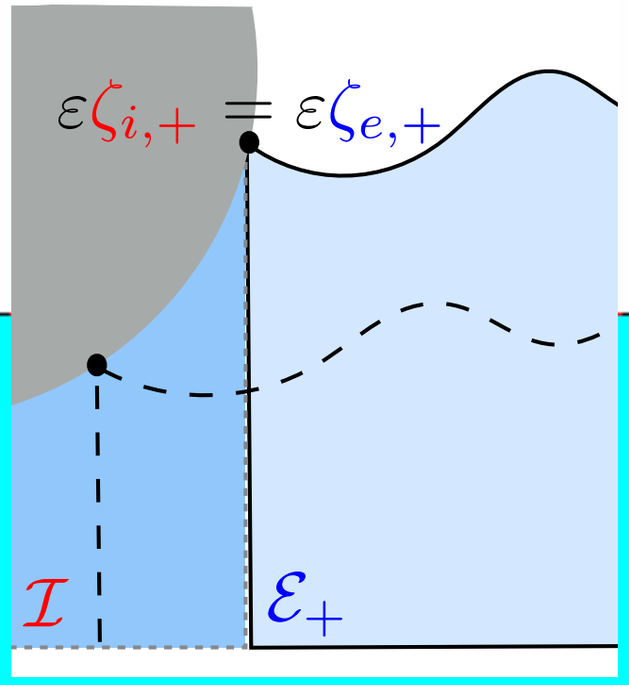
unknown: $\zeta = \zeta_{\text{sol}}$+solid motion

unknown: P

Free waves area:

unknown: ζ

known: $P = P_{\text{atm}}$



Transmission conditions at contact points:

$q = q$

$P = P, \qquad \zeta = \zeta$

unknown: free contact points

$\varepsilon\zeta$

$$u = \begin{pmatrix} \zeta \\ q \end{pmatrix}$$

q

$$\partial_t u + \partial_x f[u] = \mathrm{F}[u, \partial_x(\text{pressure}, \text{ bathymetry}), \ldots)]$$

Fixed body

Beneath the floating solid:

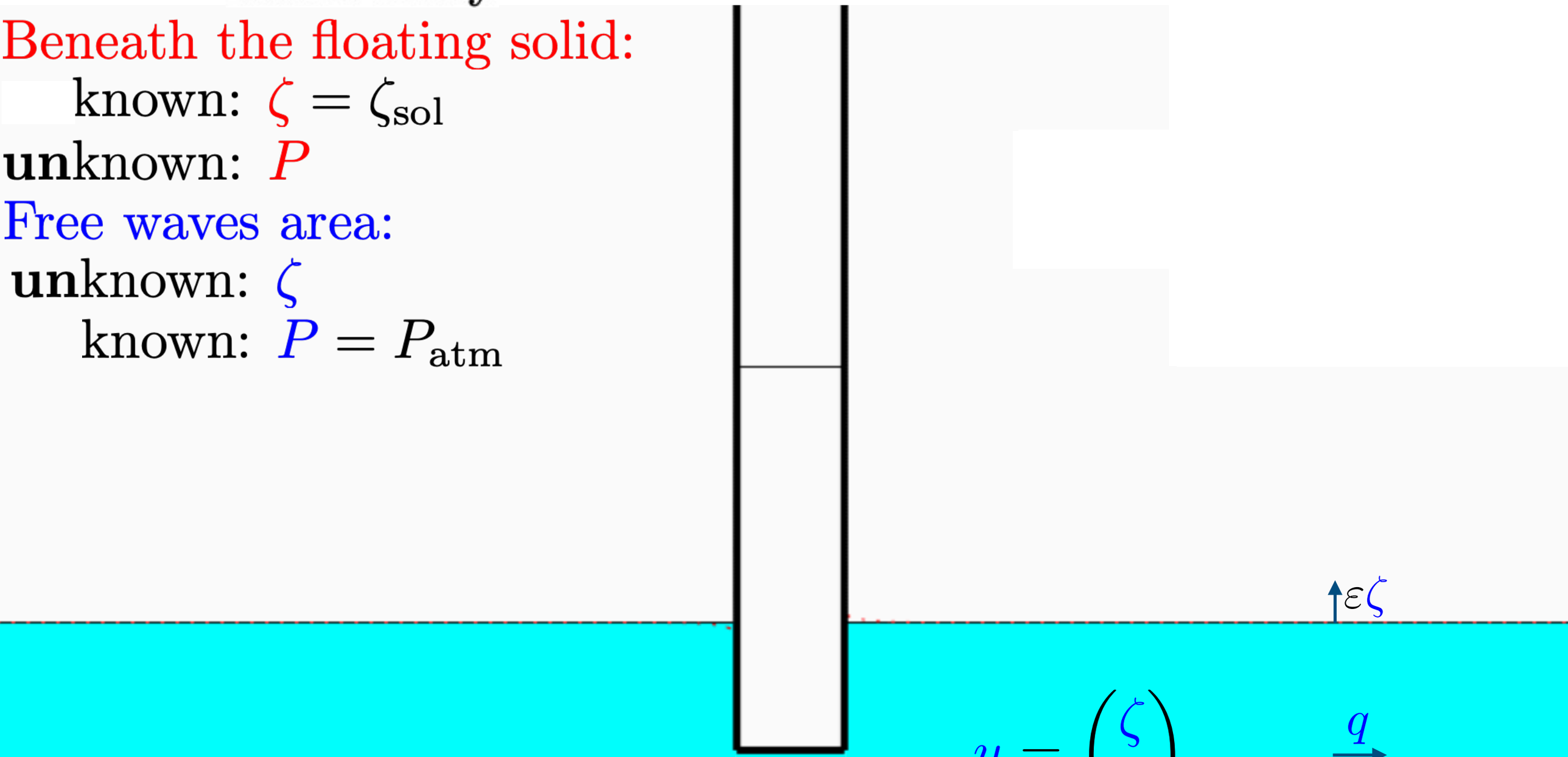
known: $\zeta = \zeta_{\text{sol}}$

unknown: P

Free waves area:

unknown: ζ

known: $P = P_{\text{atm}}$



$$\partial_t u + \partial_x f[u] = \mathrm{F}[u, \partial_x(\text{pressure}, \text{ bathymetry}), \ldots)]$$

Fixed body

Beneath the floating solid:

known: $\zeta = \zeta_{\text{sol}}$

unknown: P

Free waves area:

unknown: ζ

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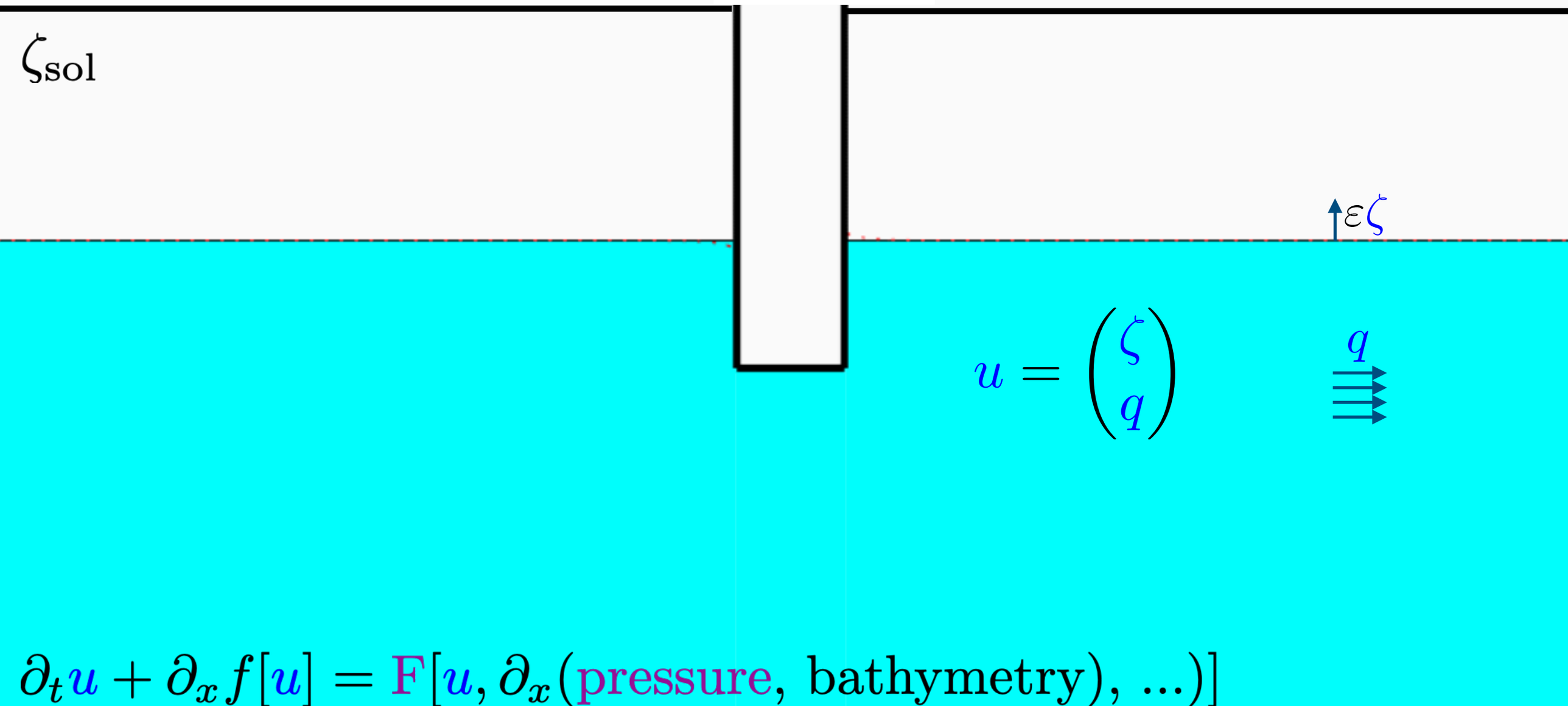


$$\partial_t u + \partial_x f[u] = \mathrm{F}[u, \partial_x(\text{pressure}, \text{ bathymetry}), \ldots)]$$

Fixed body

Beneath the floating solid:

known: $\zeta = \zeta_{\text{sol}}$

unknown: P

Free waves area:

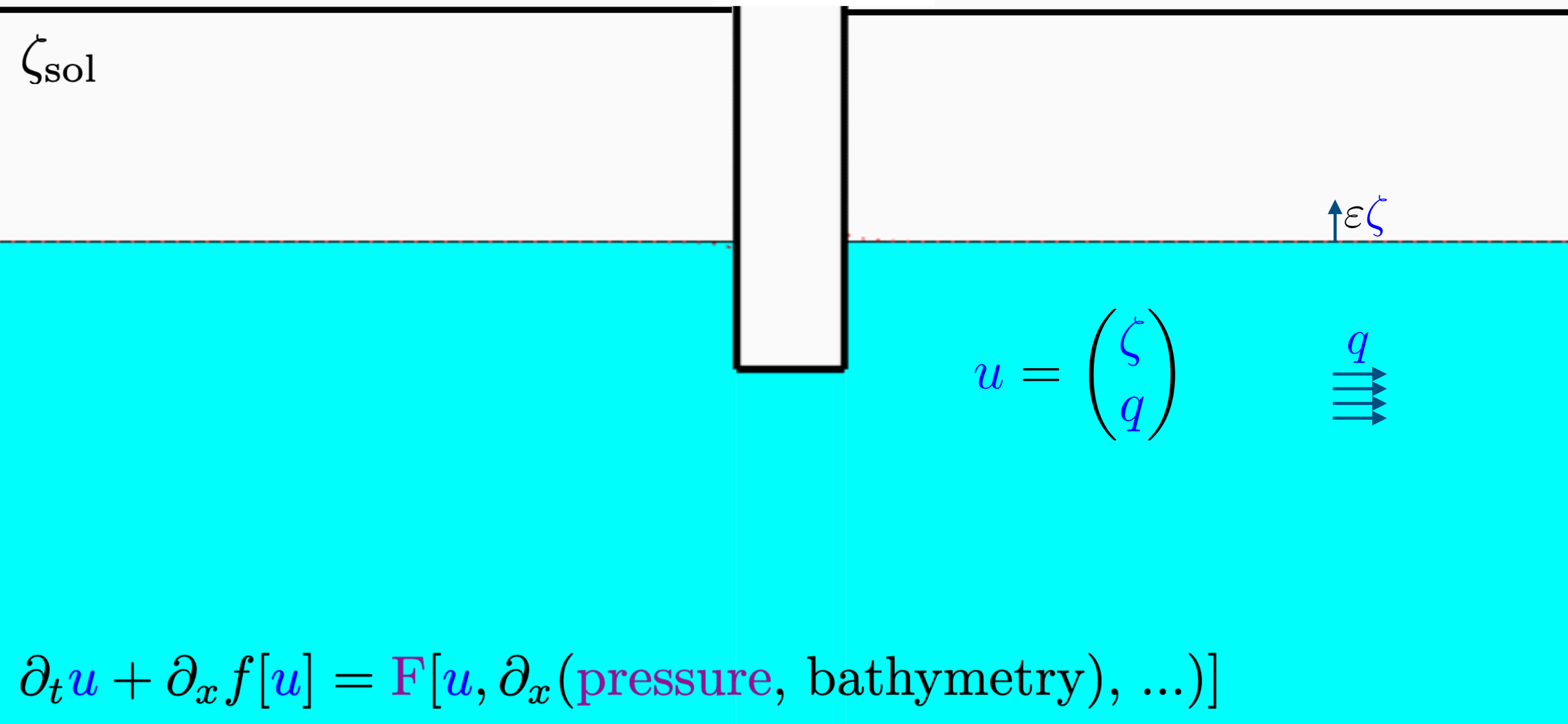
unknown: ζ

known: $P = P_{\text{atm}}$

Exclusion constraint:

$$\zeta \leq \zeta_{\text{sol}}, \quad P \geq P_{\text{atm}}$$

$$(\zeta_{\text{sol}} - \zeta)(P - P_{\text{atm}}) = 0$$



$$\partial_t u + \partial_x f[u] = \mathrm{F}[u, \partial_x(\text{pressure}, \text{ bathymetry}), \ldots)]$$

Fixed body

Beneath the floating solid:

unknown: ζ

unknown: P

Free waves area:

unknown: ζ

known: $P = P_{\text{atm}}$

Exclusion constraint:

$$\zeta \leq \zeta_{\text{sol}}, \quad P \geq P_{\text{atm}}$$

$$(\zeta_{\text{sol}} - \zeta)(P - P_{\text{atm}}) = 0$$

ζ_{sol}

Penalization parameter: λ

$\varepsilon\zeta$

$$u = \begin{pmatrix} \zeta \\ q \end{pmatrix}$$

q

$$\partial_t u + \partial_x f[u] = \mathrm{F}[u, \partial_x(\text{pressure}, \text{ bathymetry}), \ldots)]$$

Fixed body

Beneath the floating solid:
unknown: ζ

Free waves area:
unknown: ζ

Exclusion constraint:

$$\zeta \leq \zeta_{\text{sol}}, \quad P \geq P_{\text{atm}}$$
$$(\zeta_{\text{sol}} - \zeta)(P - P_{\text{atm}}) = 0$$

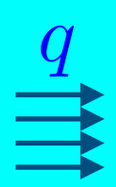
$$P_\lambda - P_{\text{atm}} = \frac{\varepsilon}{\lambda^2}(\zeta_\lambda - \zeta_{\text{sol}})_+$$

ζ_{sol}

Relaxed pressure:
Penalization parameter: λ



$$u = \begin{pmatrix} \zeta \\ q \end{pmatrix}$$



$$\partial_t u + \partial_x f[u] = \mathrm{F}[u, \partial_x(\text{pressure}, \text{ bathymetry}), \ldots)]$$

Fixed body

Beneath the floating solid:
unknown: ζ

Free waves area:
unknown: ζ

Exclusion constraint:

$$\zeta \leq \zeta_{\text{sol}}, \quad P \geq P_{\text{atm}}$$
$$(\zeta_{\text{sol}} - \zeta)(P - P_{\text{atm}}) = 0$$

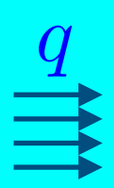
$$P_\lambda - P_{\text{atm}} = \frac{\varepsilon}{\lambda^2}(\zeta_\lambda - \zeta_{\text{sol}})_+$$

ζ_{sol}

Relaxed pressure:
Penalization parameter: λ



$$u = \begin{pmatrix} \zeta \\ q \end{pmatrix}$$



$$\partial_t u_\lambda + \partial_x f[u_\lambda] = \mathrm{F}[u_\lambda, \partial_x(\mathrm{P}_\lambda, \text{bathymetry}), \ ...)]$$

$$\partial_t u + \partial_x f[u] = \mathrm{F}[u, \partial_x(\text{pressure}, \text{ bathymetry}), \ ...)]$$

Fixed body

Beneath the floating solid:
unknown: ζ

Free waves area:
unknown: ζ

Exclusion constraint:

$$\zeta \leq \zeta_{\text{sol}}, \quad P \geq P_{\text{atm}}$$
$$(\zeta_{\text{sol}} - \zeta)(P - P_{\text{atm}}) = 0$$

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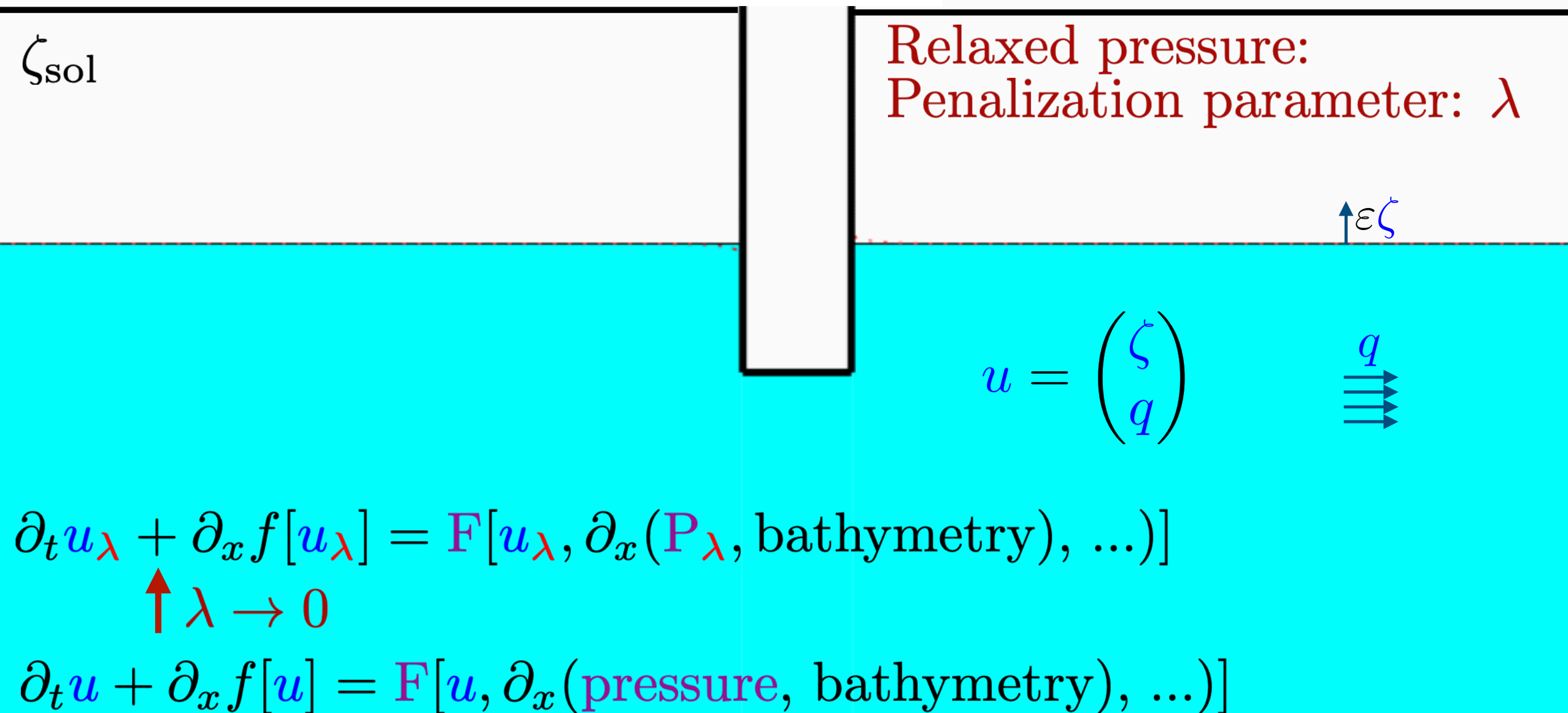
Relaxed pressure:
Penalization parameter: λ



$$\partial_t u_\lambda + \partial_x f[u_\lambda] = \mathrm{F}[u_\lambda, \partial_x(\mathrm{P}_\lambda, \text{bathymetry}), \ ...)]$$
$$\uparrow \lambda \to 0$$
$$\partial_t u + \partial_x f[u] = \mathrm{F}[u, \partial_x(\text{pressure}, \text{ bathymetry}), \ ...)]$$

Fixed body

Beneath the floating solid:
unknown: ζ

Free waves area:
unknown: ζ

Exclusion constraint:

$$\zeta \leq \zeta_{\text{sol}}, \quad P \geq P_{\text{atm}}$$
$$(\zeta_{\text{sol}} - \zeta)(P - P_{\text{atm}}) = 0$$

$$(\zeta_{\text{sol}} - \zeta_\lambda)_+(P_\lambda - P_{\text{atm}}) = 0$$
$$P_\lambda - P_{\text{atm}} = \frac{\varepsilon}{\lambda^2}(\zeta_\lambda - \zeta_{\text{sol}})_+$$

ζ_{sol}

Relaxed pressure:
Penalization parameter: λ

$\varepsilon\zeta$

$$u = \begin{pmatrix} \zeta \\ q \end{pmatrix}$$

q

$$\partial_t u_\lambda + \partial_x f[u_\lambda] = \mathrm{F}[u_\lambda, \partial_x(\mathrm{P}_\lambda, \text{bathymetry}), \ ...)]$$

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Free waves area:
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Relaxed pressure:
Penalization parameter: λ

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$$\partial_t u + \partial_x f[u] = \mathrm{F}[u, \partial_x(\text{pressure}, \text{ bathymetry}), \ ...)]$$

Moving body

Beneath the floating solid:

unknown: ζ ζ_{sol}(solid motion)

Free waves area:

unknown: ζ

Exclusion constraint:

$$\zeta \leq \zeta_{\text{sol}}, \quad P \geq P_{\text{atm}}$$

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$$P_\lambda - P_{\text{atm}} = \frac{\varepsilon}{\lambda^2}(\zeta_\lambda - \zeta_{\text{sol}})_+$$

Relaxed pressure:
Penalization parameter: λ

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$\varepsilon\zeta$

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$\uparrow \lambda \to 0$

$$\partial_t u + \partial_x f[u] = \mathrm{F}[u, \partial_x(\text{pressure}, \text{ bathymetry}), \ ...)]$$

Moving body

Beneath the floating solid:

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Free waves area:

unknown: ζ

Exclusion constraint:

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$\uparrow \lambda \to 0$

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Moving body

Beneath the floating solid:

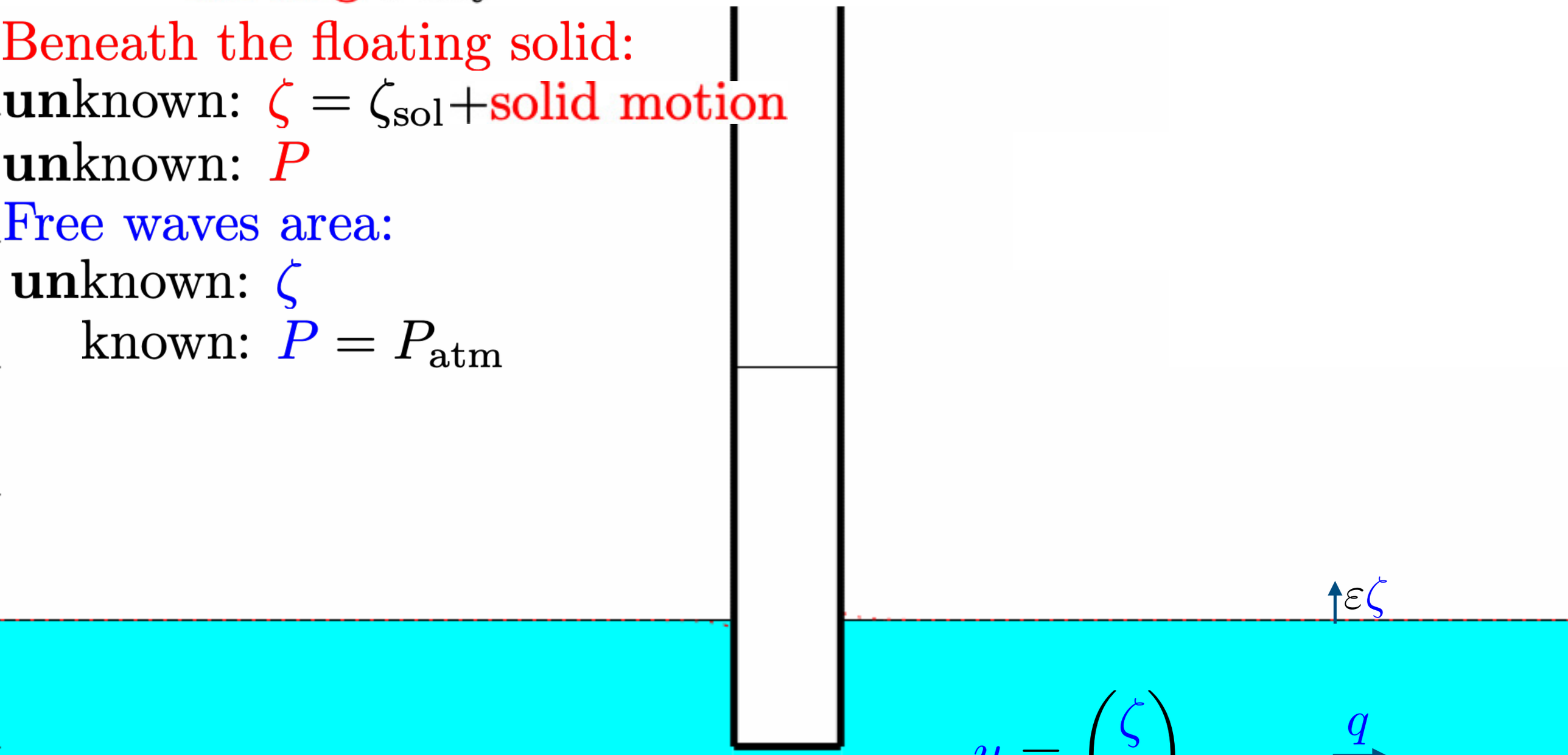
unknown: $\zeta = \zeta_{\text{sol}}$+solid motion

unknown: P

Free waves area:

unknown: ζ

known: $P = P_{\text{atm}}$



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Moving body

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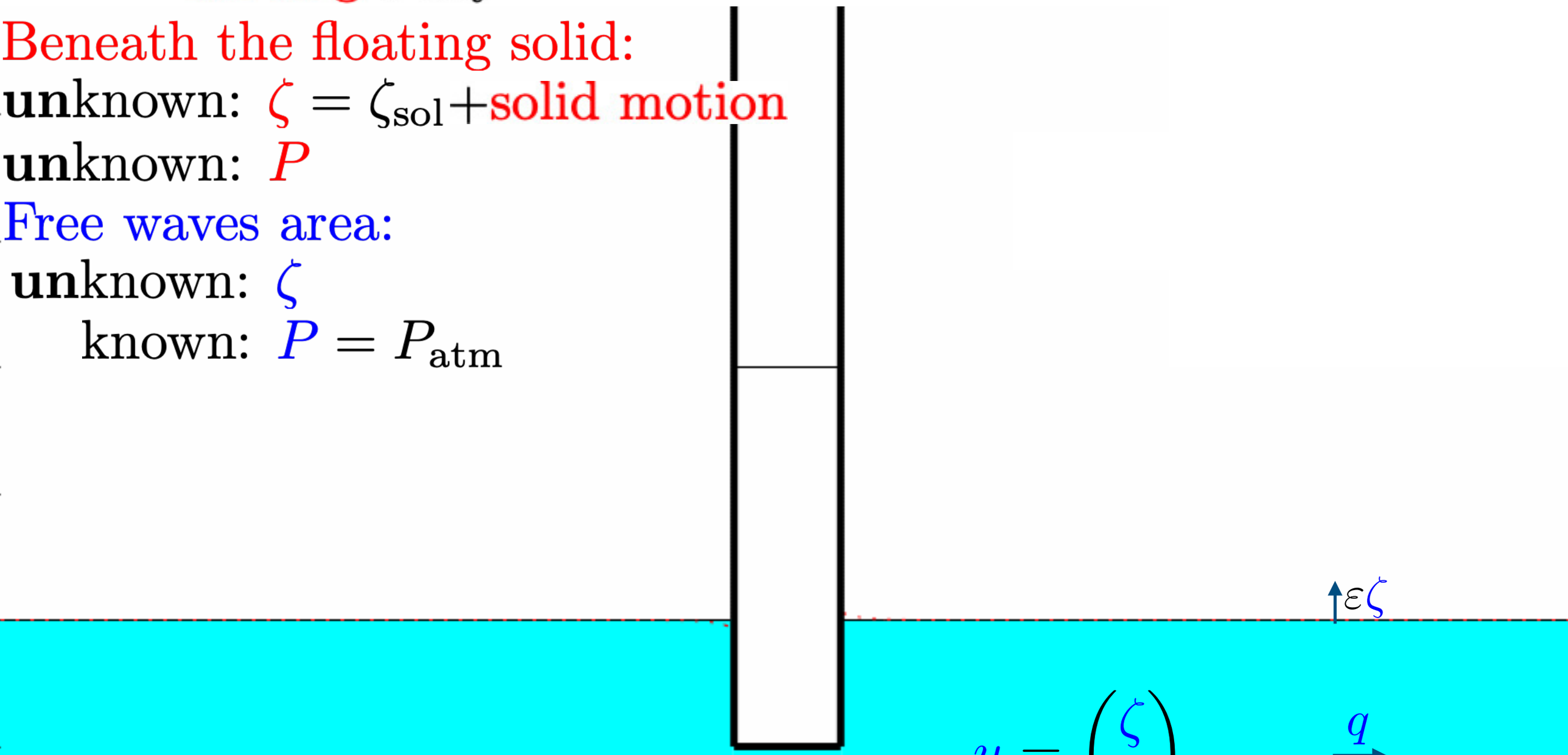


$$\partial_t u + \partial_x f[u] = \mathrm{F}[u, \partial_x(\text{pressure}, \text{ bathymetry}), \ldots)]$$

Moving body

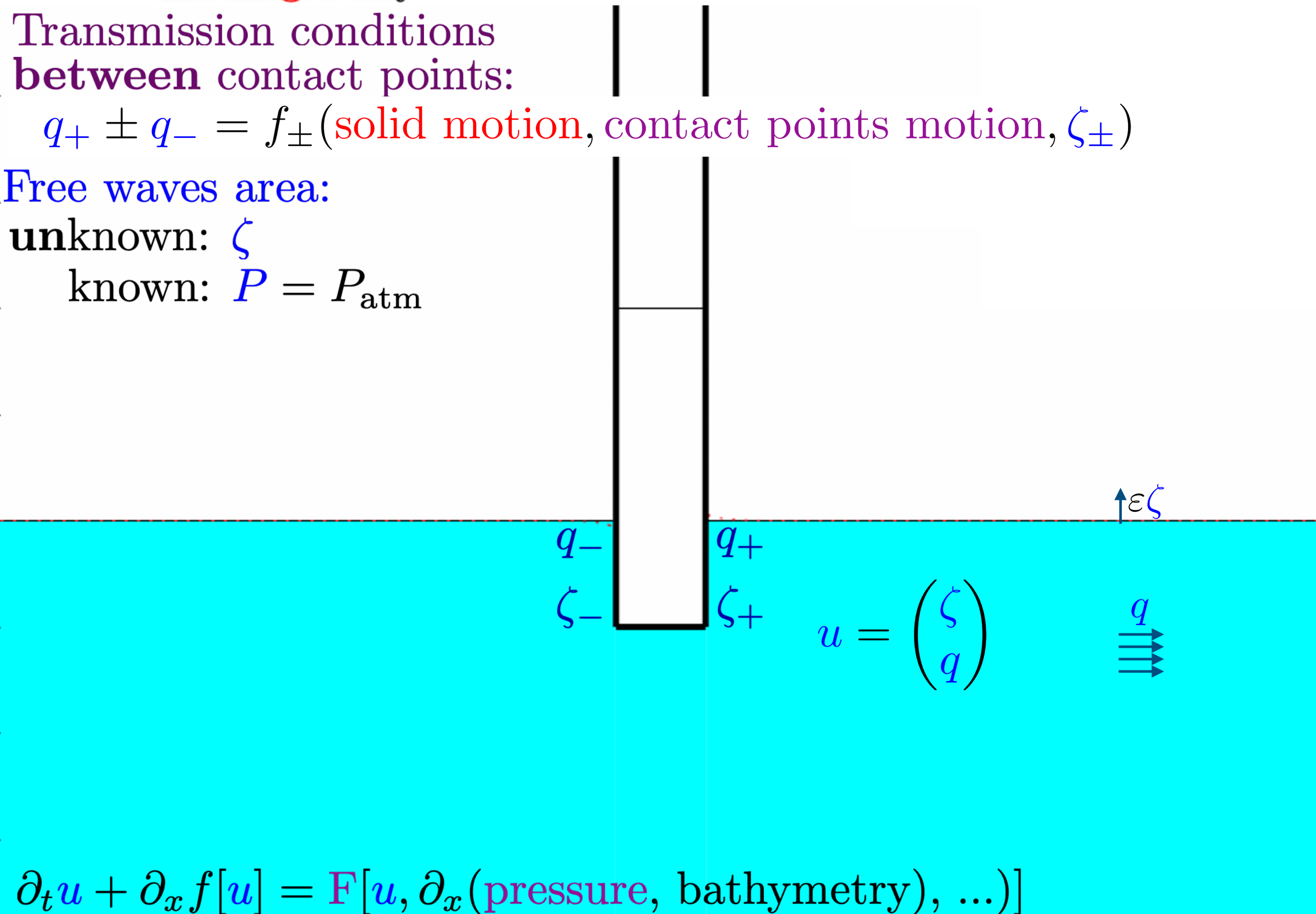
Transmission conditions
between contact points:

$$q_+ \pm q_- = f_\pm(\text{solid motion}, \text{contact points motion}, \zeta_\pm)$$

Free waves area:

unknown: ζ

known: $P = P_{\text{atm}}$



$$\partial_t u + \partial_x f[u] = \mathrm{F}[u, \partial_x(\text{pressure}, \text{ bathymetry}), \ldots)]$$

Moving body

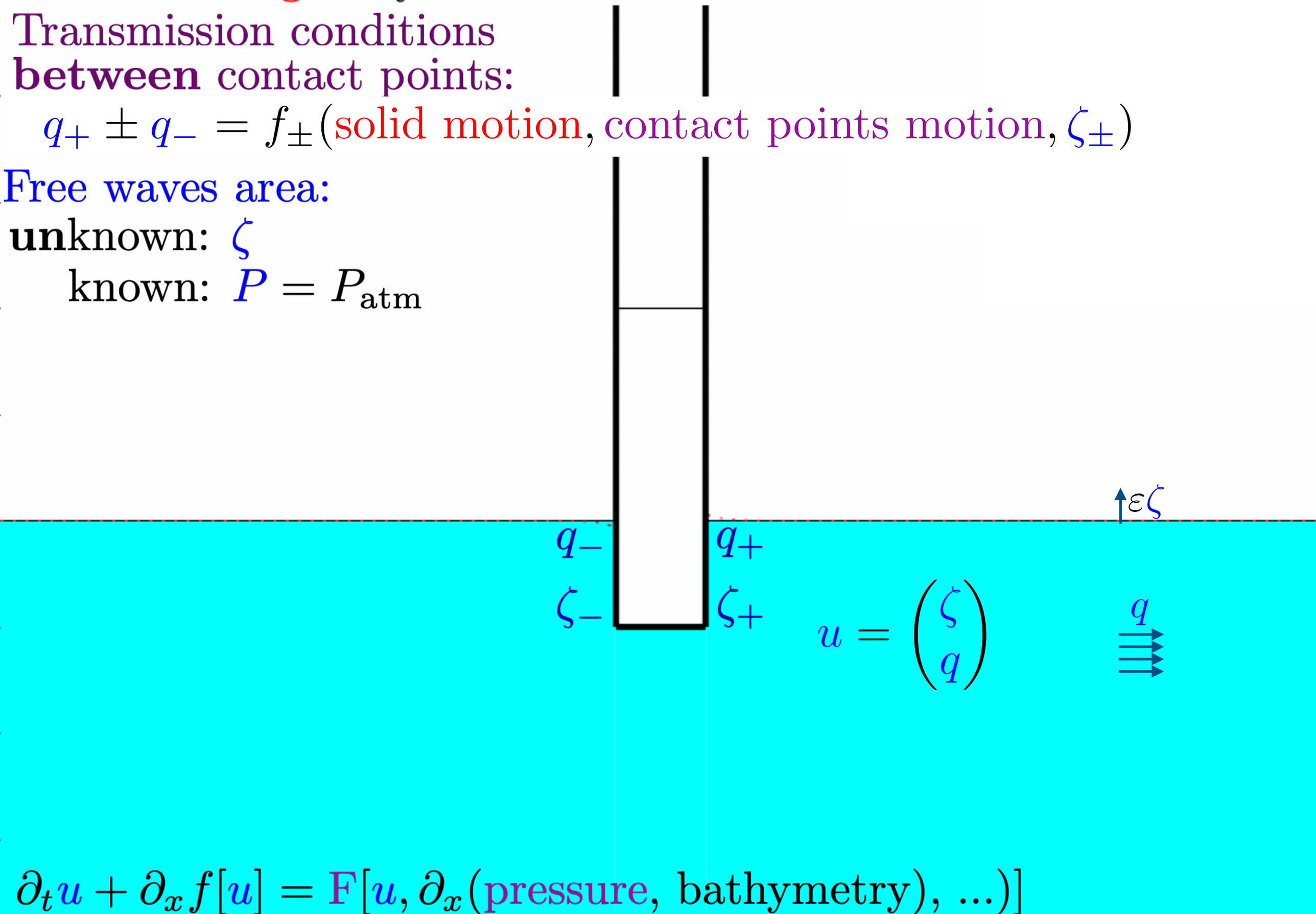
Transmission conditions
between contact points:

$$q_+ \pm q_- = f_\pm(\text{solid motion}, \text{contact points motion}, \zeta_\pm)$$

Free waves area:

unknown: ζ

known: $P = P_{\text{atm}}$



$$\partial_t u + \partial_x f[u] = \mathrm{F}[u, \partial_x(\text{pressure}, \text{ bathymetry}), \ldots)]$$

Moving body

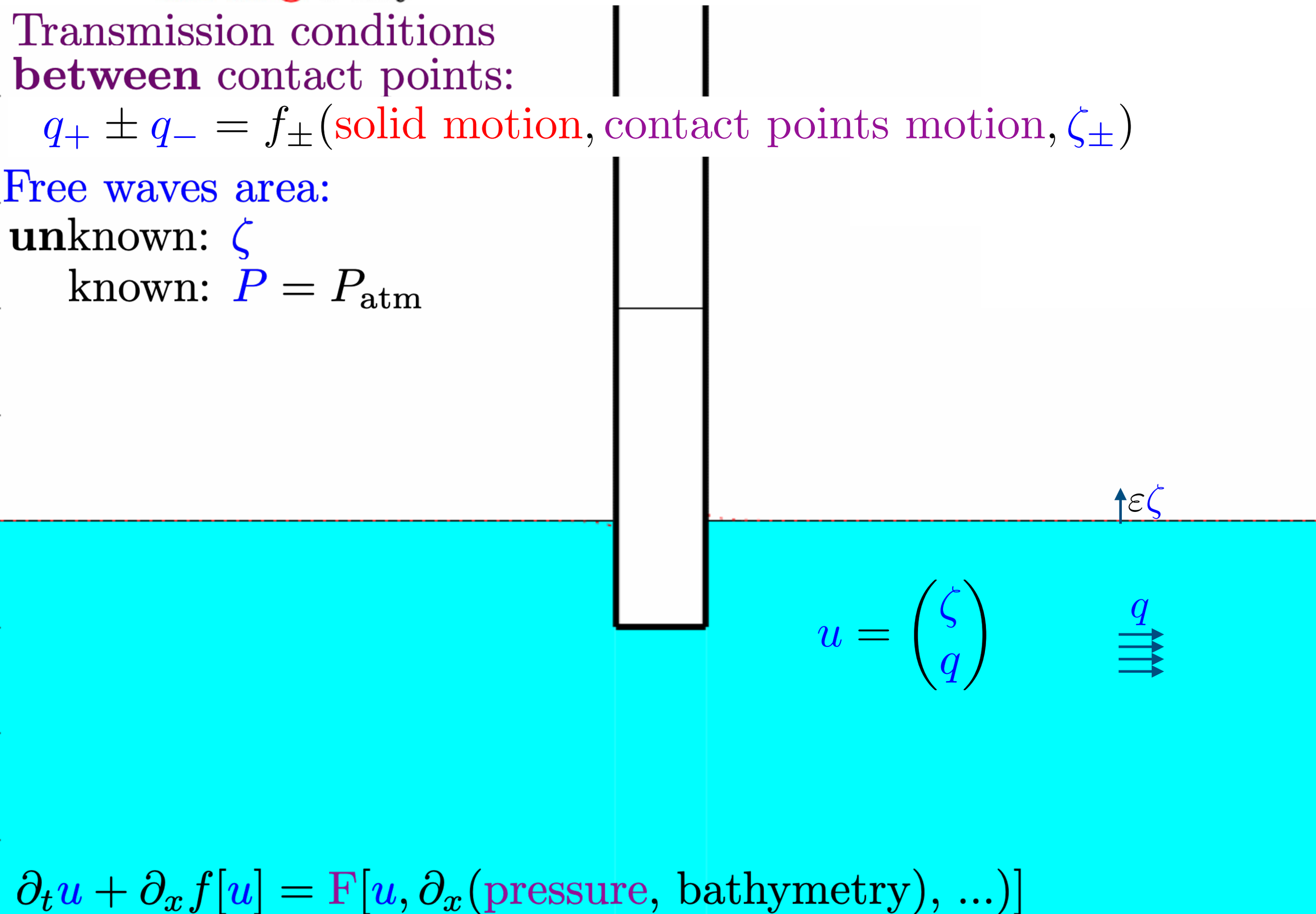
Transmission conditions
between contact points:

$$q_+ \pm q_- = f_\pm(\text{solid motion}, \text{contact points motion}, \zeta_\pm)$$

Free waves area:

unknown: ζ

known: $P = P_{\text{atm}}$



$$\partial_t u + \partial_x f[u] = \mathrm{F}[u, \partial_x(\text{pressure}, \text{ bathymetry}), \ldots)]$$

Moving body

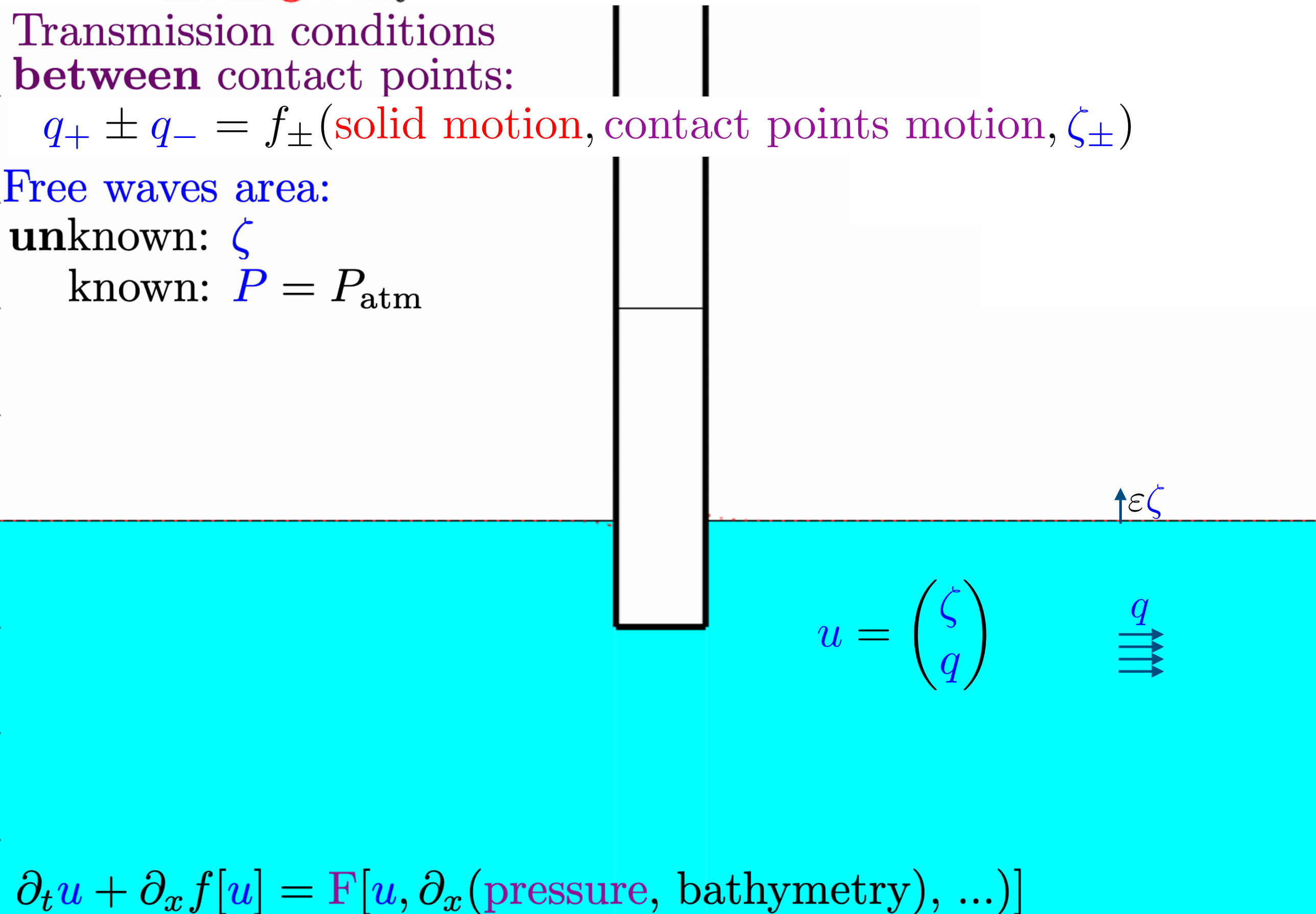
Transmission conditions
between contact points:

$$q_+ \pm q_- = f_\pm(\text{solid motion}, \text{contact points motion}, \zeta_\pm)$$

Free waves area:

unknown: ζ

known: $P = P_{\text{atm}}$



$$\partial_t u + \partial_x f[u] = \mathrm{F}[u, \partial_x(\text{pressure}, \text{ bathymetry}), \ldots)]$$

Moving body

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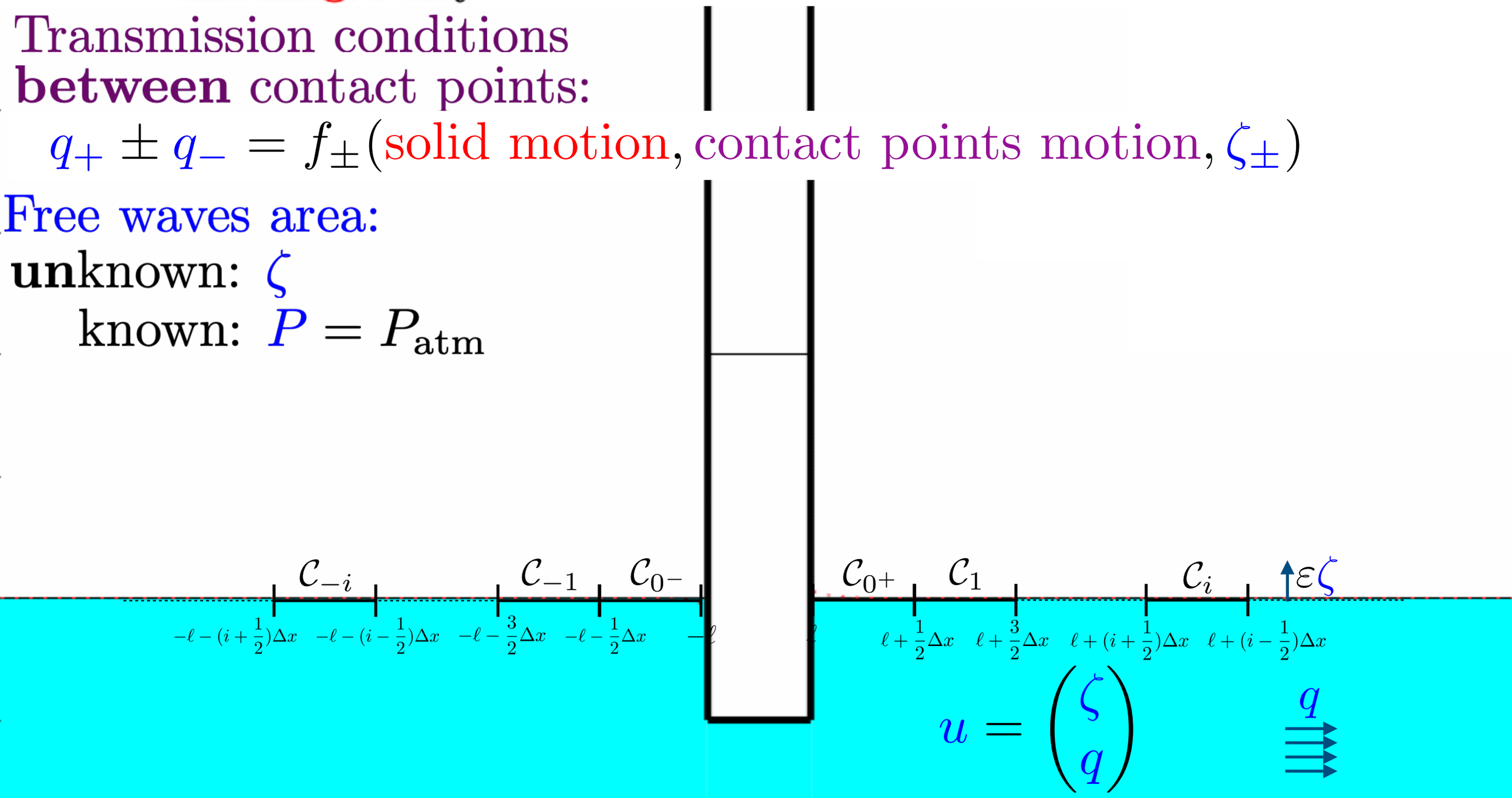

$$u = \begin{pmatrix} \zeta \\ q \end{pmatrix}$$

q

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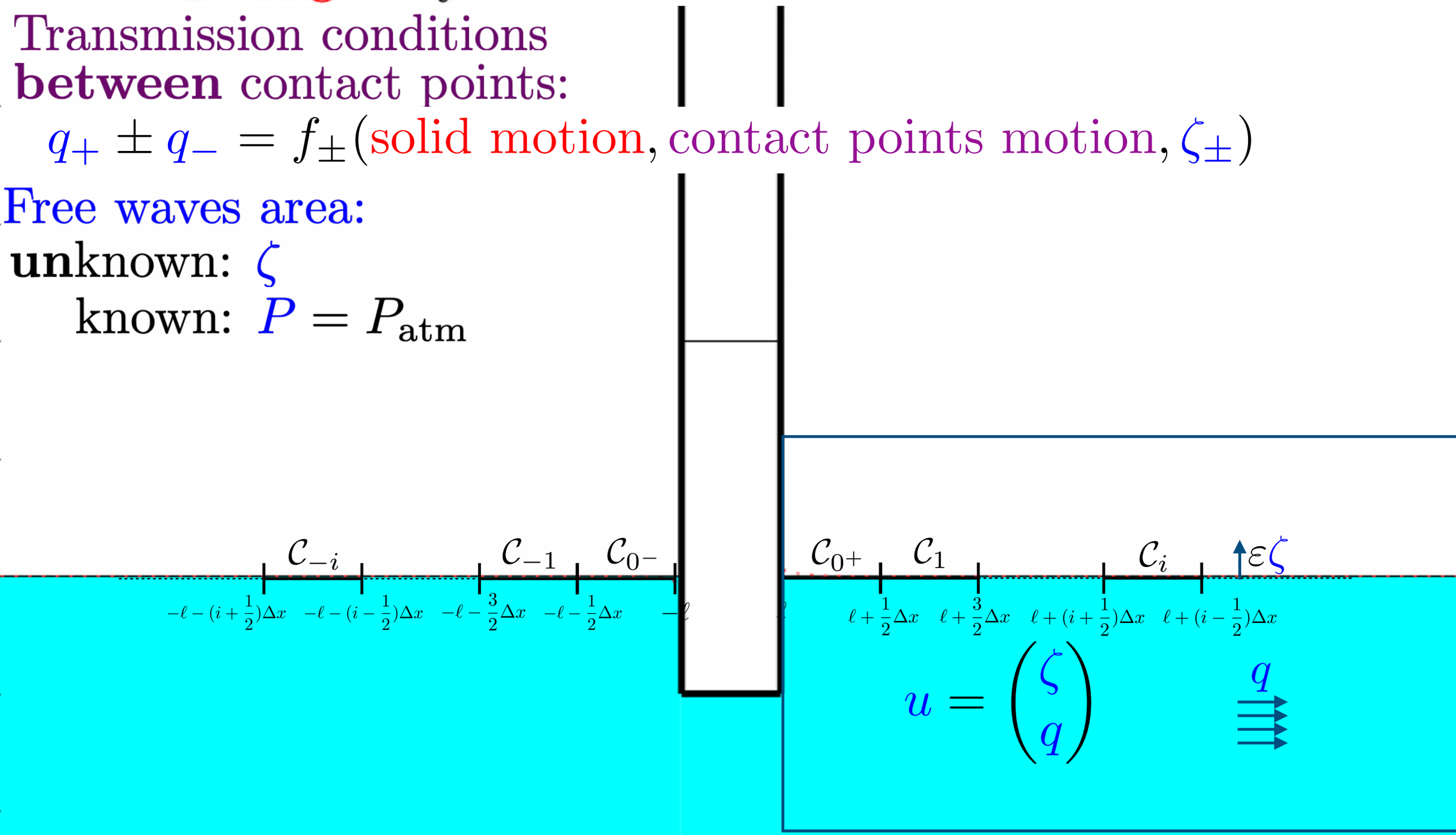

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$\mathcal{C}_{-i}$ $\mathcal{C}_{-1}$ $\mathcal{C}_{0-}$ $\mathcal{C}_{0+}$ $\mathcal{C}_1$ $\mathcal{C}_i$ $\varepsilon\zeta$

$-\ell-(i+\frac{1}{2})\Delta x$ $-\ell-(i-\frac{1}{2})\Delta x$ $-\ell-\frac{3}{2}\Delta x$ $-\ell-\frac{1}{2}\Delta x$ $-\ell$ ℓ $\ell+\frac{1}{2}\Delta x$ $\ell+\frac{3}{2}\Delta x$ $\ell+(i+\frac{1}{2})\Delta x$ $\ell+(i-\frac{1}{2})\Delta x$

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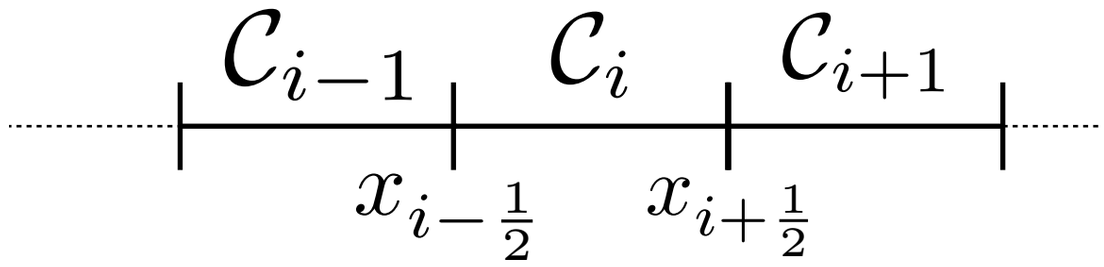
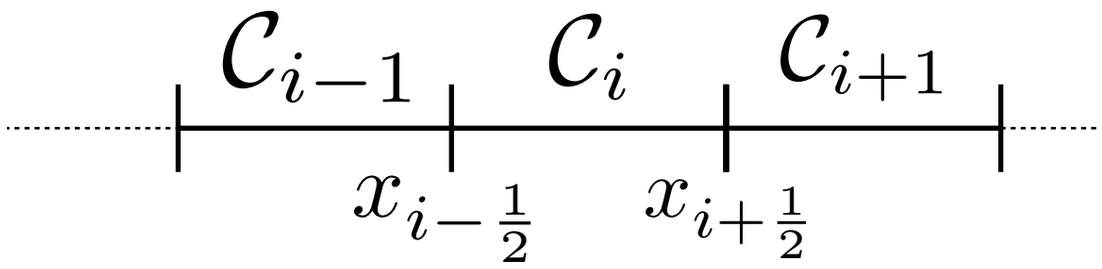


$$\partial_t u + \partial_x f[u] = 0$$

$$u_i^n := \frac{1}{\Delta x}\int_{x_{i-\frac{1}{2}}}^{x_{i+\frac{1}{2}}} u(n\Delta t, x)dx$$



$$\partial_t u + \partial_x f[u] = 0$$

$$\frac{1}{\Delta x \Delta t}\int_{x_{i-\frac{1}{2}}}^{x_{i+\frac{1}{2}}}\int_{n\Delta t}^{(n+1)\Delta t}$$

$$\frac{u_i^{n+1} - u_i^n}{\Delta t} + \frac{1}{\Delta t}\int_{n\Delta t}^{(n+1)\Delta t} \frac{f[u]_{|_{x_{i+\frac{1}{2}}}} - f[u]_{|_{x_{i-\frac{1}{2}}}}}{\Delta x} = 0$$

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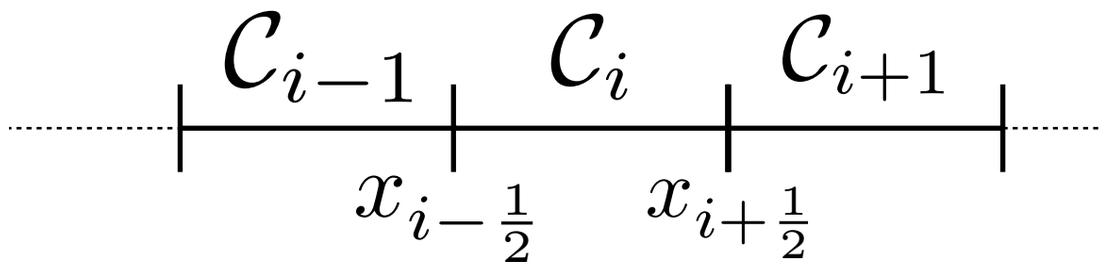

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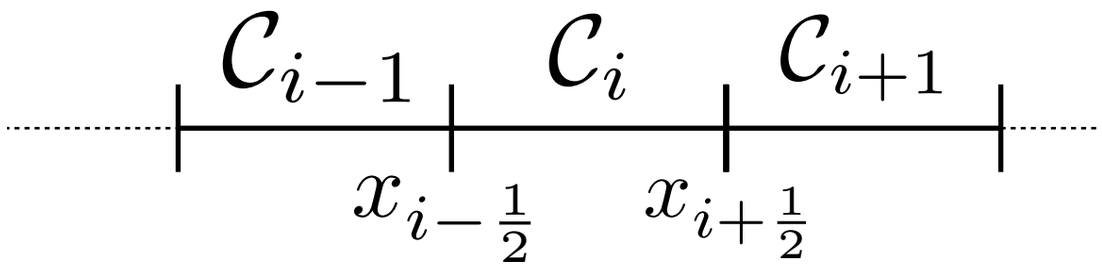

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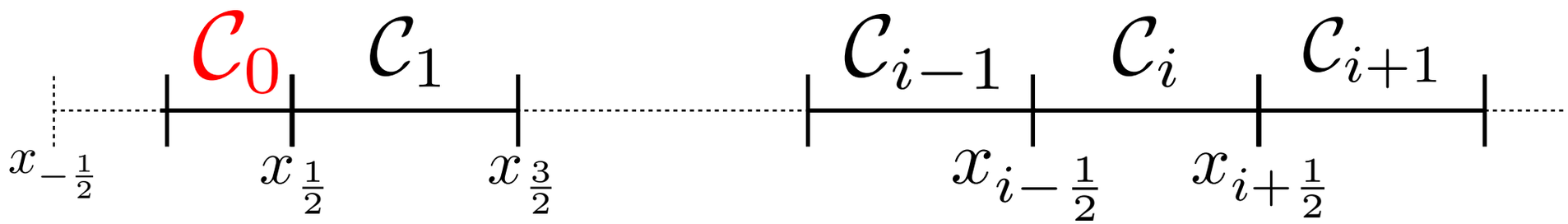

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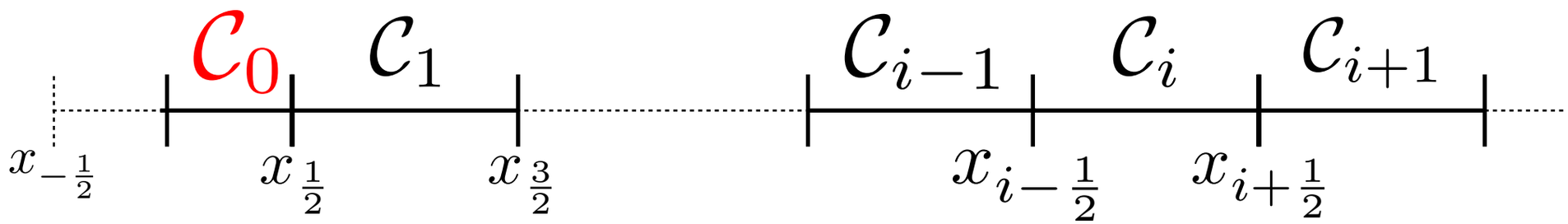

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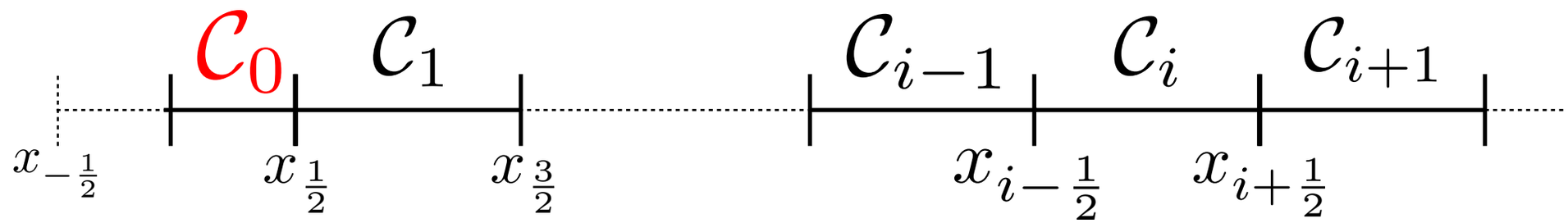

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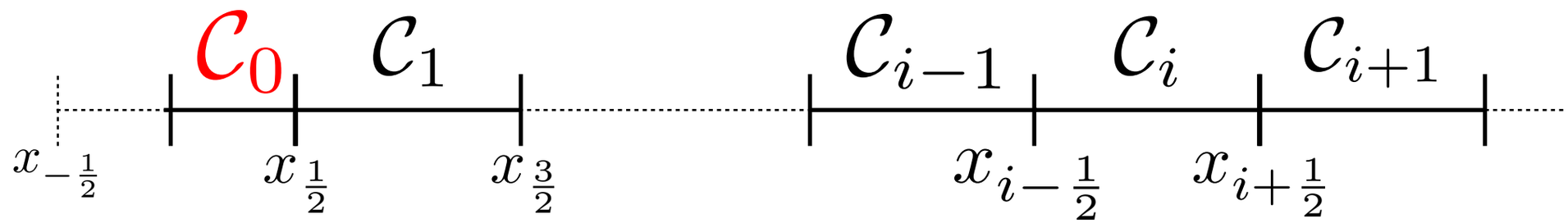

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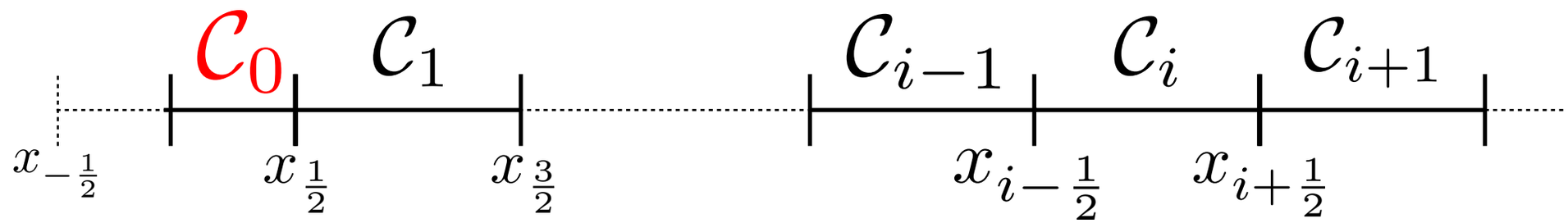

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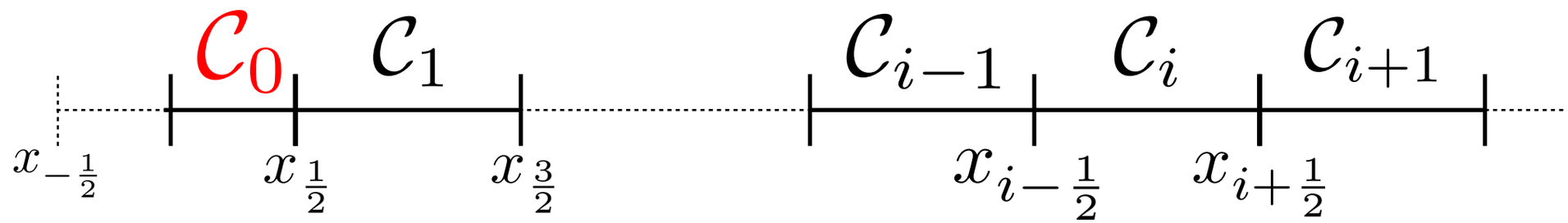

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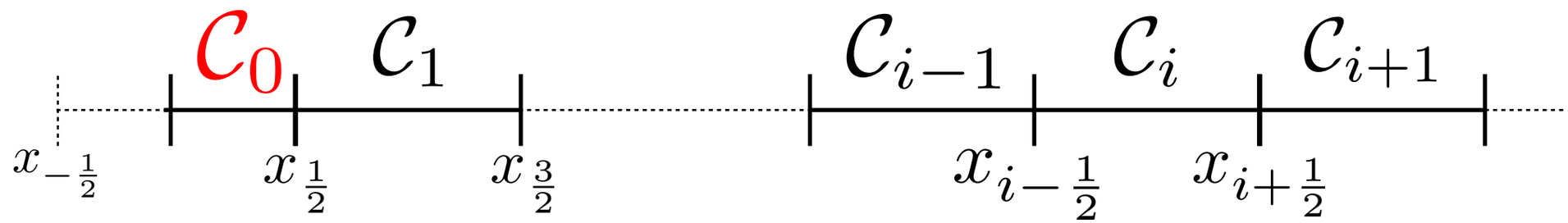

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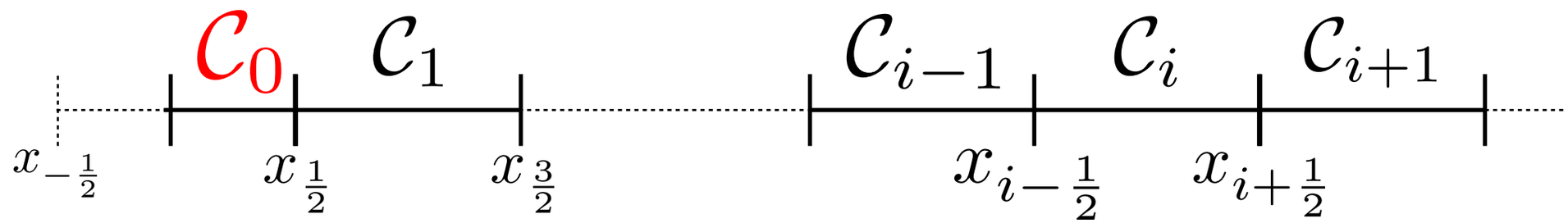

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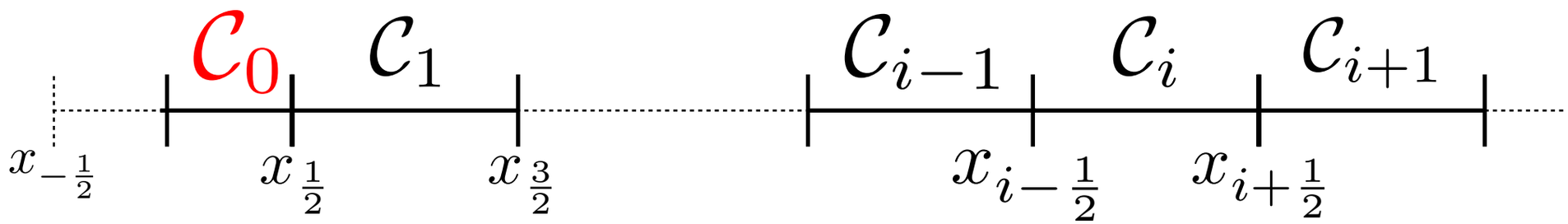

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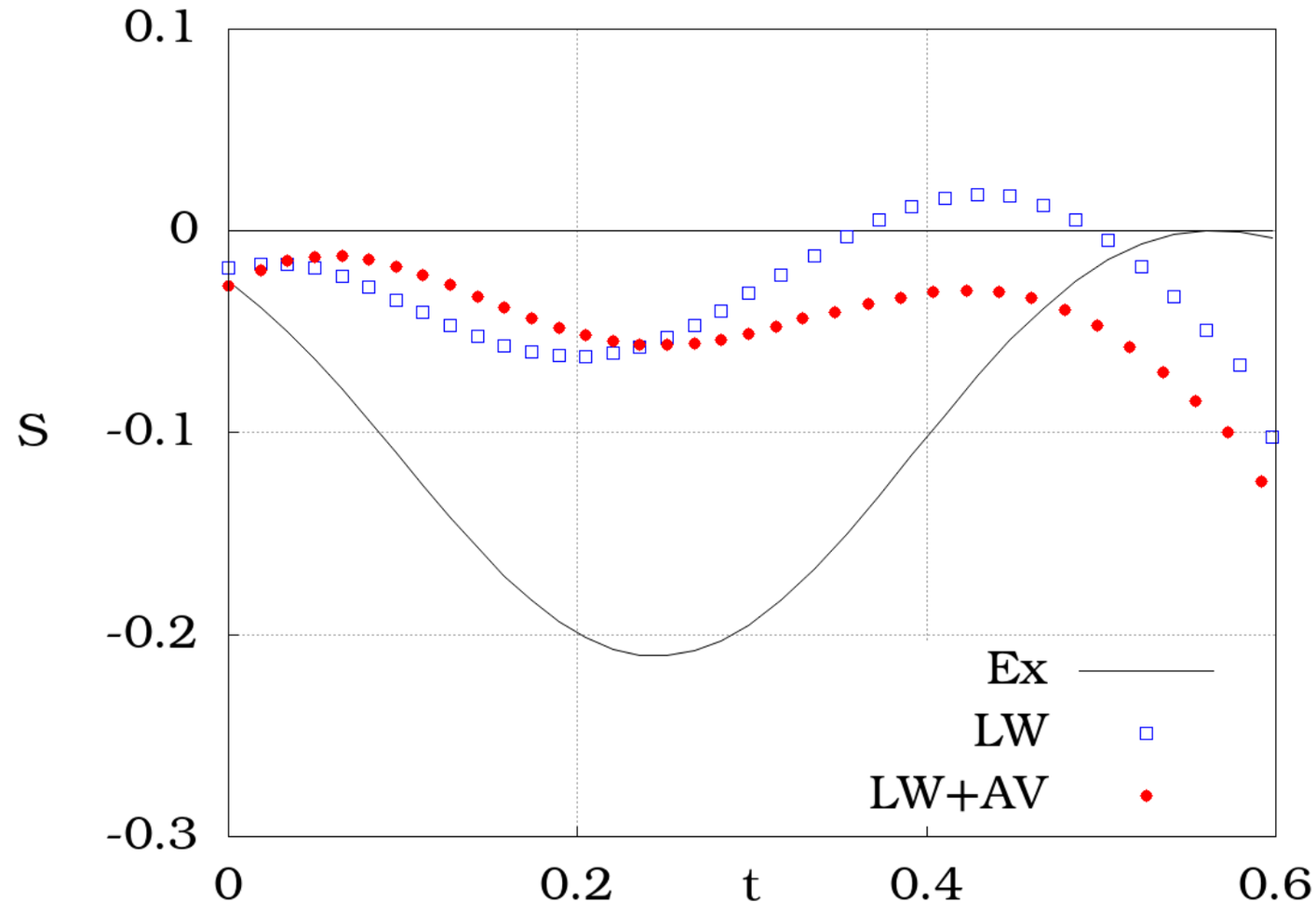

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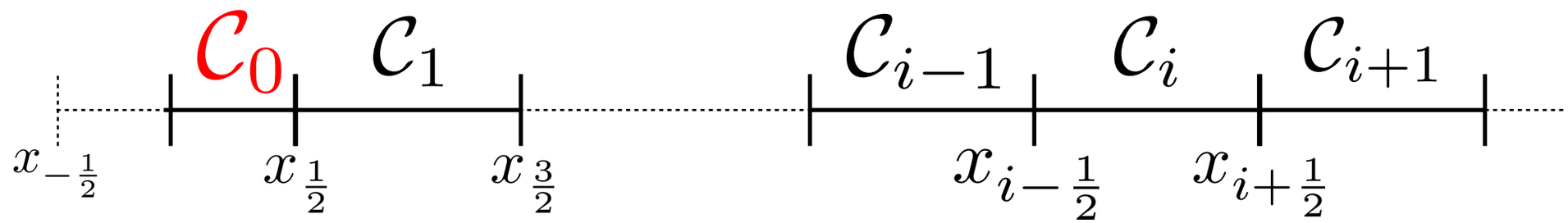

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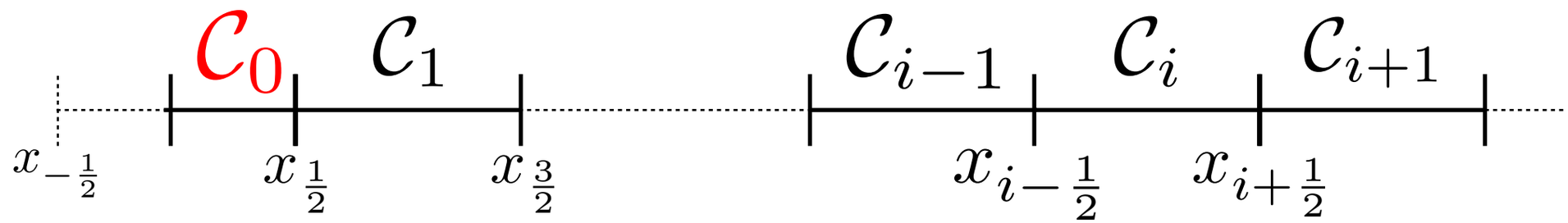

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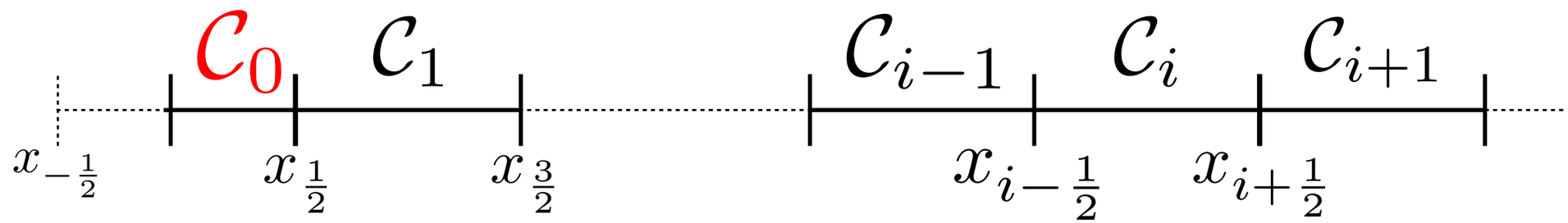

$$u = \begin{pmatrix}\zeta \\ q\end{pmatrix} \quad \begin{matrix}\zeta_{|x=0} = \text{unknown} \\ q_{|x=0} = \text{given}\end{matrix}$$

Boundary layer

$$\partial_t u + \partial_x f[u] - \Delta x \partial_x \left(\frac{\lambda[u]}{2}\partial_x u\right) = 0$$

$$\mathcal{F}[u_i^n, u_{i+1}^n] =: -\frac{\lambda}{2}(u_{i+1}^n - u_i^n) + \mathfrak{F}[u_i^n, u_{i+1}^n] \approx \frac{1}{\Delta t}\int_{n\Delta t}^{(n+1)\Delta t} f[u]_{|x_{i+\frac{1}{2}}}$$

ζ_0^n =????

$q_0^n = q_{|x=0}(n\Delta t)$

$$\frac{u_i^{n+1} - u_i^n}{\Delta t} + \frac{\mathcal{F}[u_i^n, u_{i+1}^n] - \mathcal{F}[u_{i-1}^n, u_i^n]}{\Delta x} = 0$$

$$\frac{\varphi_-[\zeta_0^{n+1}, q_{|x=0}((n+1)\Delta t)] - \varphi_-[u_0^n]}{\Delta t} - \lambda_{\text{R.I}}^-[u_0^n]\frac{\varphi_-[u_1^n] - \varphi_-[u_0^n]}{\Delta x} = 0$$

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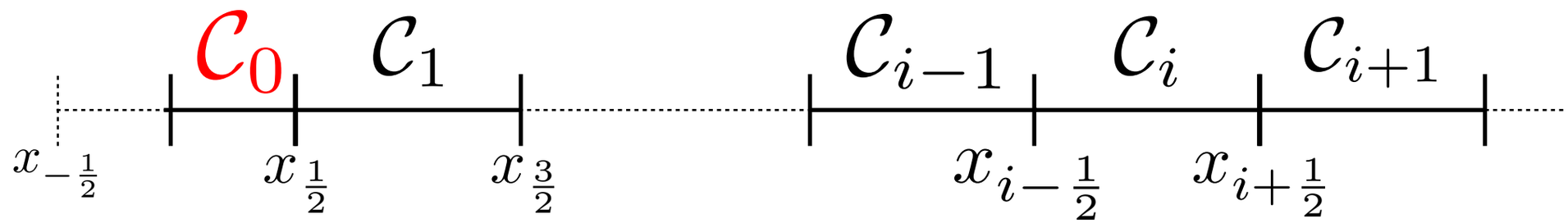

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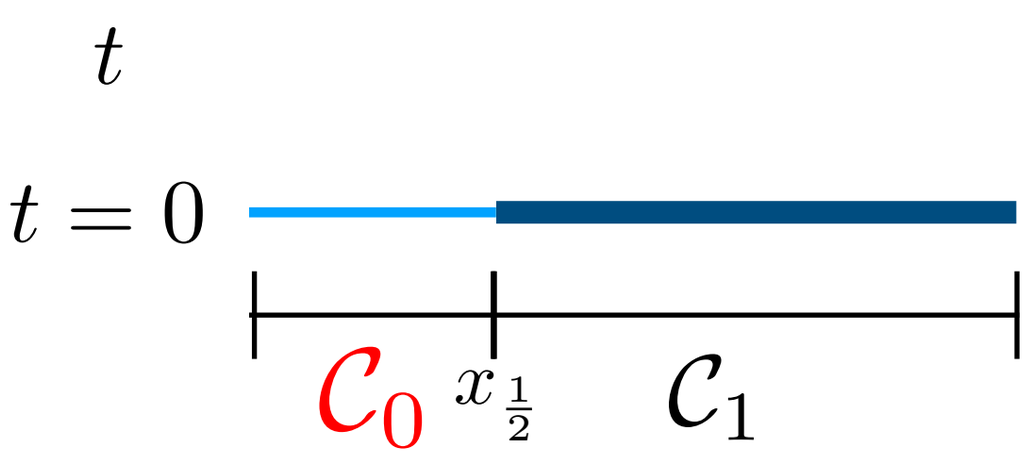

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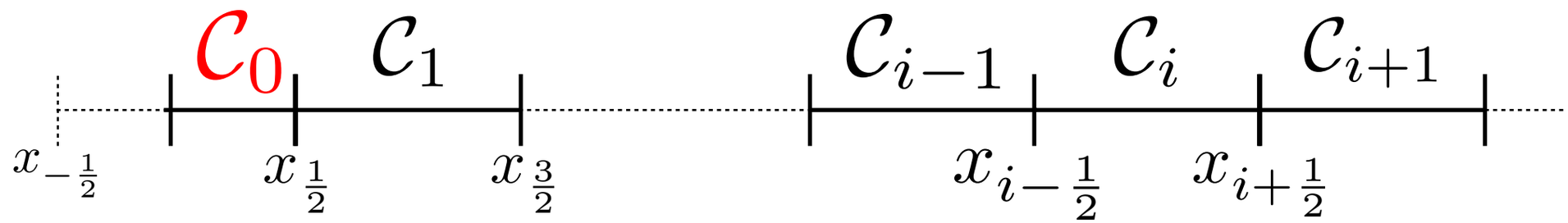

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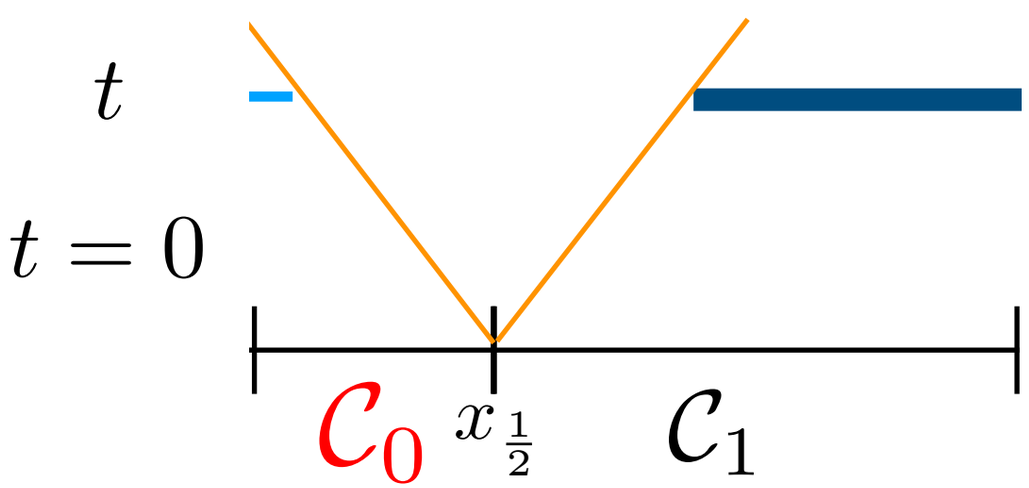

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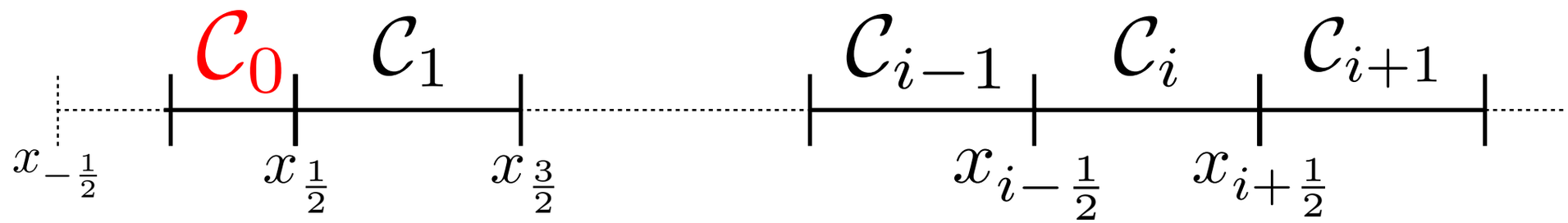

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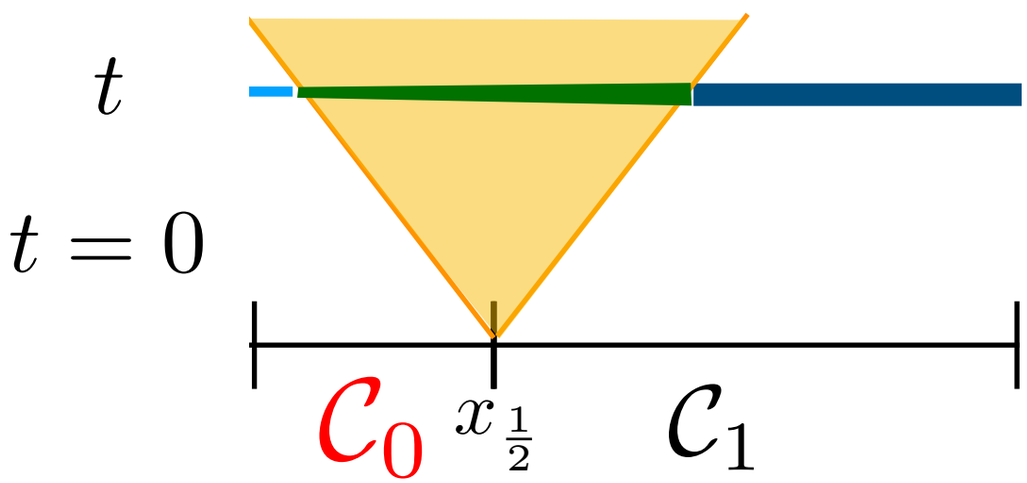

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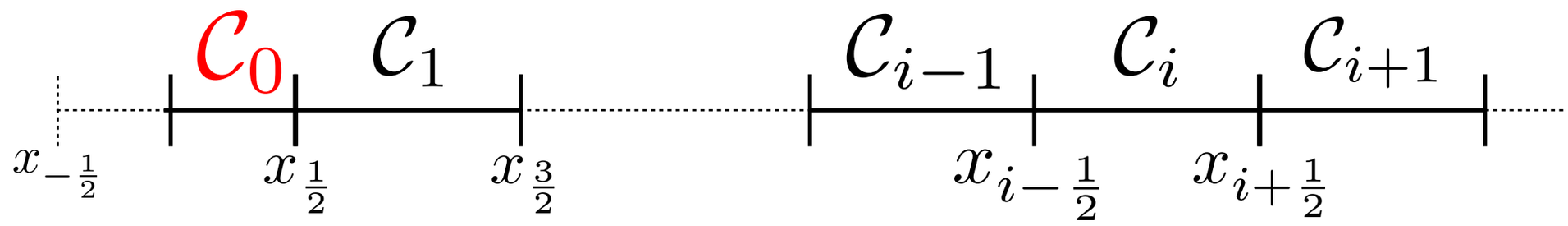

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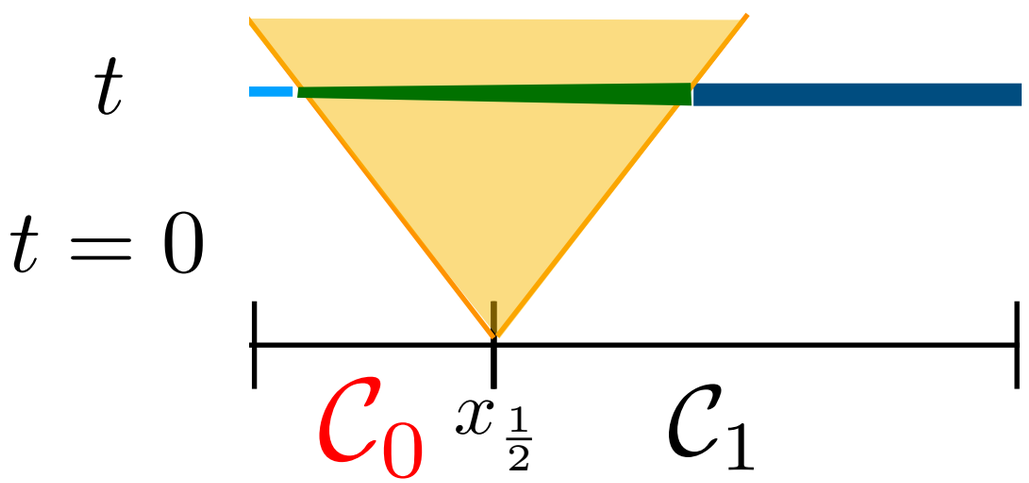

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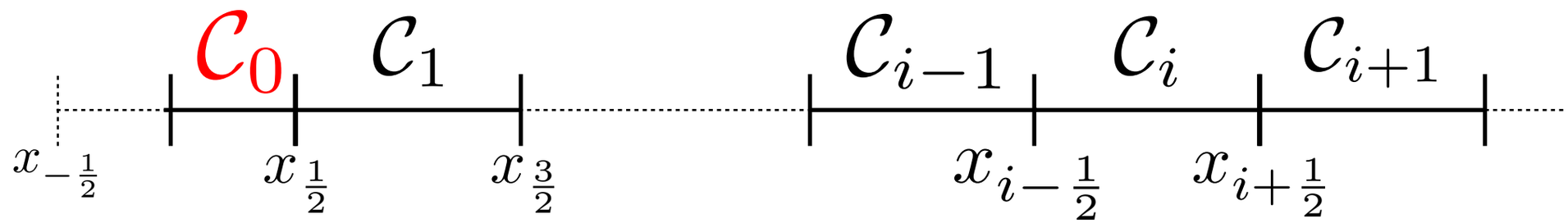

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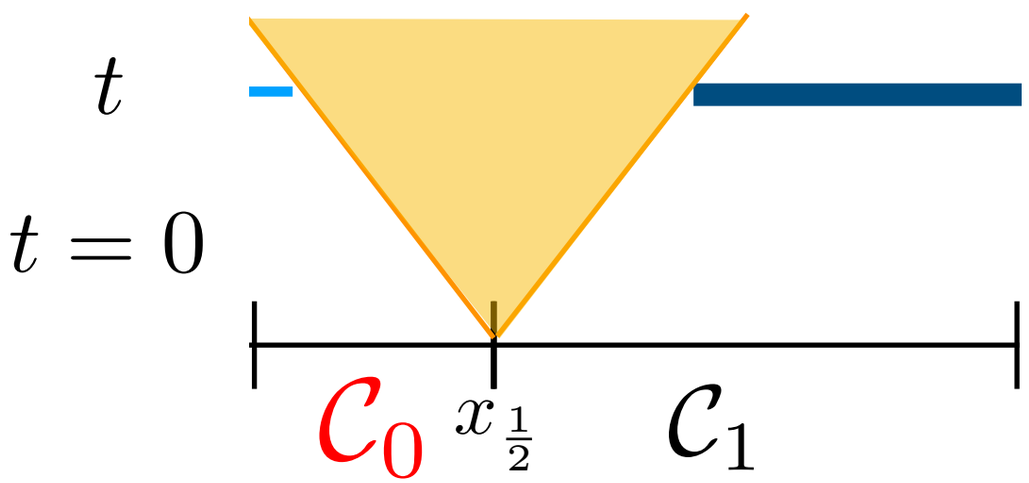

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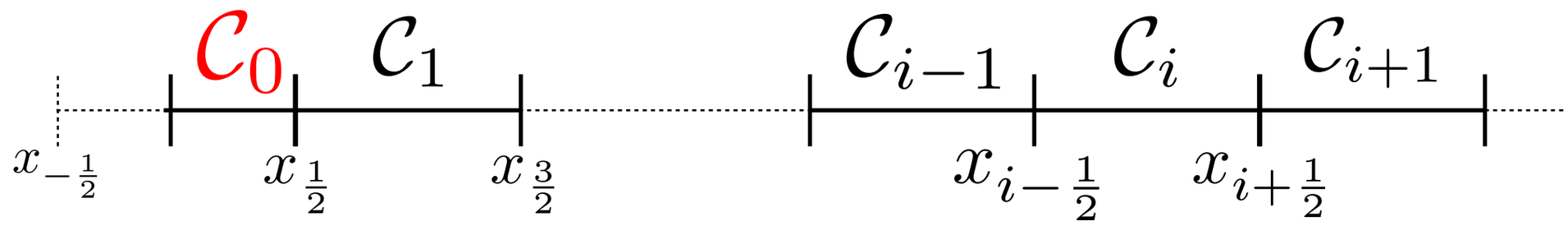

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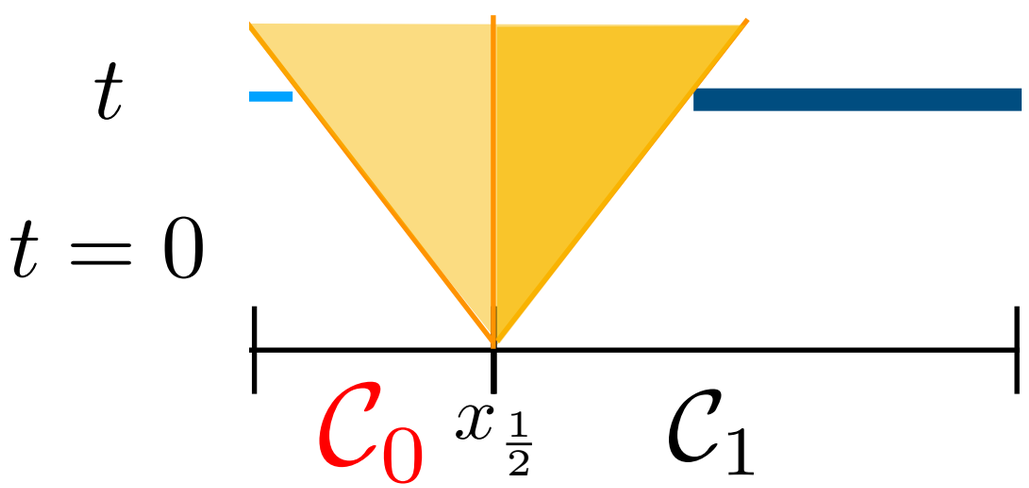

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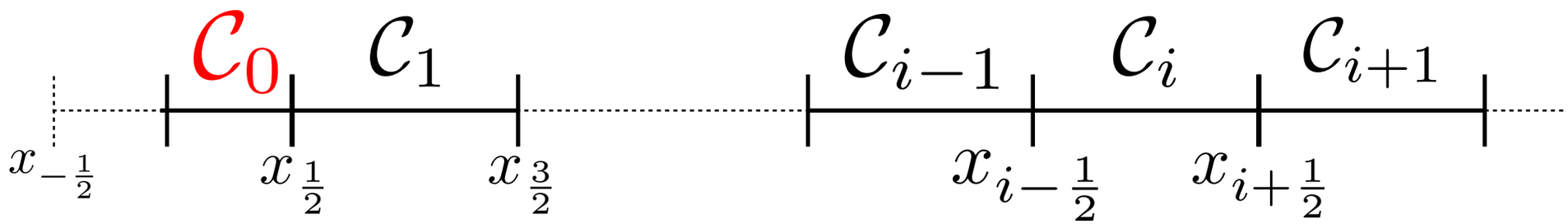

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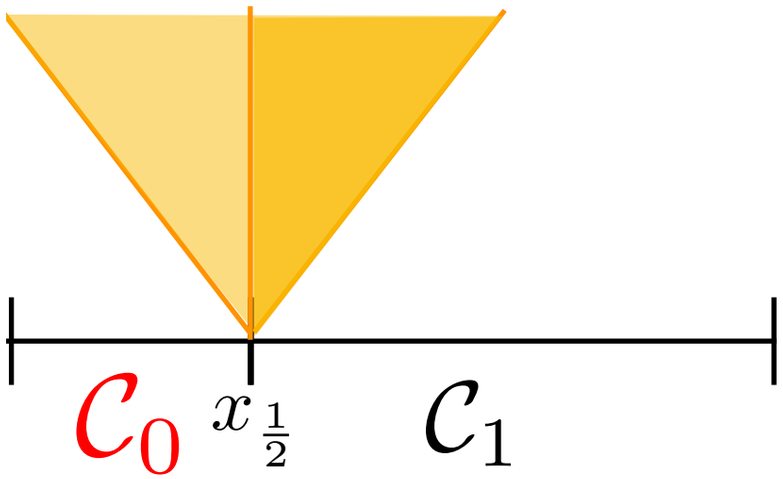

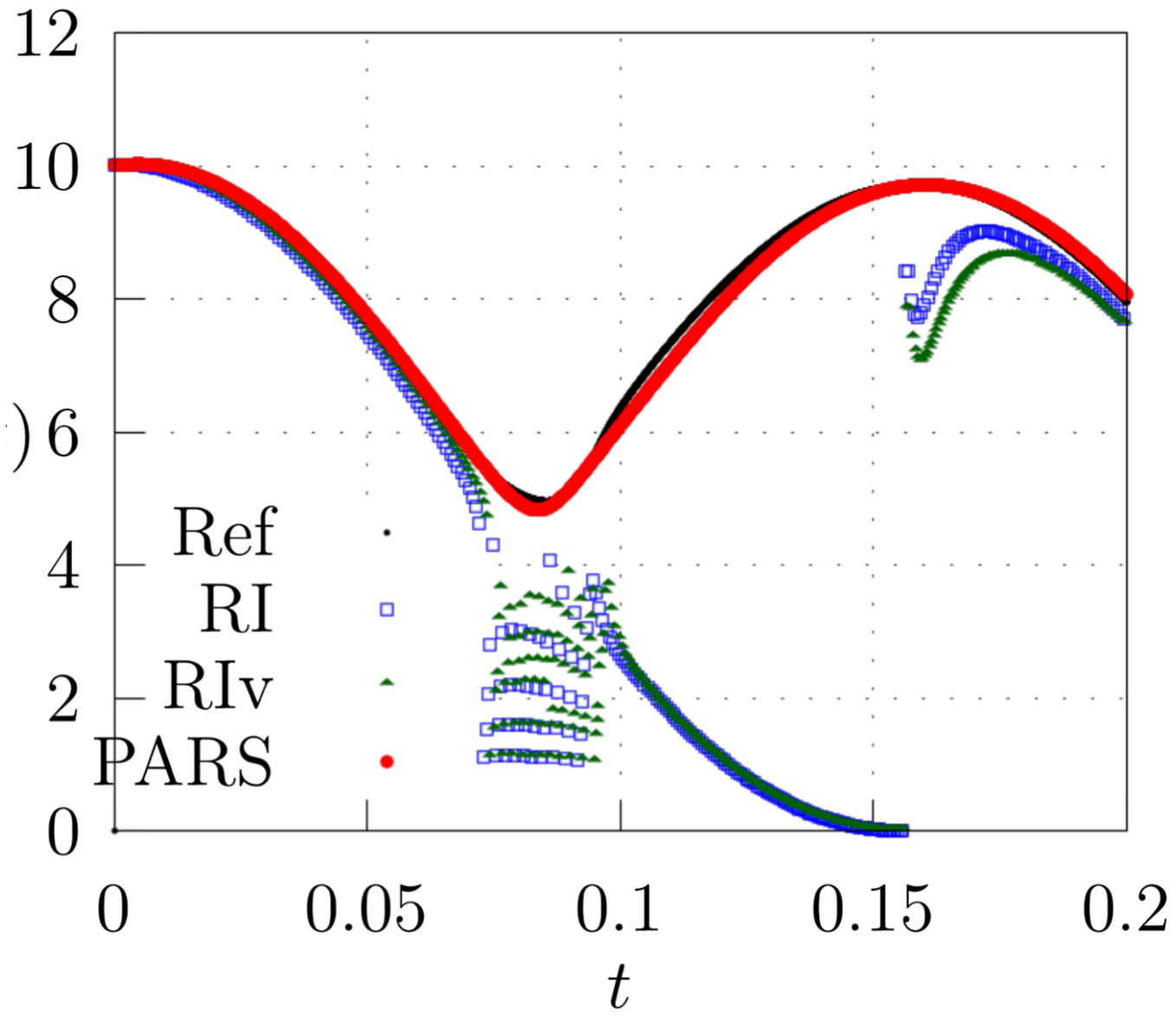

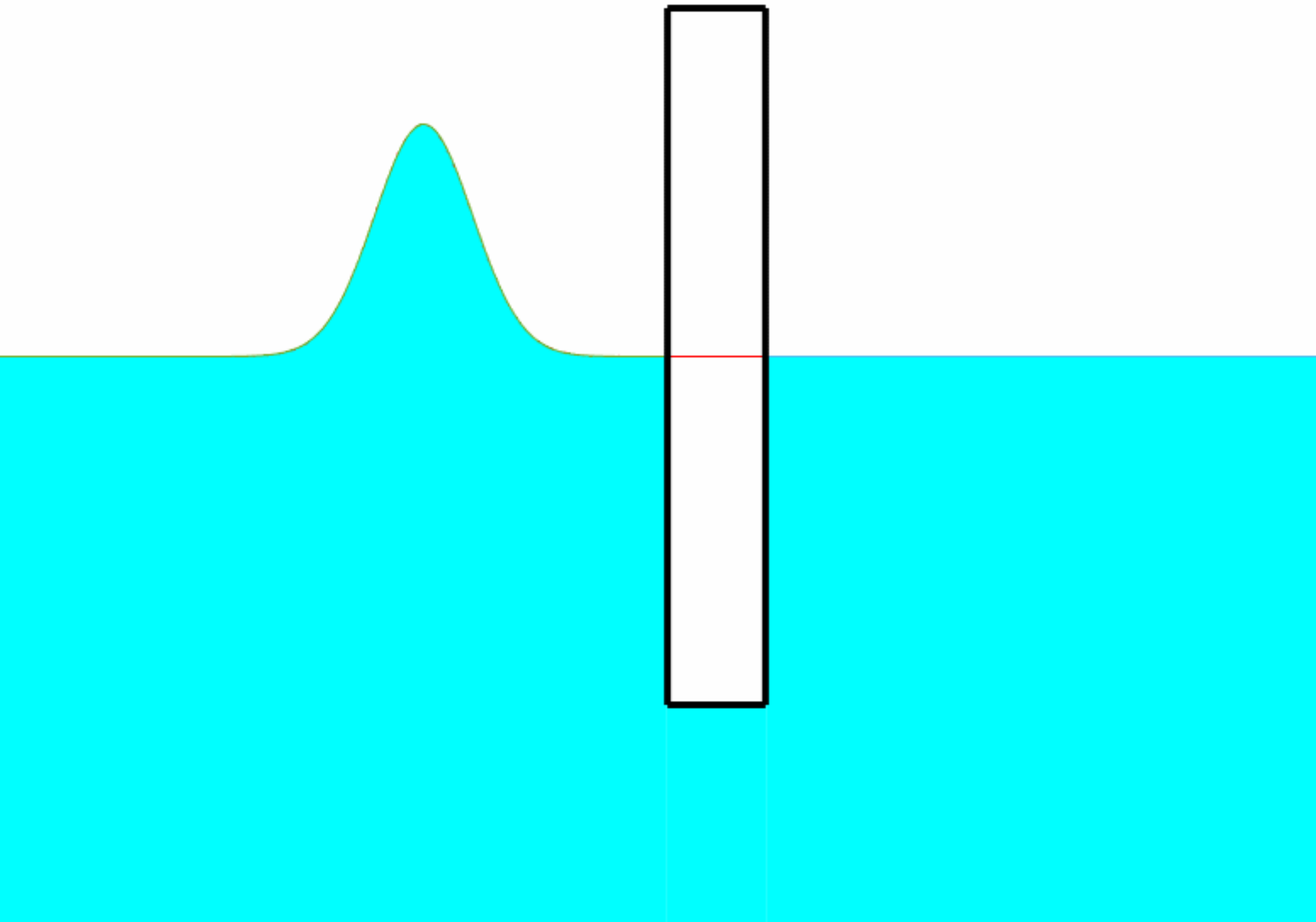

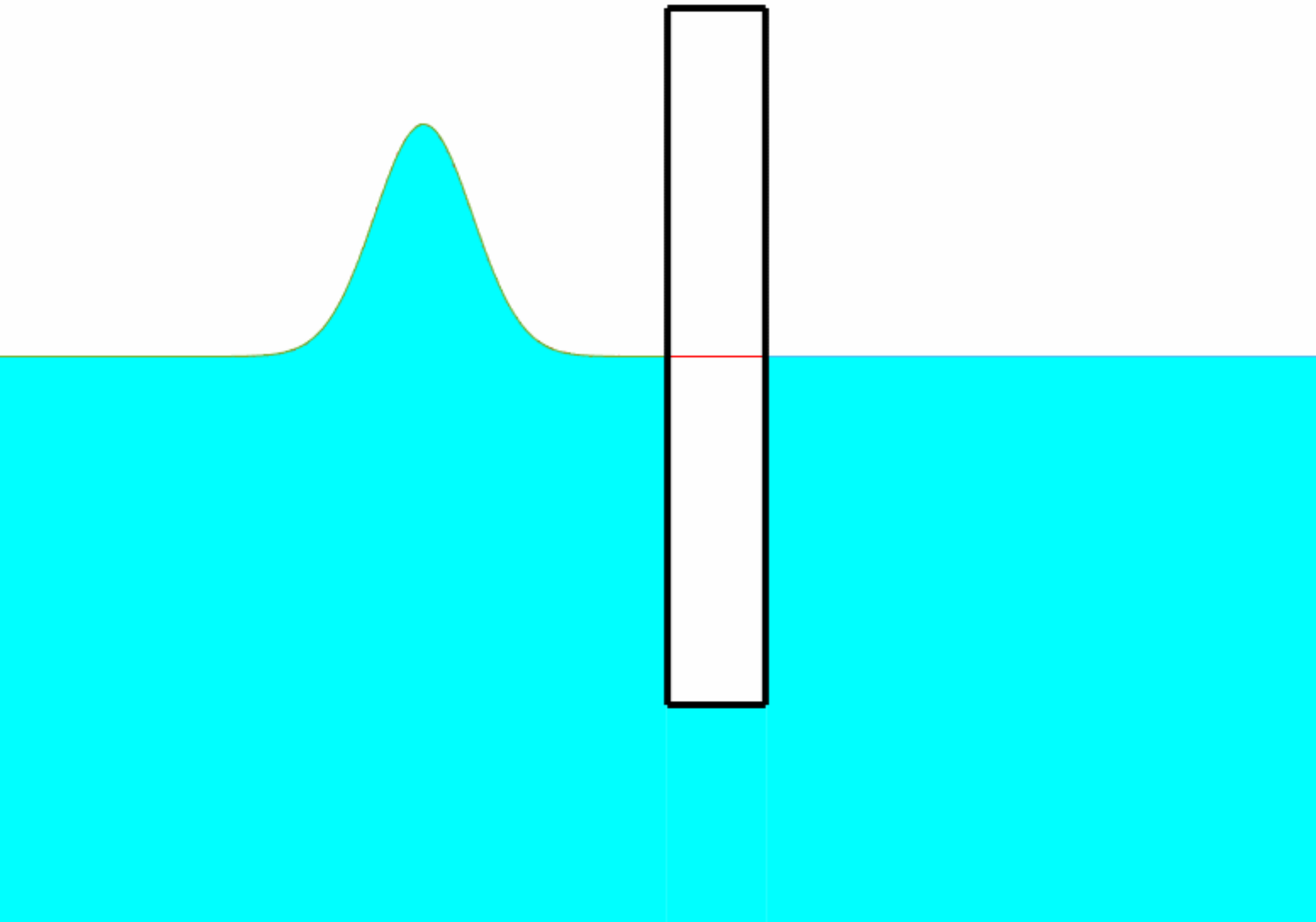

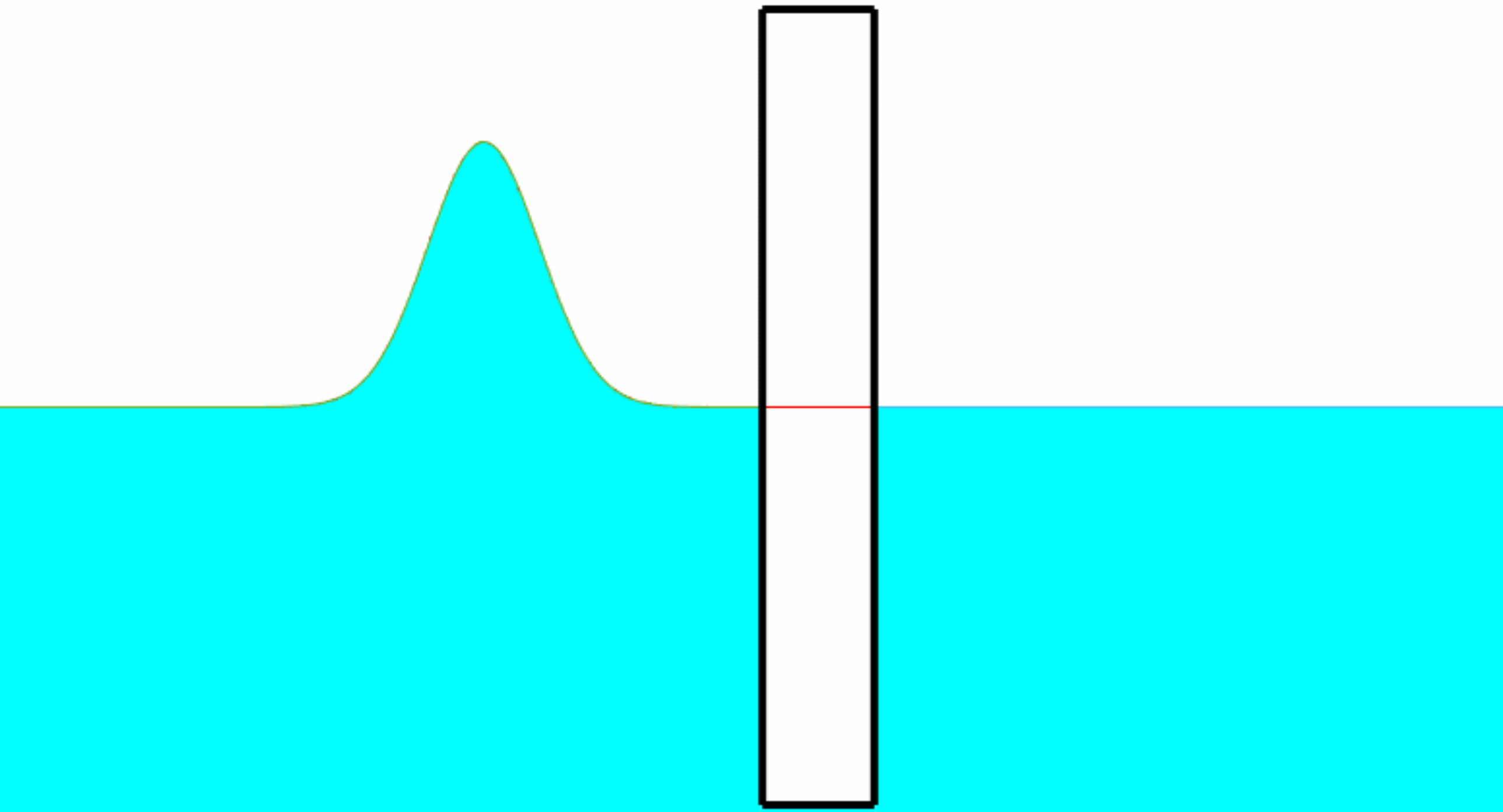

$$(1 - K\partial_x^2)\partial_t u + \partial_x f[u] = \mathrm{F}$$

Inria

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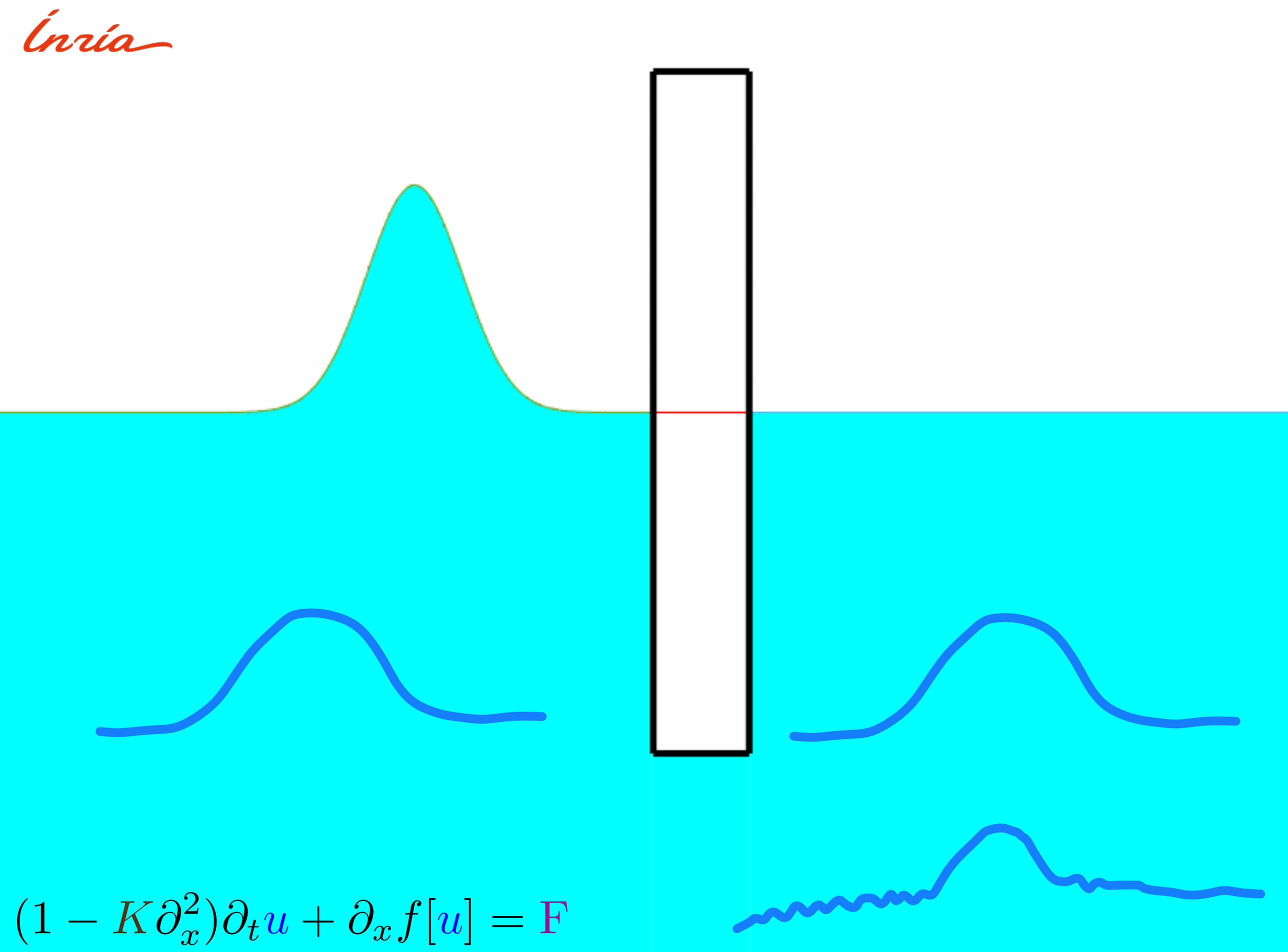

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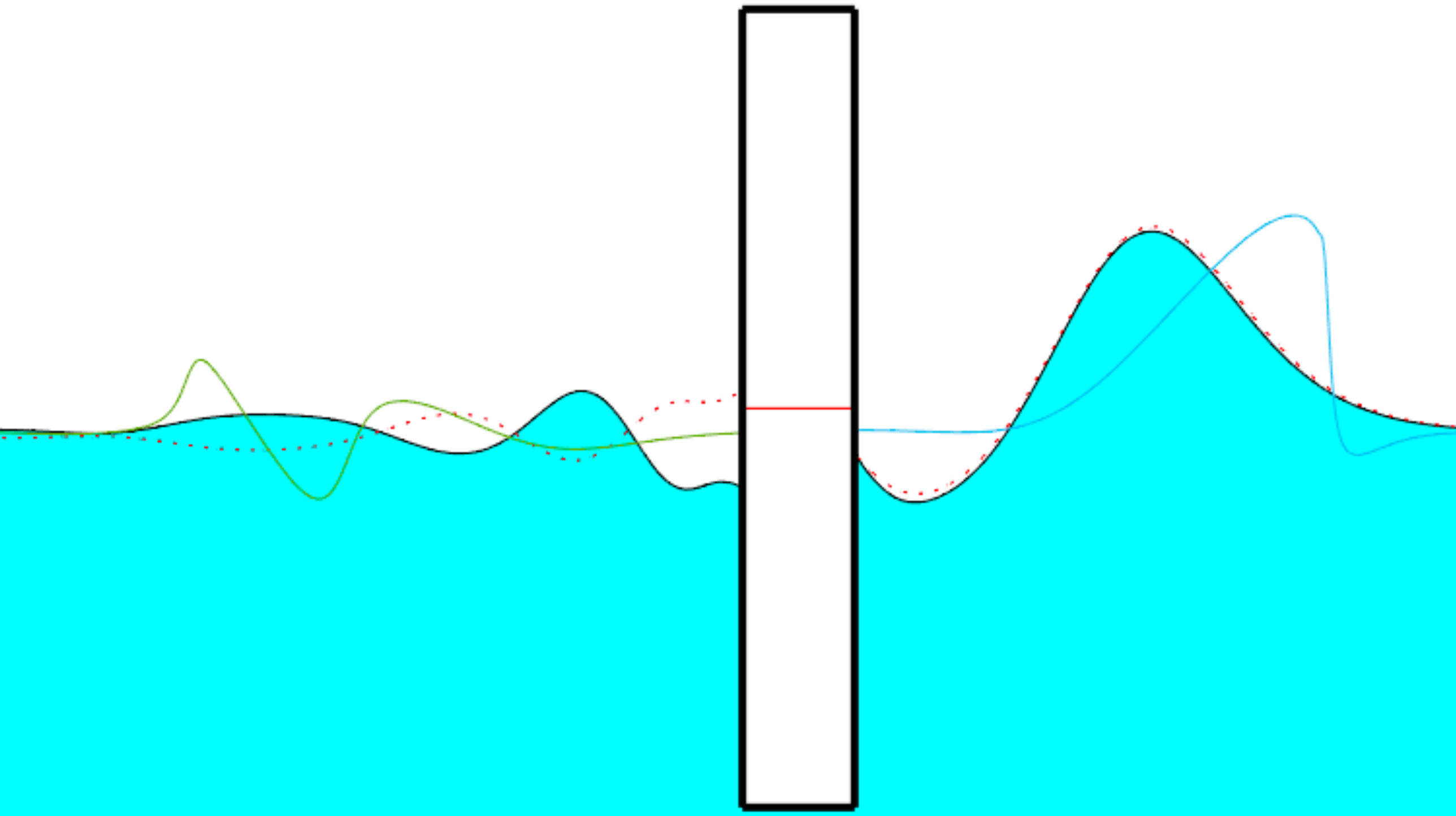

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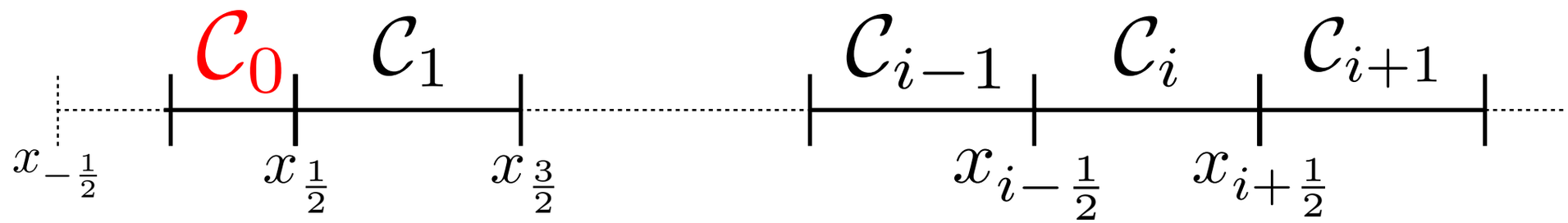

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$$\zeta_0^n = ????$$
$$q_0^n = q_{|x=0}(n\Delta t)$$

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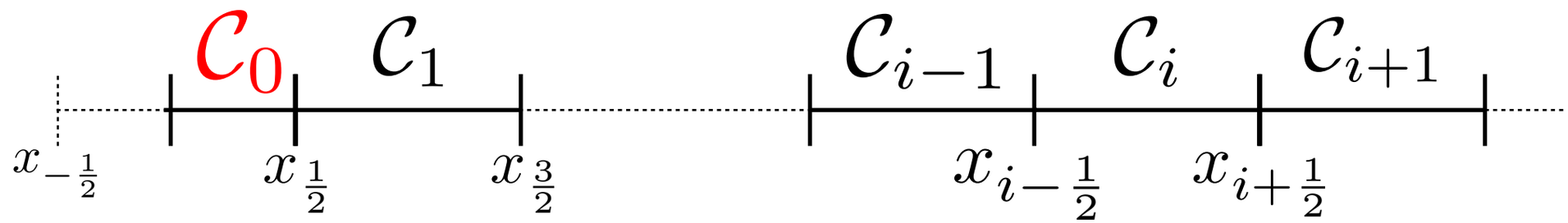

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1- How to compute "$(1 - K\partial_x^2)^{-1}$" ?

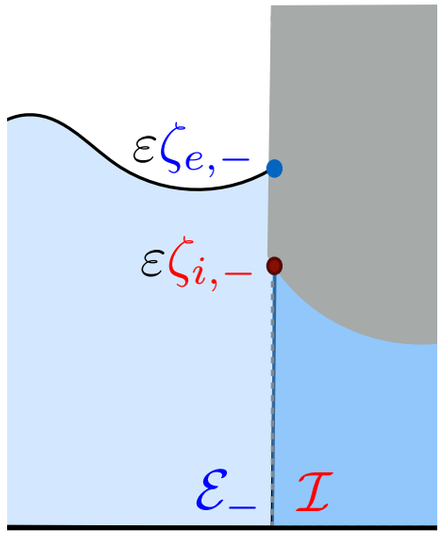
2- Missing boundary condition ζ_0^n ?

3- Non-local numerical dissipation λ_κ ?

Floating object: theoretical maths

Floating object: CFD [G. Beck, A. Riquier, 2026]

Floating object: theoretical maths



[D. Bresch, D. Lannes, and G. Métivier, 2021]

[G. Beck, D.Lannes, 2021]

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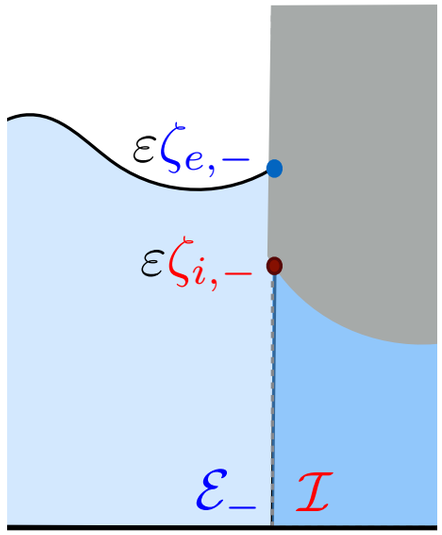

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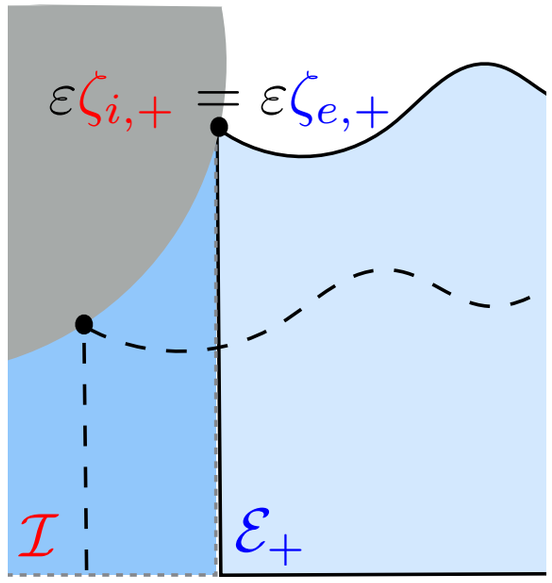

[F.John, 1949] [D.Lannes, 2017]
[D.Lannes, M. Oen Paulsen, 2025]
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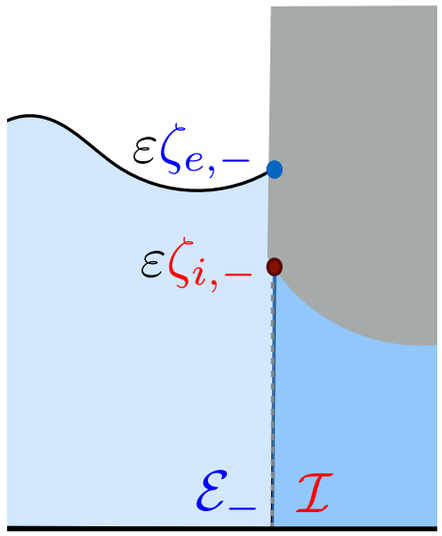

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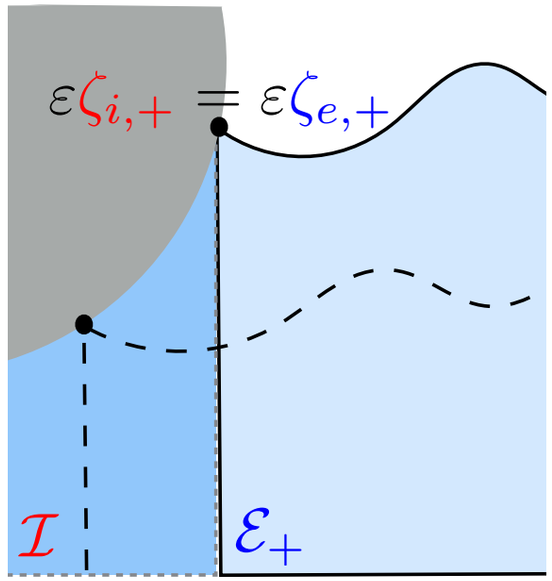

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Floating object: numerics

[G. Beck, D.Lannes, L. Weynans 2021-2025] [E. Godlewski, M. Parisot, J. Sainte-Marie, F. Wahl, 2020]
[G. Beck, E. Contentin, L. Martaud, 2027] [A. Haidar, F. Marche, F. Vilar, 2025]

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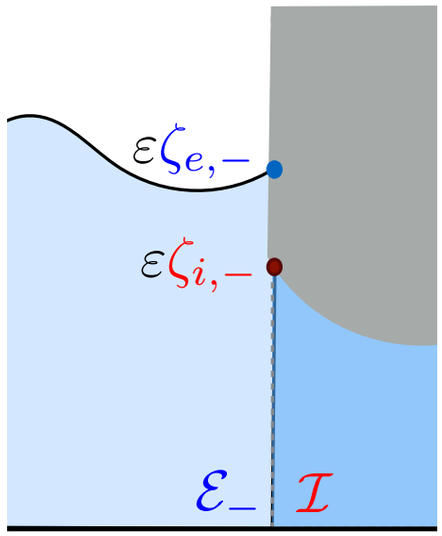
Floating object: theoretical maths



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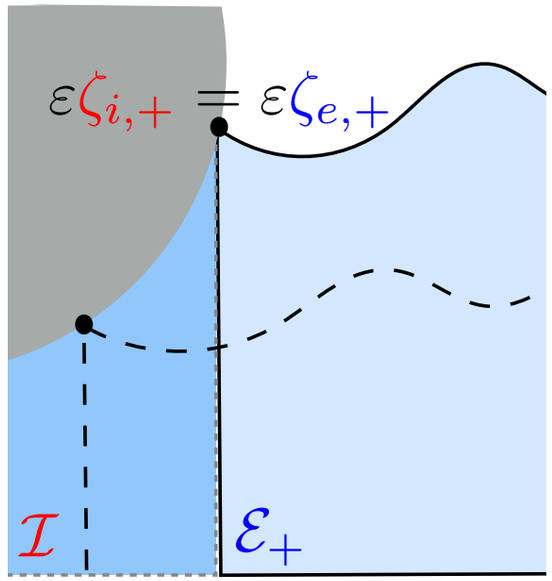

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Floating object: numerics

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Floating object: CFD [G. Beck, A. Riquier, 2026]

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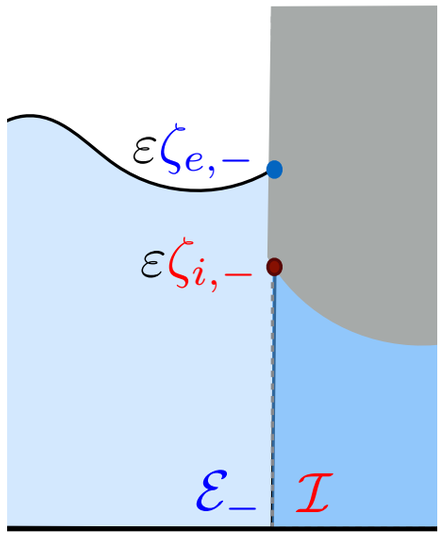

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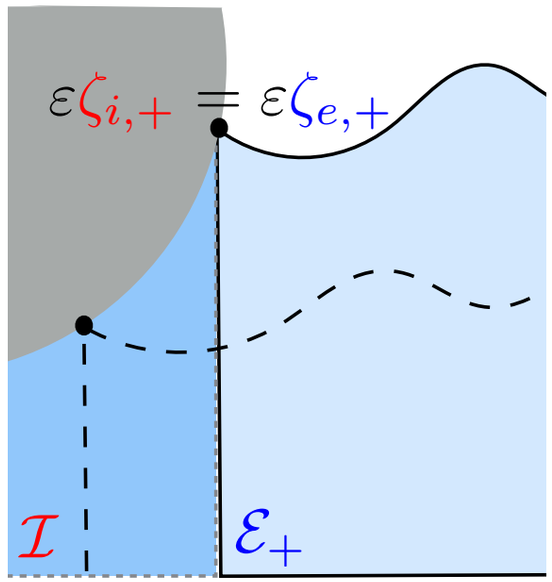

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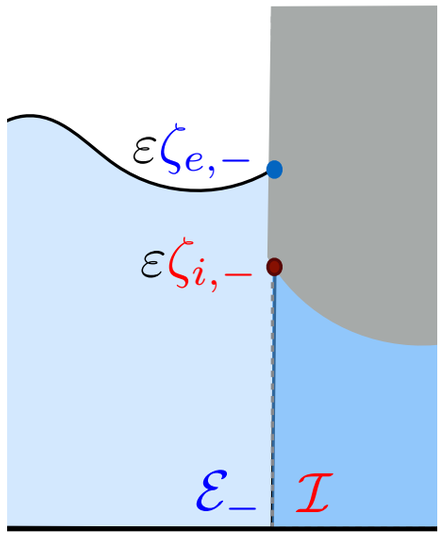

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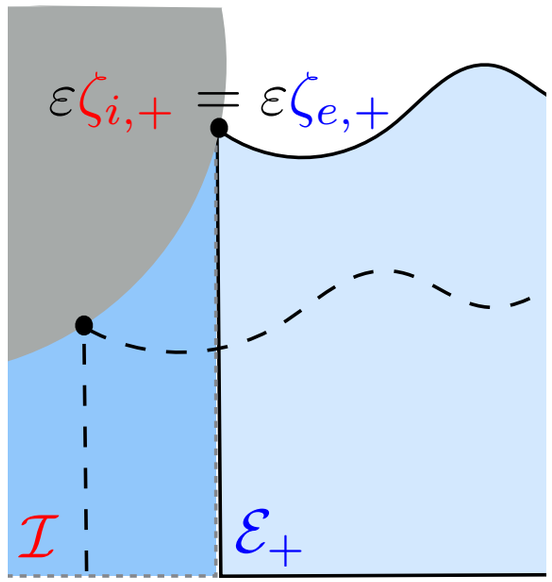

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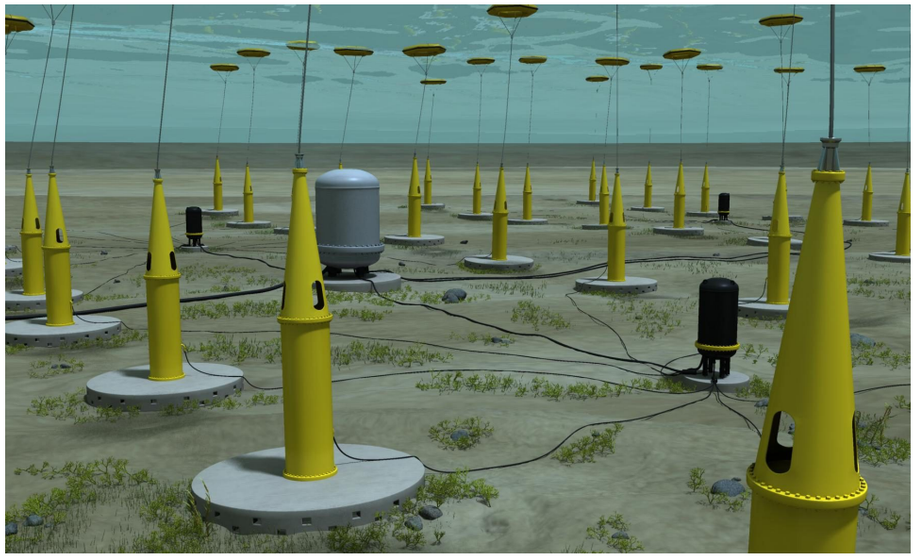

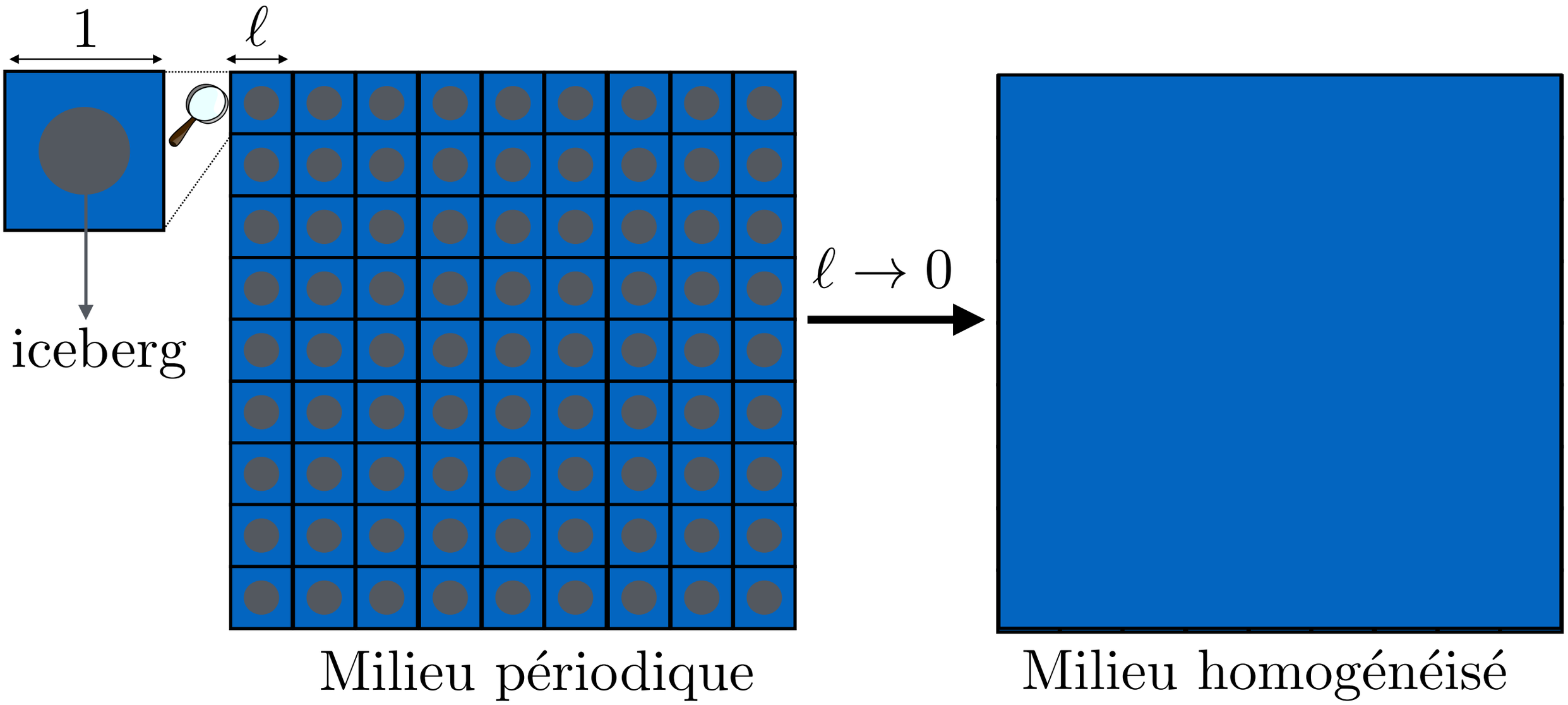
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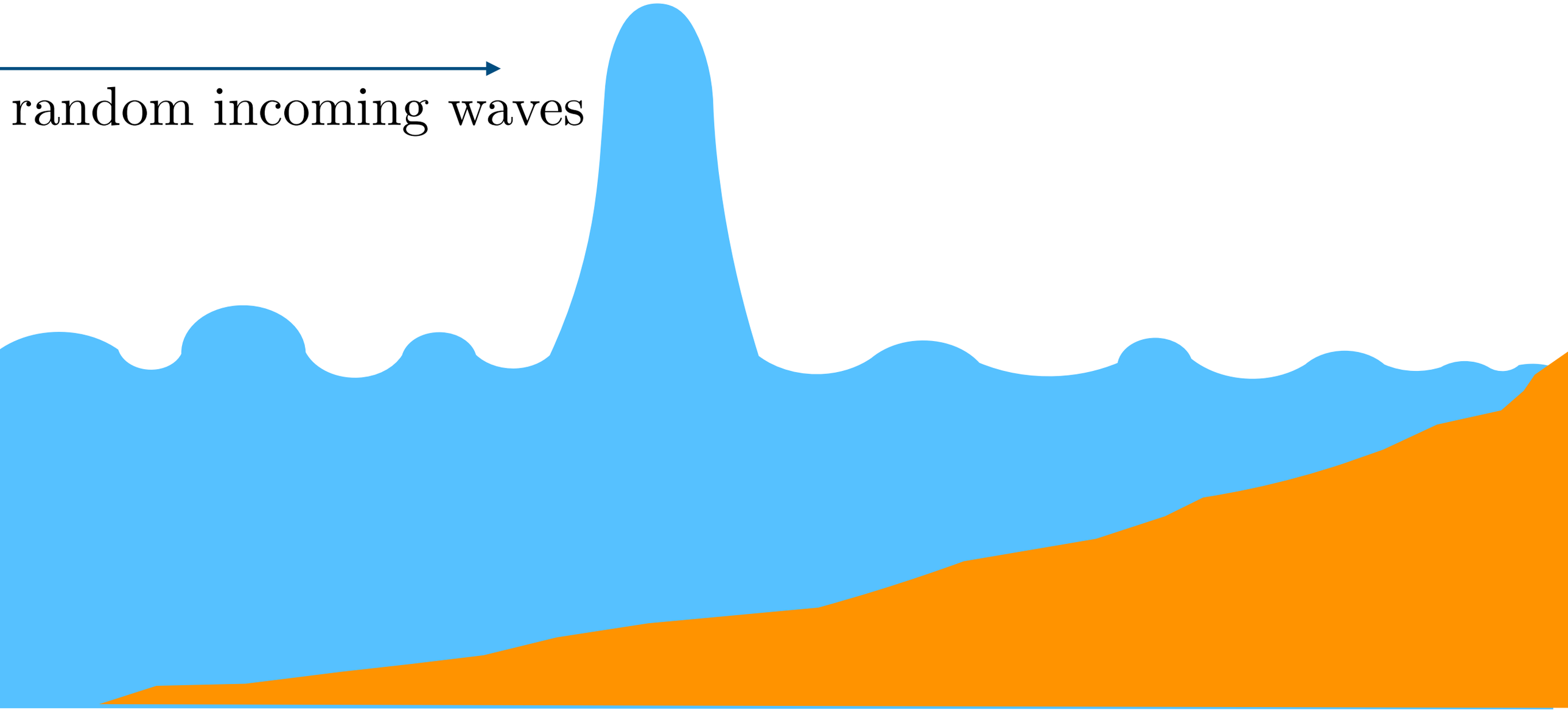
« Creativity is making the complicated simple »

(Charles MINGuS)

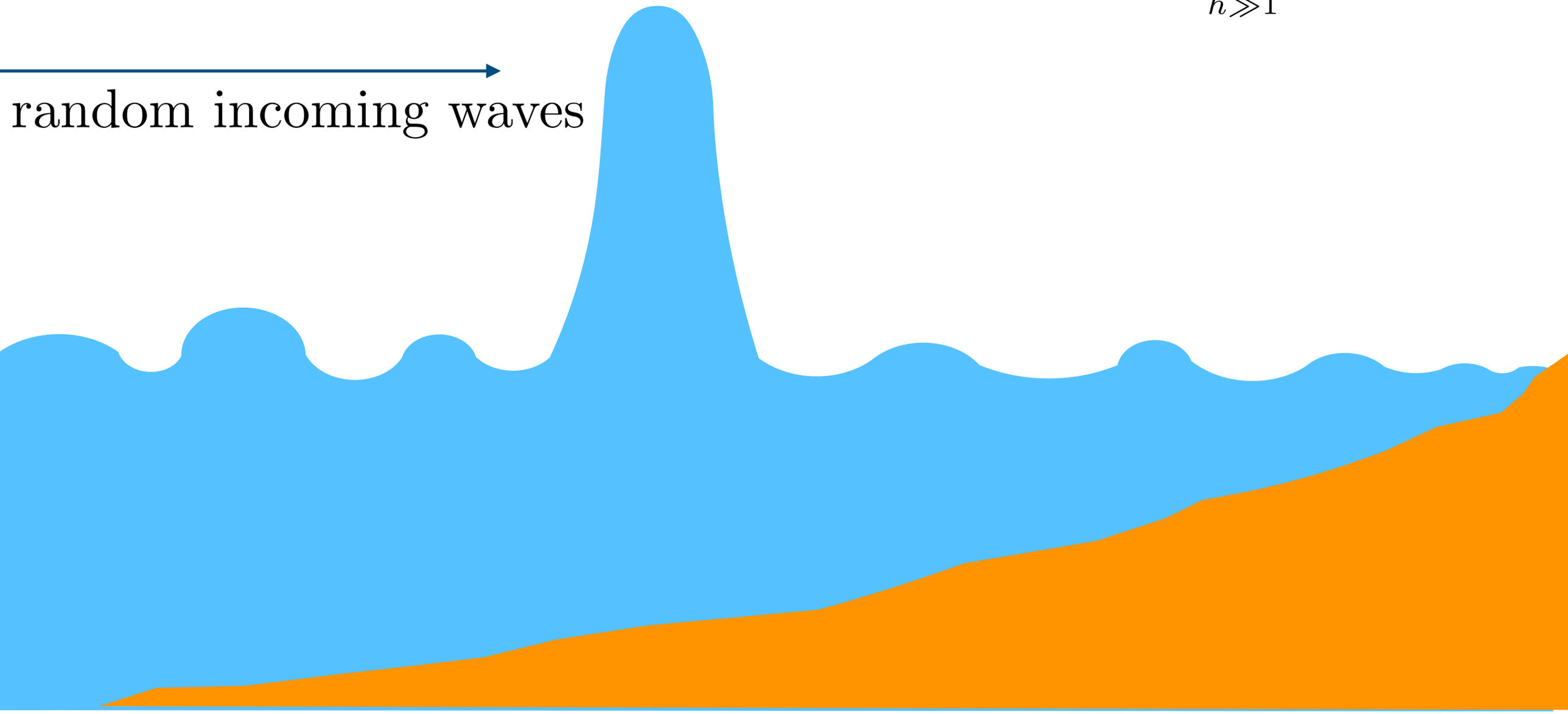
random incoming waves



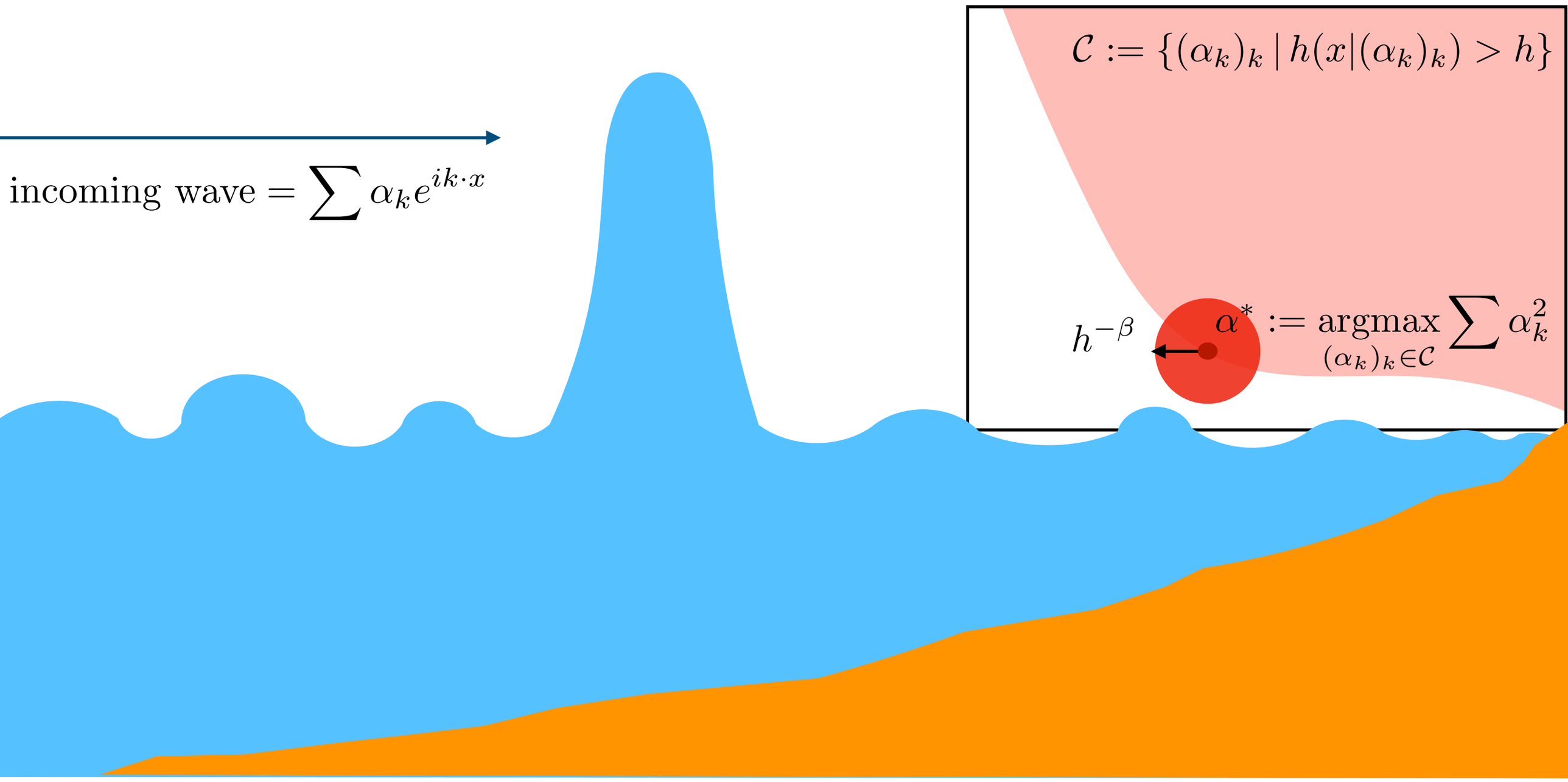
Large deviations: $\ln \mathbb{P}\Big[\,|\text{ water column }| > h\Big] \underset{h\gg 1}{=} -\mathcal{L}(h) + o(1)$

$$\mathcal{L}(h) \underset{h\gg 1}{\to} \infty$$

random incoming waves



Large deviations: $\ln \mathbb{P}\Big[\text{ water column } > h\Big] \underset{h\gg 1}{=} -\mathcal{L}(h) + o(1)$



Large deviations: $\ln \mathbb{P}\Big[\text{ water column } > h\Big] \underset{h\gg 1}{=} \ln \mathbb{P}\Big[\text{ water column } \in B(\alpha^*, h^{-\beta})\Big] + o(1)$

