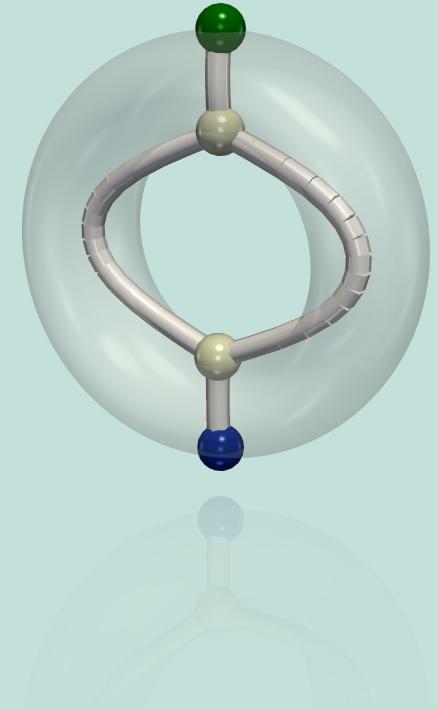


An introduction to Topological Data Analysis



Julien Tierny



About dimension reduction

About dimension reduction

- **Data reduction**
 - $n \times n$ image: n^2 -dimensional vector

About dimension reduction

- **Data reduction**

- nxn image: n^2 -dimensional vector
- Modern simulations
 - Time-varying
 - Multiphysics
 - Ensemble runs

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- **How to reduce data?**

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- **Dimensionality reduction**
- **How to reduce data?**

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- **How to reduce data?**

- **Dimensionality reduction**

- Projections down to “human” dimensions
- Interactive analysis and exploration

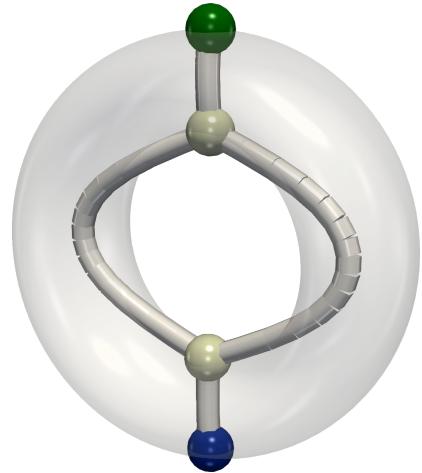
About dimension reduction

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 - nxn image: n^2 -dimensional vector
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- **Dimensionality reduction**
 - Projections down to “human” dimensions
 - Interactive analysis and exploration
- **How to preserve the important information?**

What is Topological Data Analysis?

- **Context**

- Data
- On “meshes”, or “meshable” things



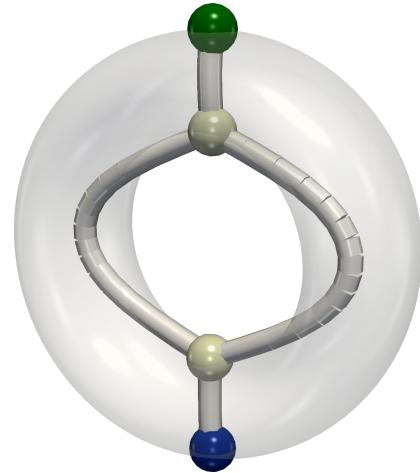
What is Topological Data Analysis?

- **Context**

- Data
- On “meshes”, or “meshable” things

- **Swiss-army knife for feature extraction**

- Points, curves, surfaces, volumes, ...
- Robustness
- Multi-scale nature
- From raw data to features of interest



Piecewise linear setting

- Input PL scalar data
 - $f : \mathcal{M} \rightarrow \mathbb{R}$



Piecewise linear setting

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 - $f : \mathcal{M} \rightarrow \mathbb{R}$



Piecewise linear setting

- Input PL scalar data
 - $f : \mathcal{M} \rightarrow \mathbb{R}$
- Topological abstractions



Piecewise linear setting

- Input PL scalar data
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- Topological abstractions
 - Critical points



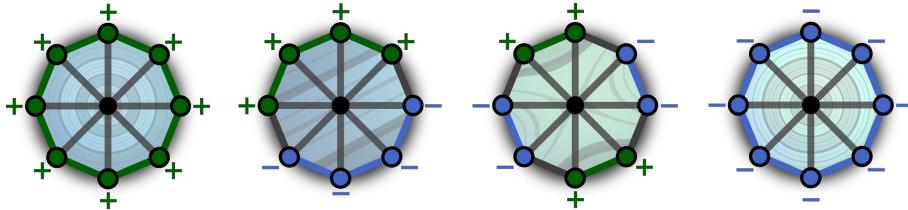
Piecewise linear setting

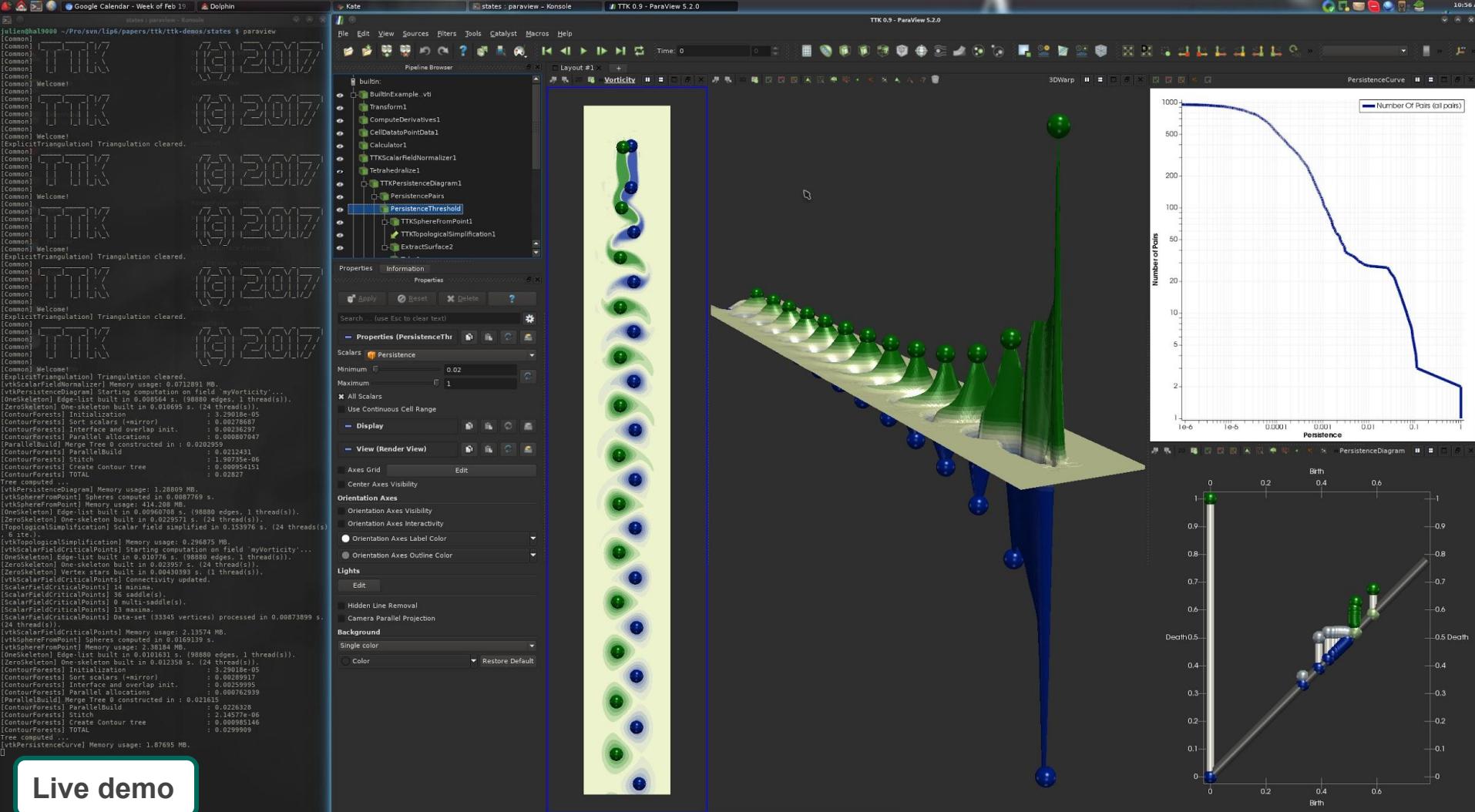
- Input PL scalar data
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 - Critical points



Critical point extraction

- Local link inspection
 - Banchoff 1970





Piecewise linear setting

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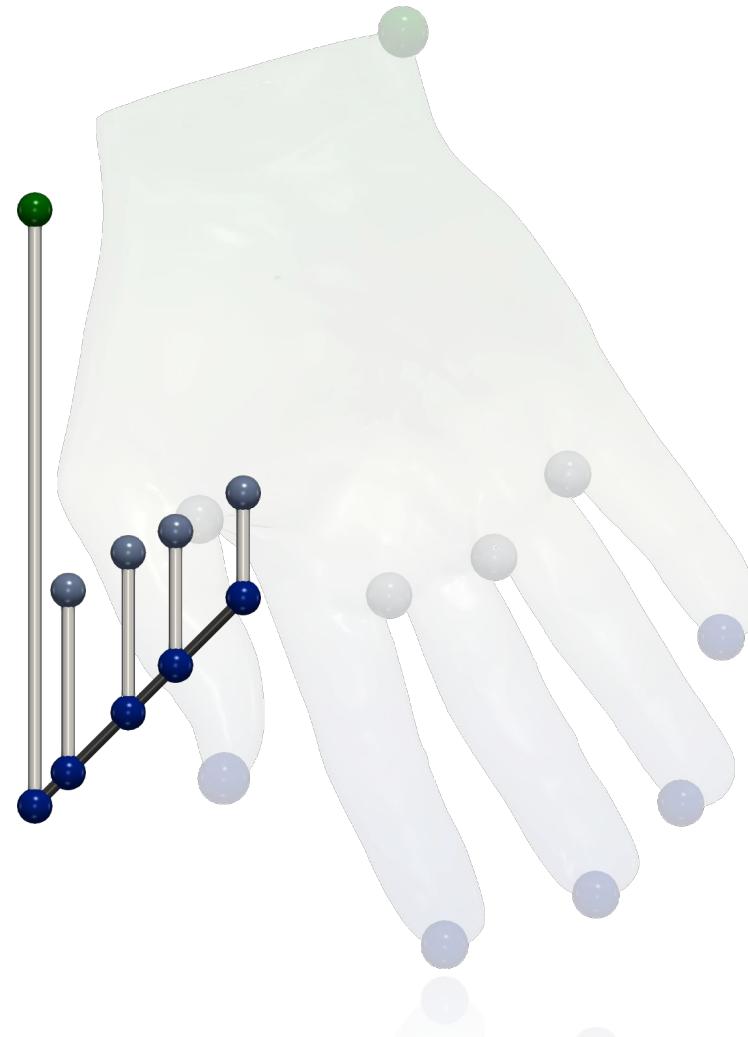
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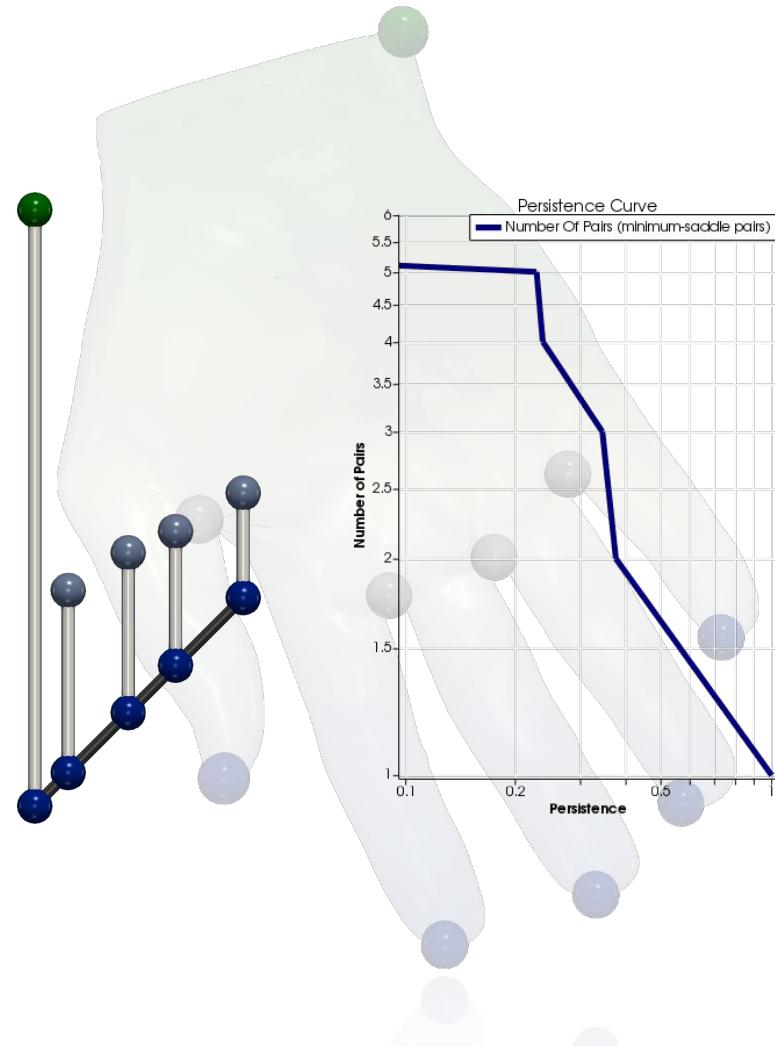
Piecewise linear setting

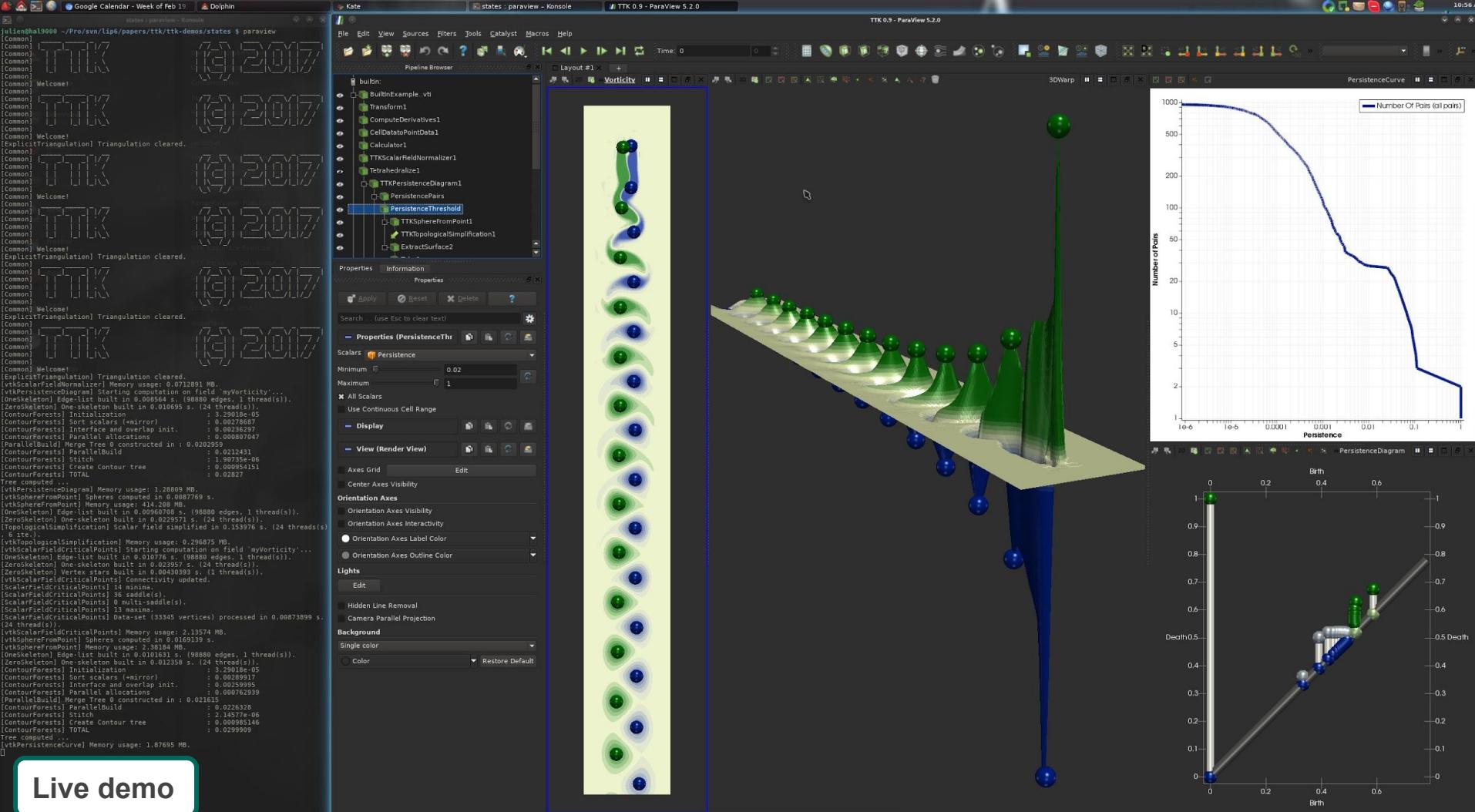
- Input PL scalar data
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 - Persistence diagrams



Piecewise linear setting

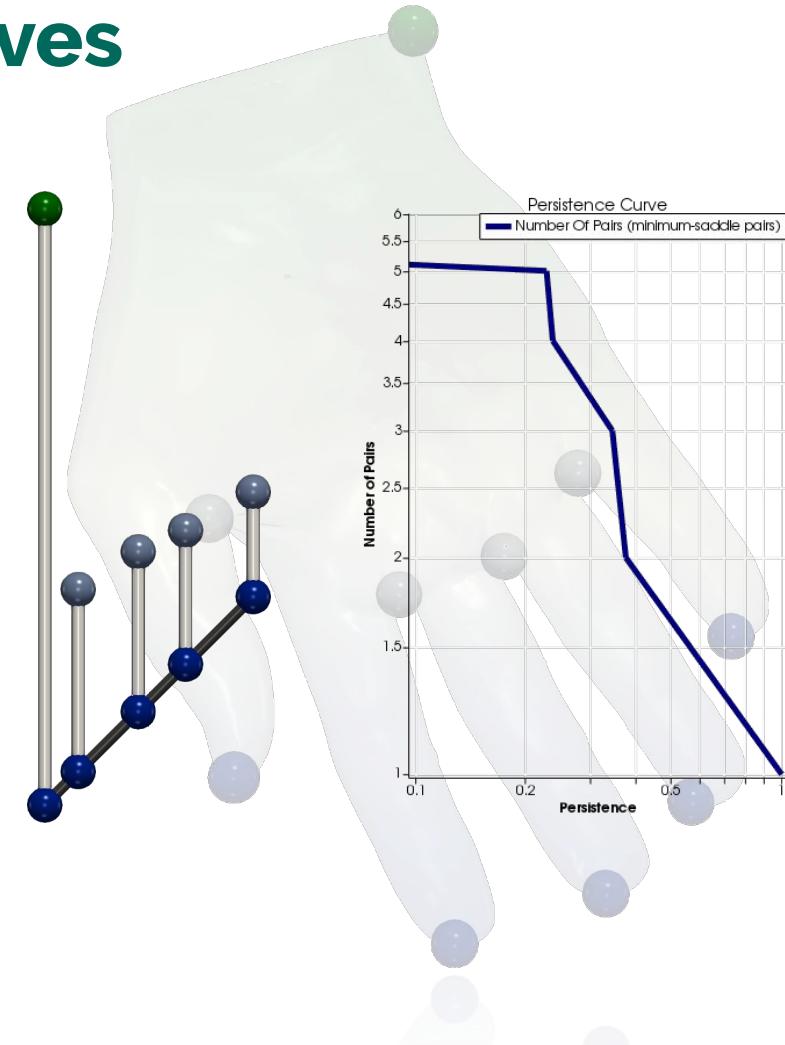
- Input PL scalar data
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 - Critical points
 - Persistence diagrams
 - Persistence curve





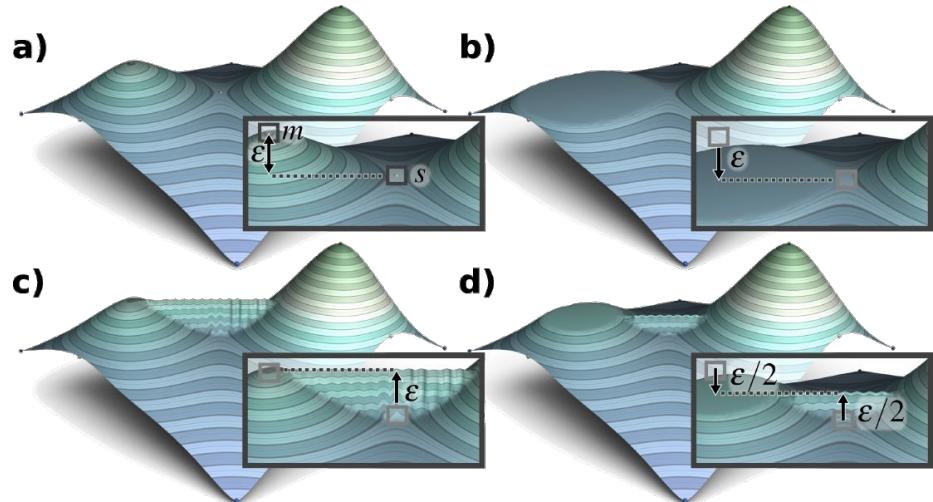
Persistence diagrams/curves

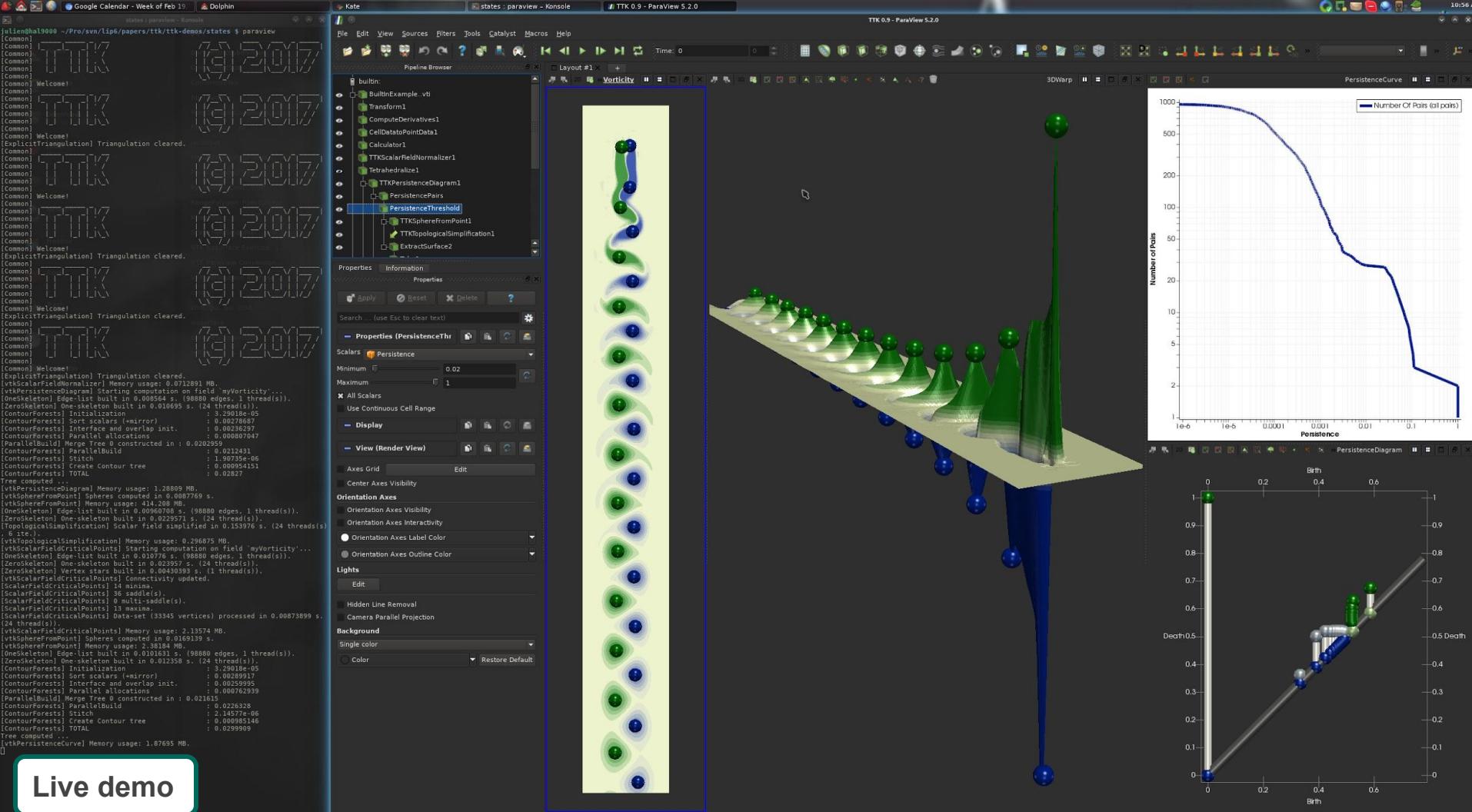
- **Arbitrary dimension**
 - Boundary matrix reduction
 - Edelsbrunner et al. 2002
- **Low-dimensions**
 - Union-Find data structures
 - Min-saddle pairs
 - Saddle-max pairs
 - Gueunet et al. 2017
 - Saddle connectors
 - Saddle-saddle pairs



Persistence simplification

- Simplify the data
 - Retain only persistent features
- Algorithms
 - Edelsbrunner et al. 2006, Attali et al. 2009, *Tierny and Pascucci 2012*, Bauer et al. 2012





Piecewise linear setting

- Input PL scalar data
 - $f : \mathcal{M} \rightarrow \mathbb{R}$
- Topological abstractions
 - Critical points
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 - Persistence curve
 - Reeb graphs



Piecewise linear setting

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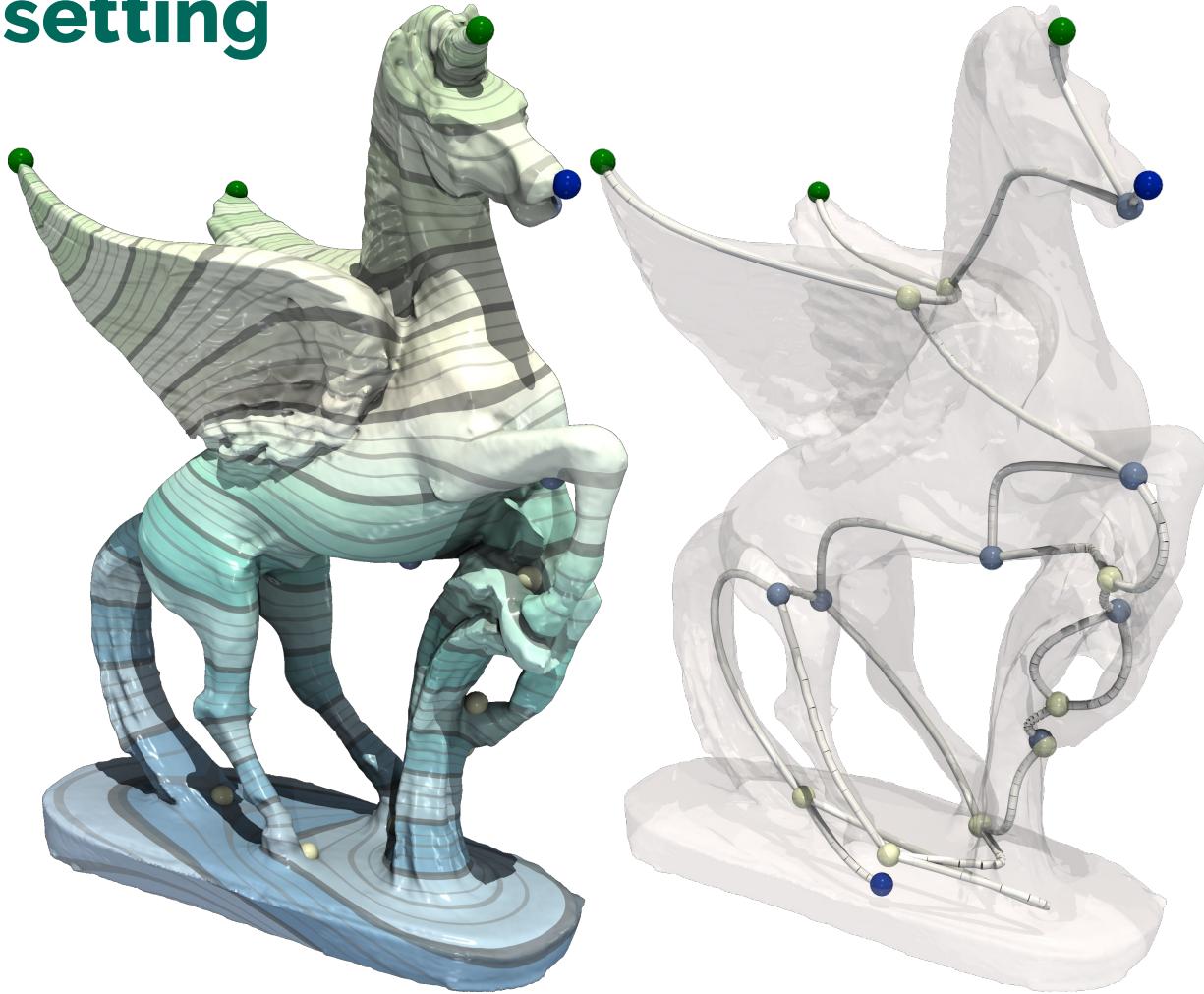
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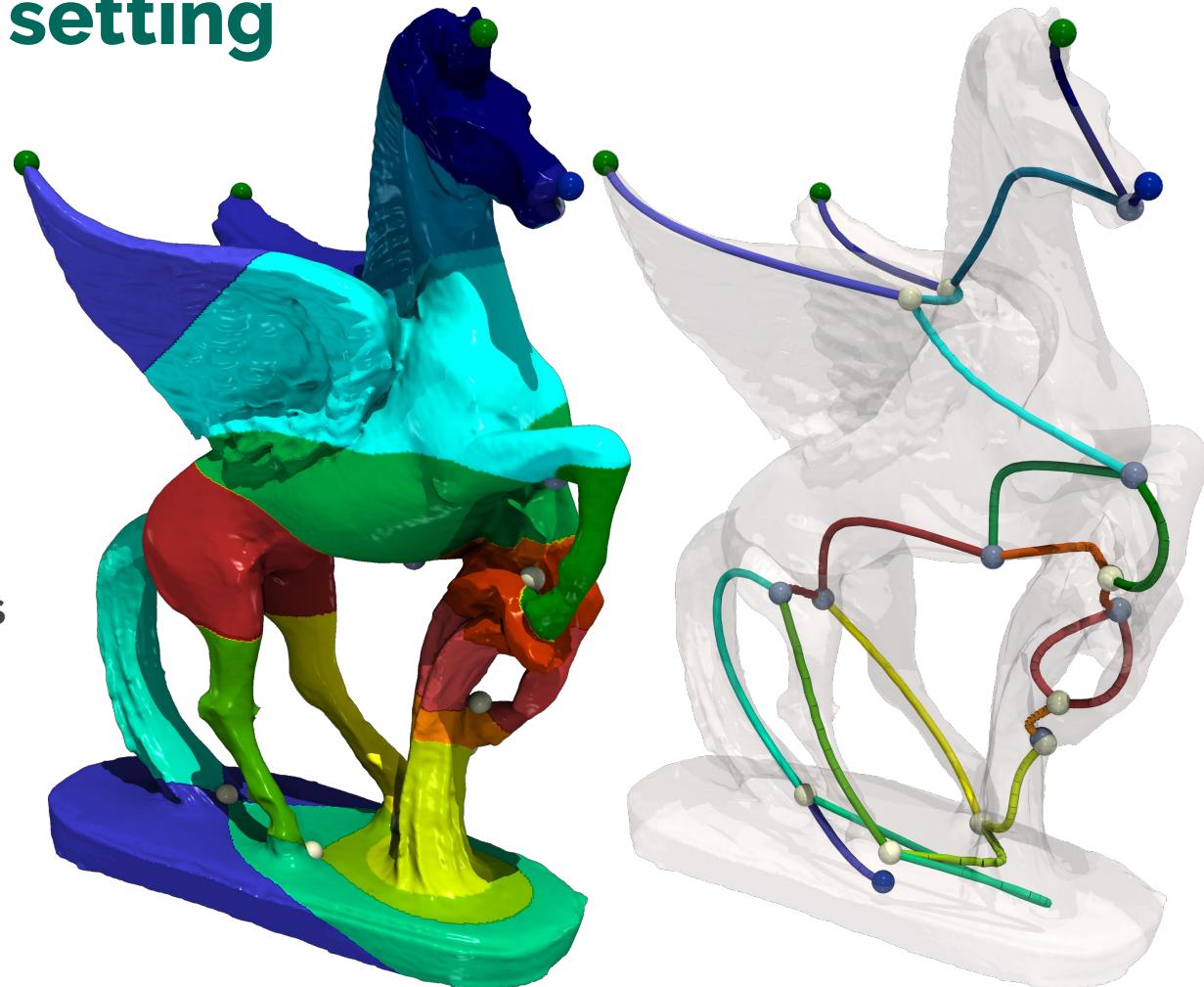
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The screenshot shows a complex visualization interface for scientific data analysis. On the left, a terminal window displays log messages from TTK and ParaView, including welcome messages, thread counts, memory usage, and various processing steps like persistence diagram computation and topological simplification. The main area features a 3D view of a dataset represented by several colored, textured surfaces (blue, green, red, orange) against a dark background. To the right of the 3D view is a Persistence Diagram, a plot of birth and death times for topological features. The bottom right corner contains a smaller 3D view of the same dataset. A 'Live demo' button is visible in the bottom left corner.

[live demo](#)

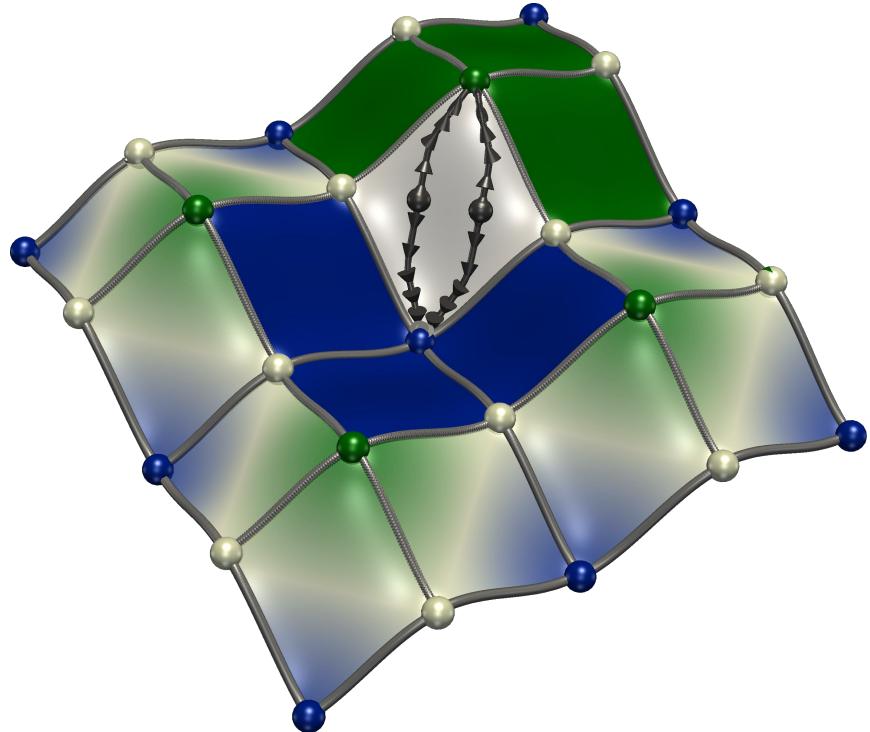
Reeb graphs

- **Vertex based contouring**
 - Shinagawa and Kunii 1991
- **Quantized range contouring**
 - Biasotti et al. 2000, Hilaga et al. 2001, Wood et al. 2004
- **Critical contouring**
 - Patane et al. 2008, *Tierny et al. 2009*, Doraiswamy and Natarajan 2013, Hajij and Rosen 2018
- **Dynamic connectivity**
 - Cole-McLaughlin et al. 2003, Pascucci et al. 2007, Doraiswamy and Natarajan 2009, Parsa 2013, *Gueunet et al. 2019*



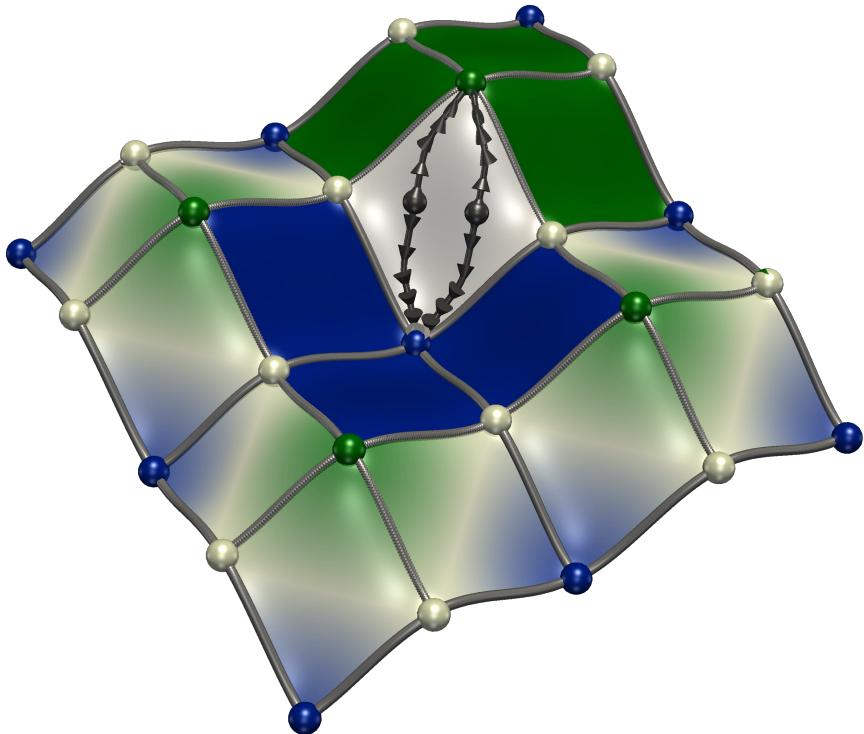
Discrete Morse Theory

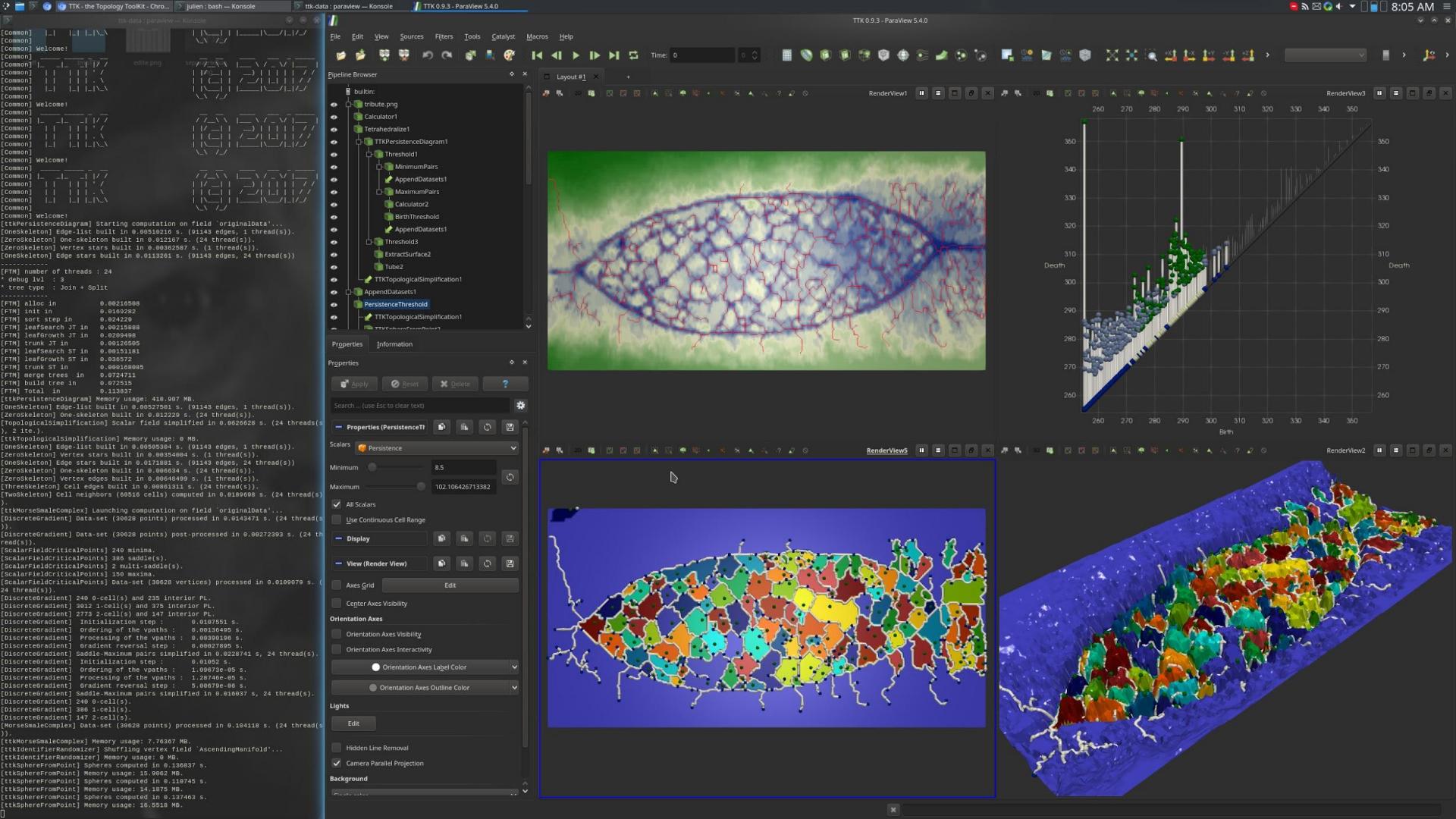
- **Morse-Smale complex**
 - Integration equivalence
 - Challenging PL computation

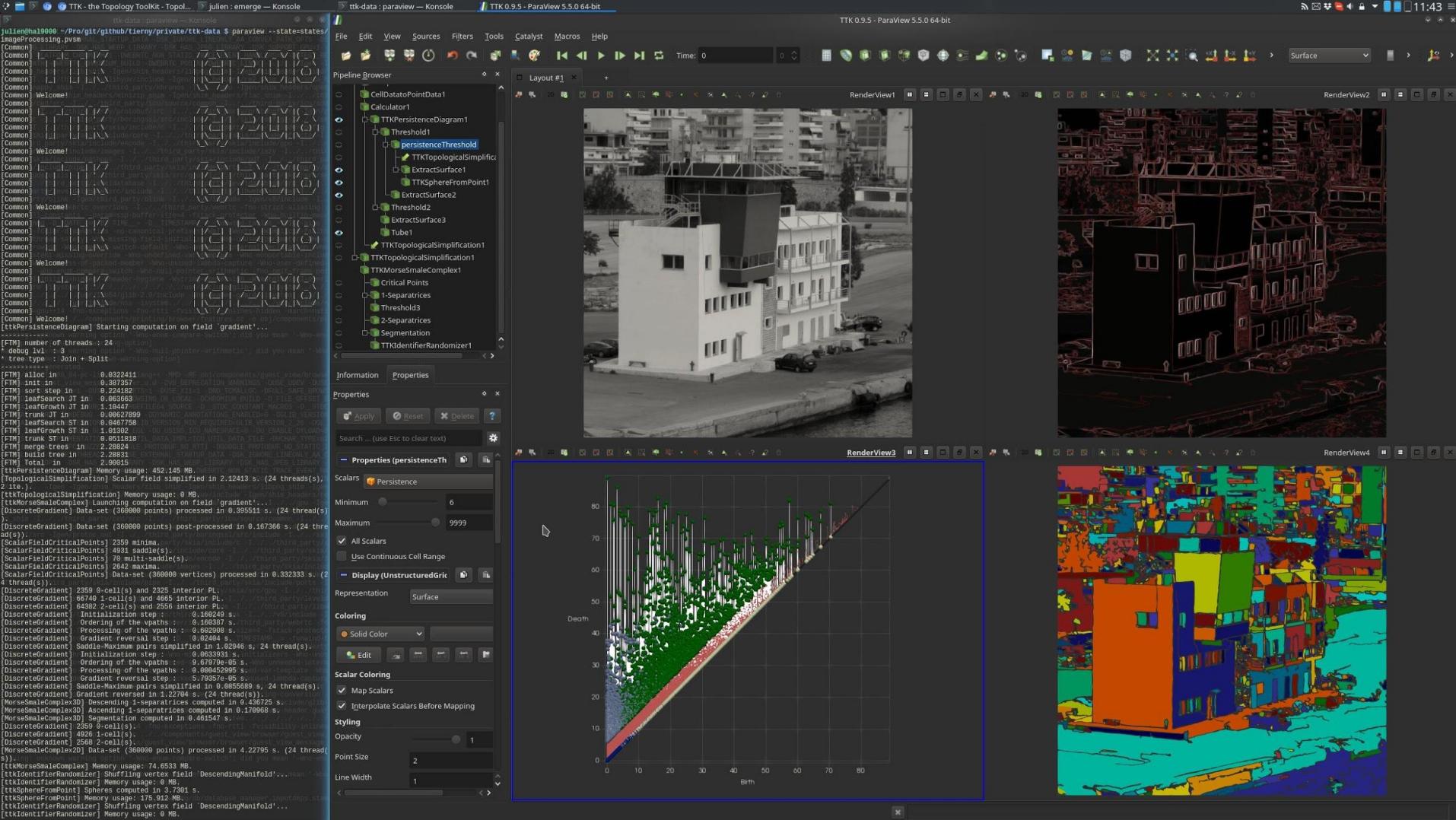


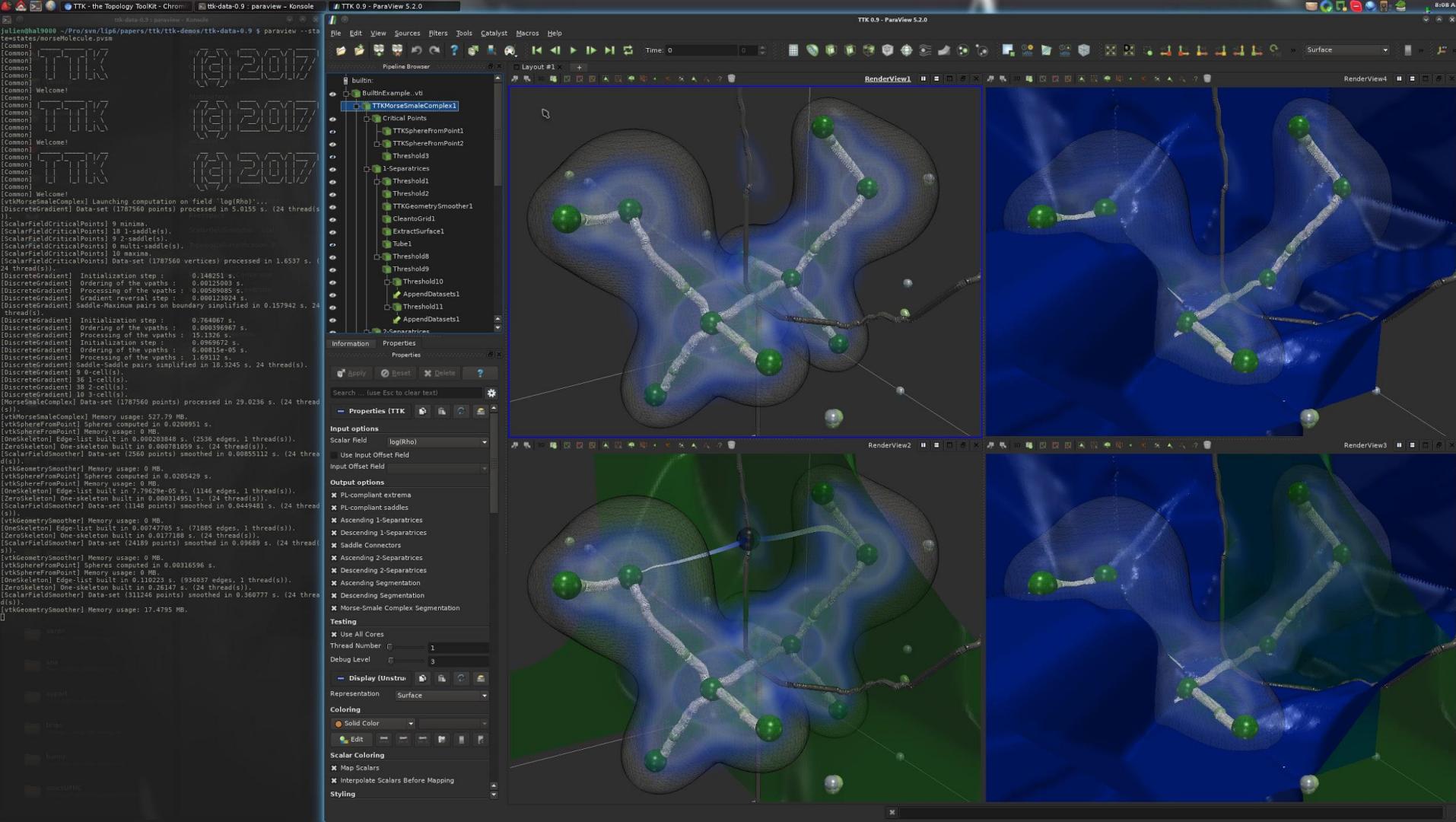
Discrete Morse Theory

- **Morse-Smale complex**
 - Integration equivalence
 - Challenging PL computation
- **Discrete Morse theory**
 - Forman 1998
 - Algorithms
 - Gyulassy 2008, Robins 2011,
Shivashankar and Natarajan 2012,
Tierny et al. 2017

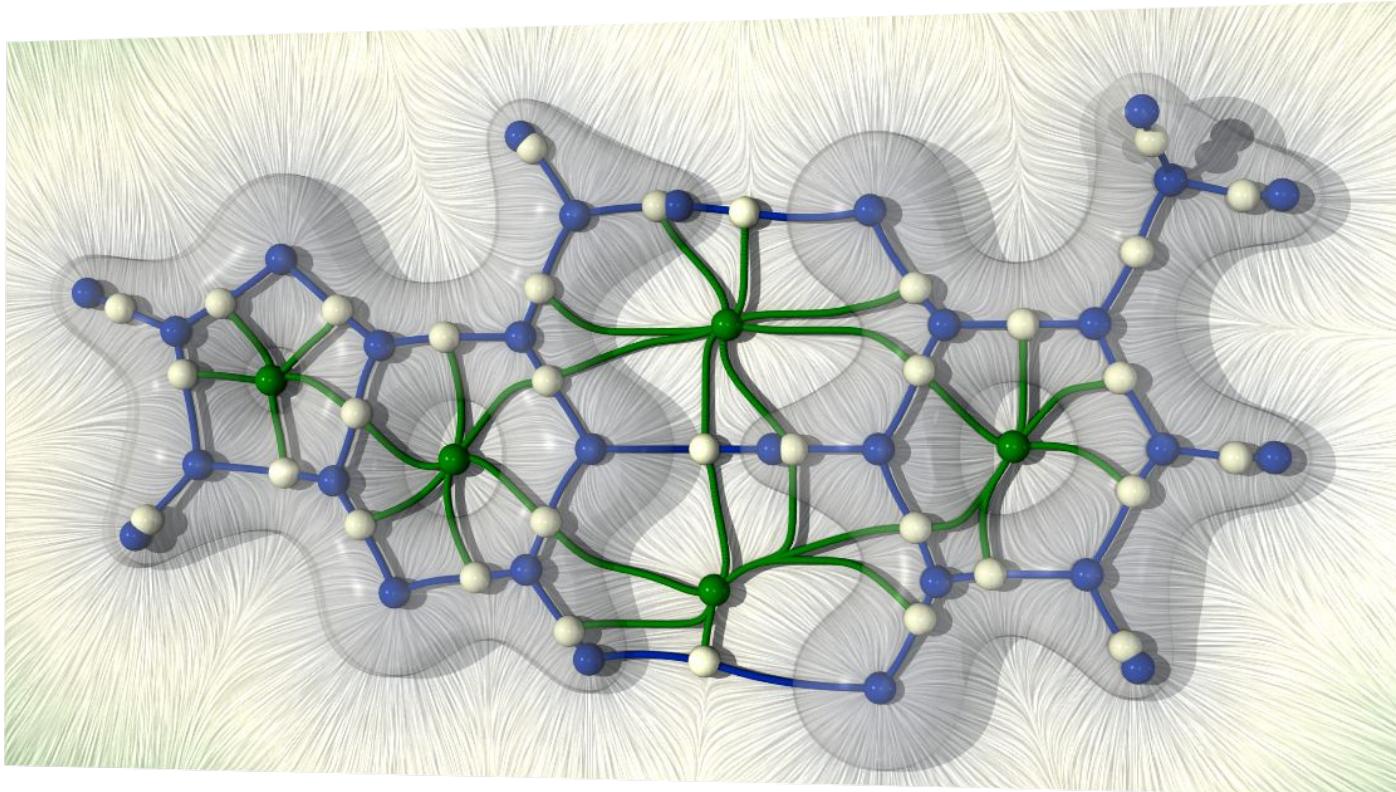






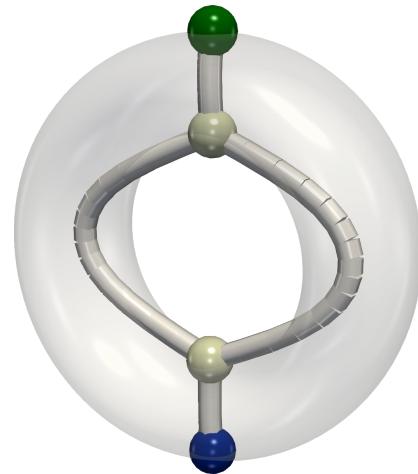


Application of the Morse-Smale complex



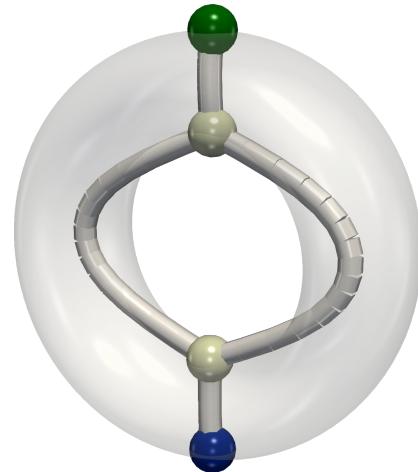
So far

- TDA for low dimensional fields
 - Data reduction by feature extraction



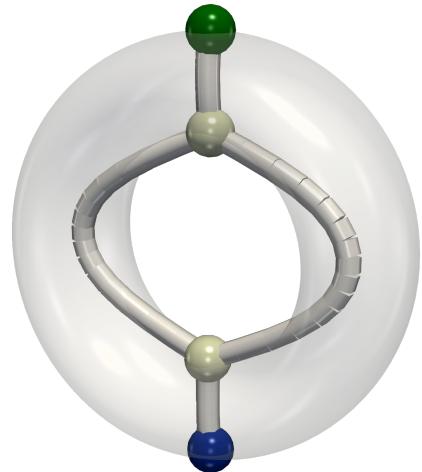
So far

- TDA for low dimensional fields
 - Data reduction by feature extraction
 - Critical points (vortices)
 - 1-Separatrices (filament structures)
 - 2-Separatrices (walls)
 - Regions of interest



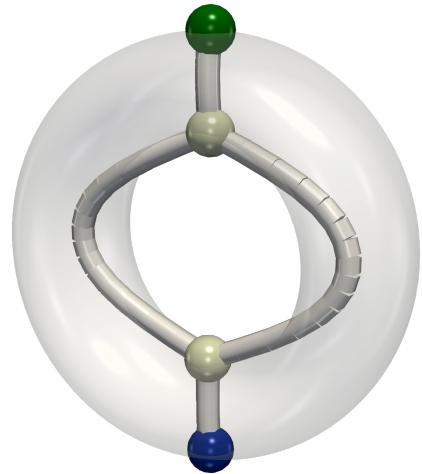
So far

- **TDA for low dimensional fields**
 - Data reduction by feature extraction
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 - Regions of interest
 - Only store topological information
 - Further analysis, measure, comparison
 - TDA driven lossy compression (Soler et al. 2018)

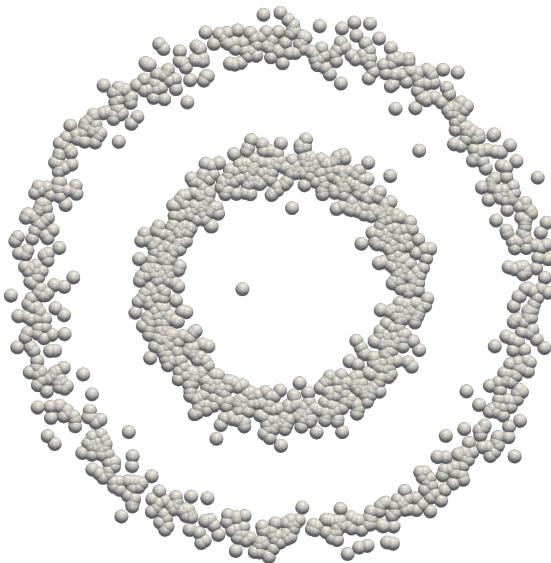


So far

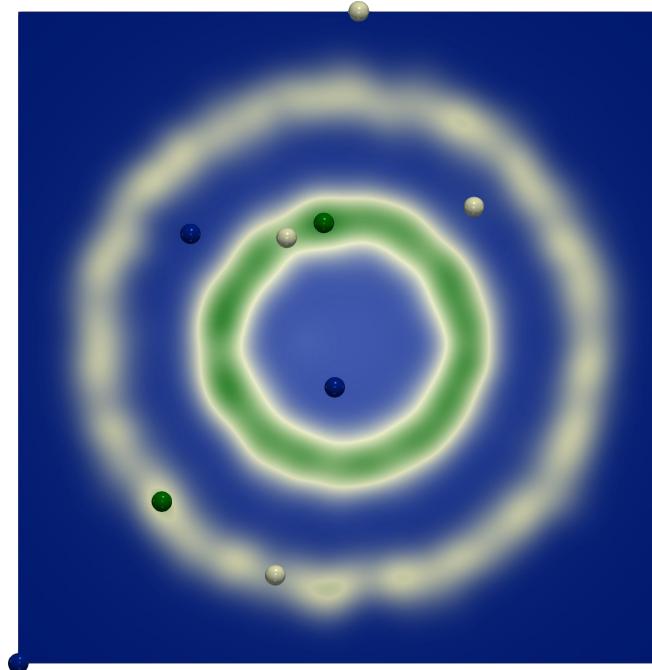
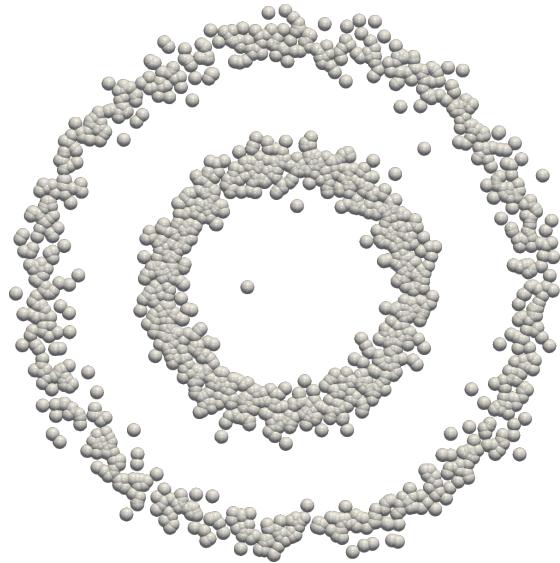
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- **How about generic high dimensional point clouds?**



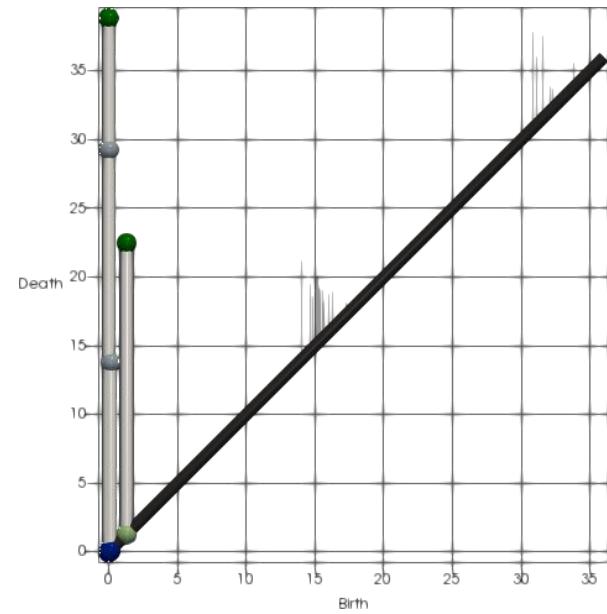
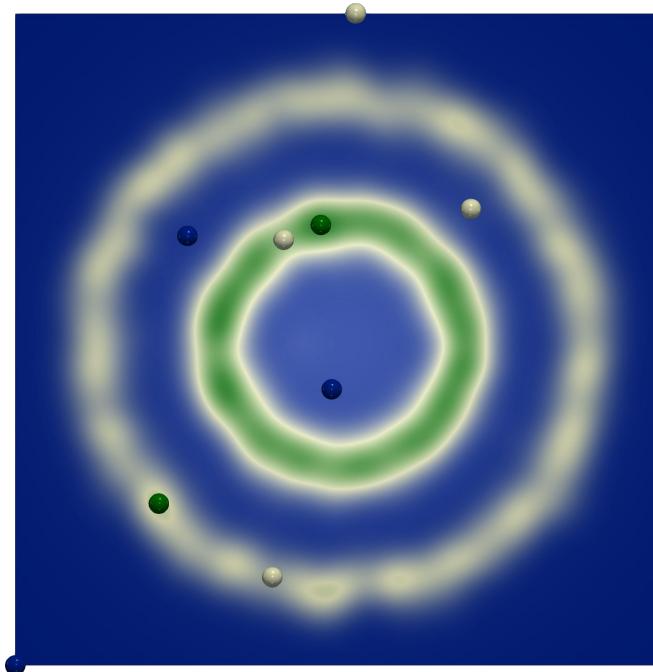
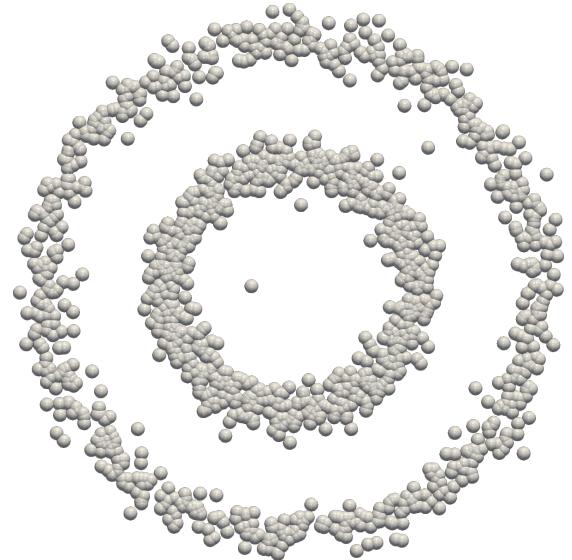
What about point cloud data?



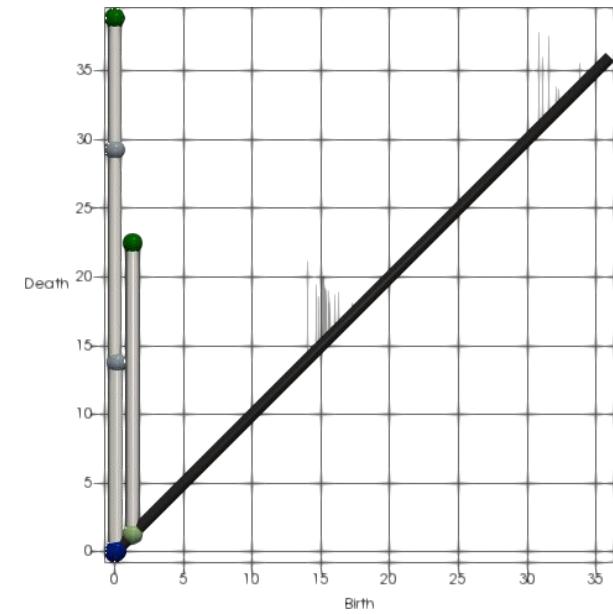
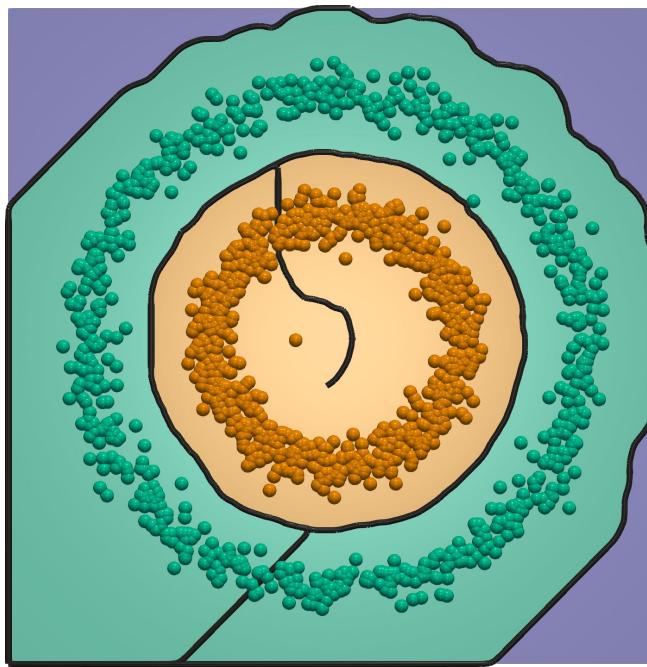
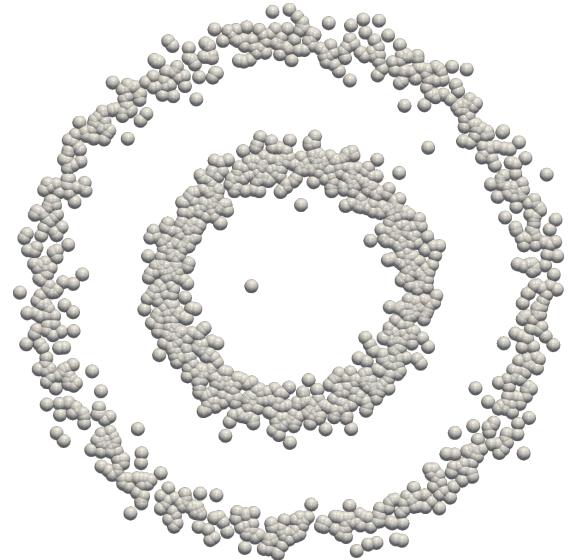
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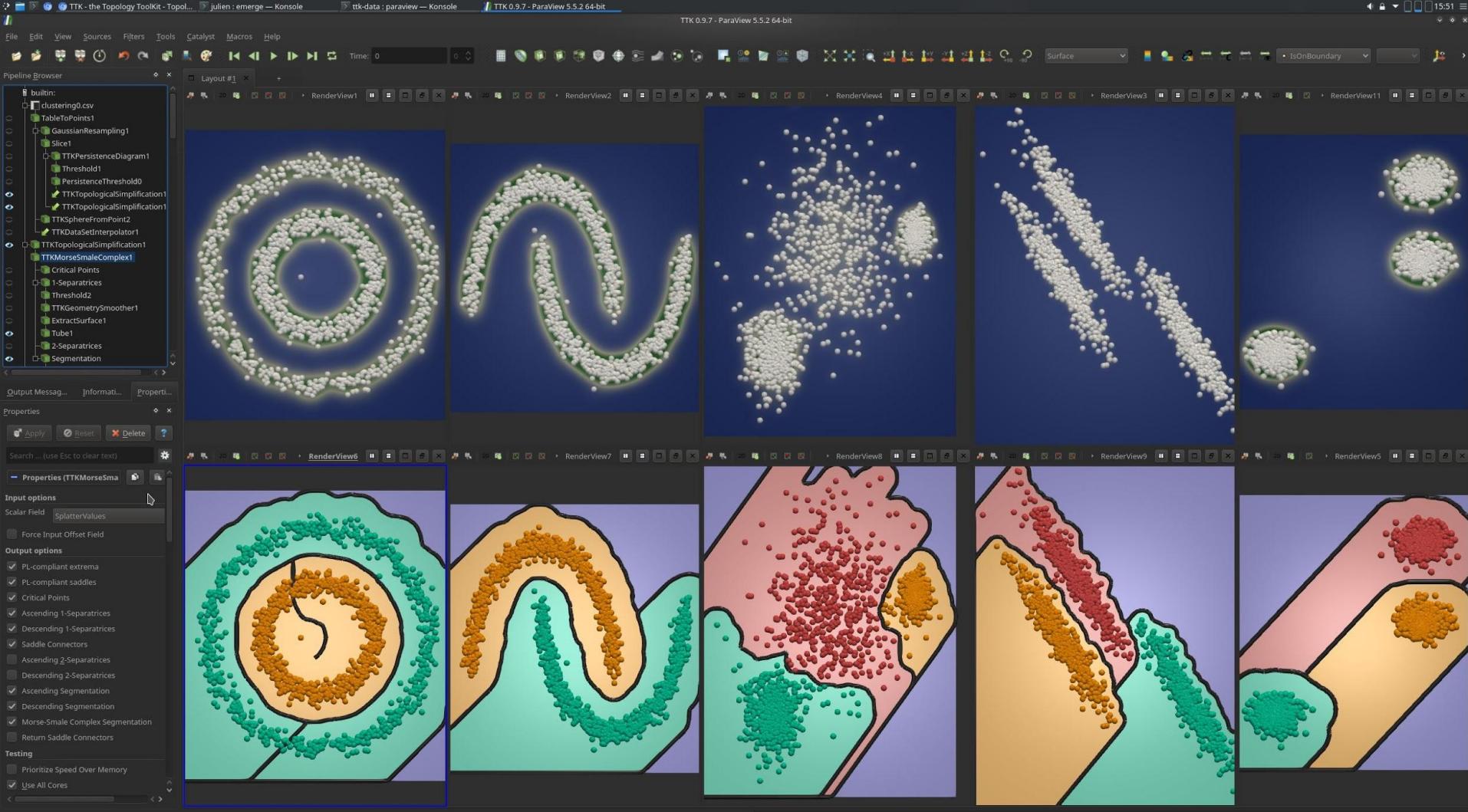


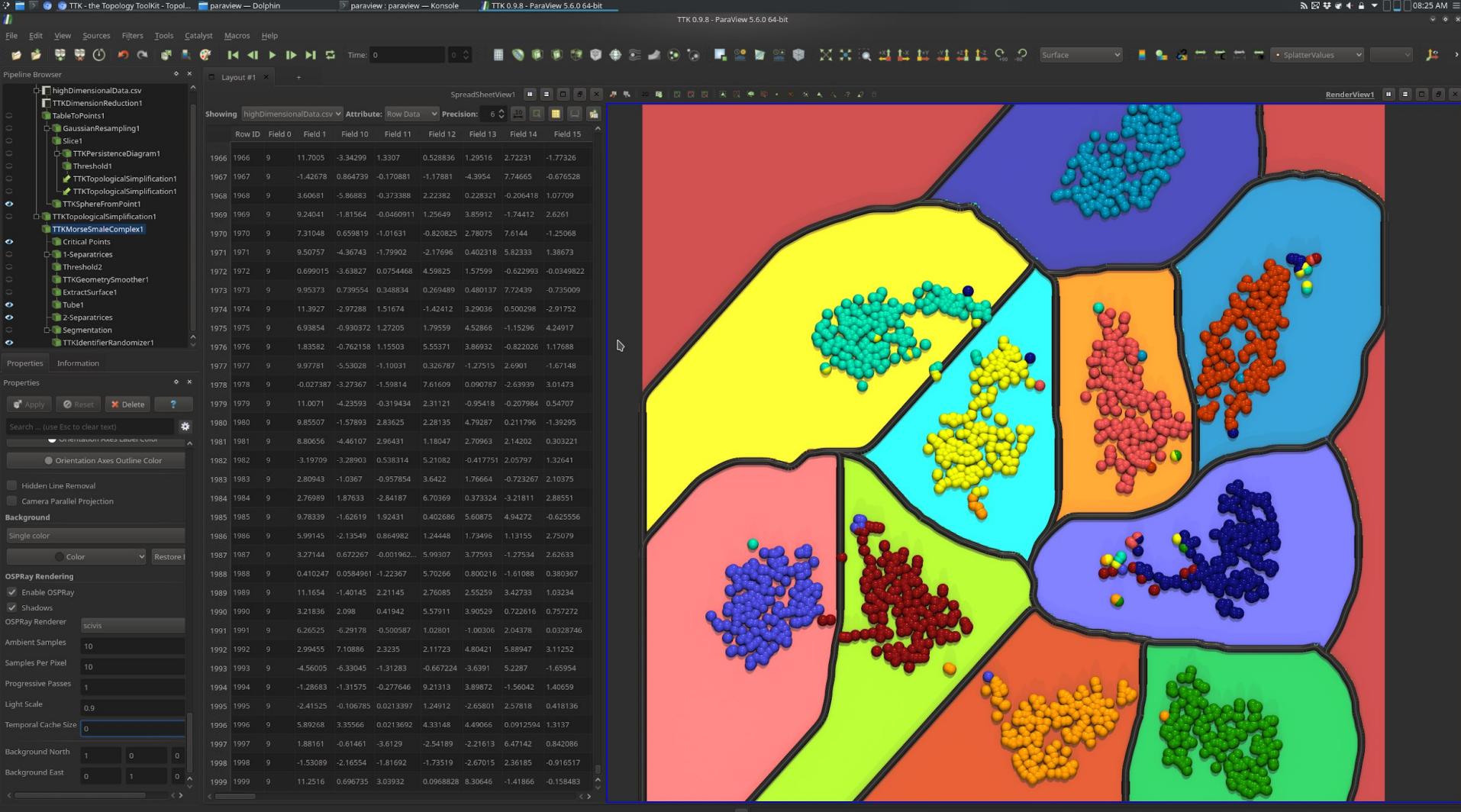
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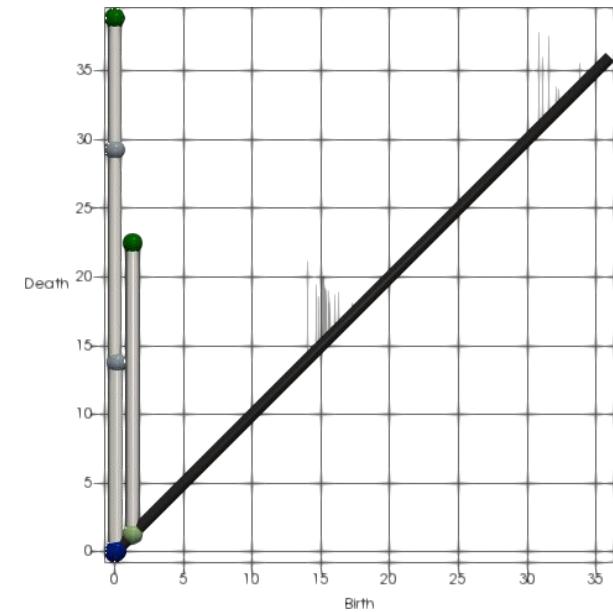
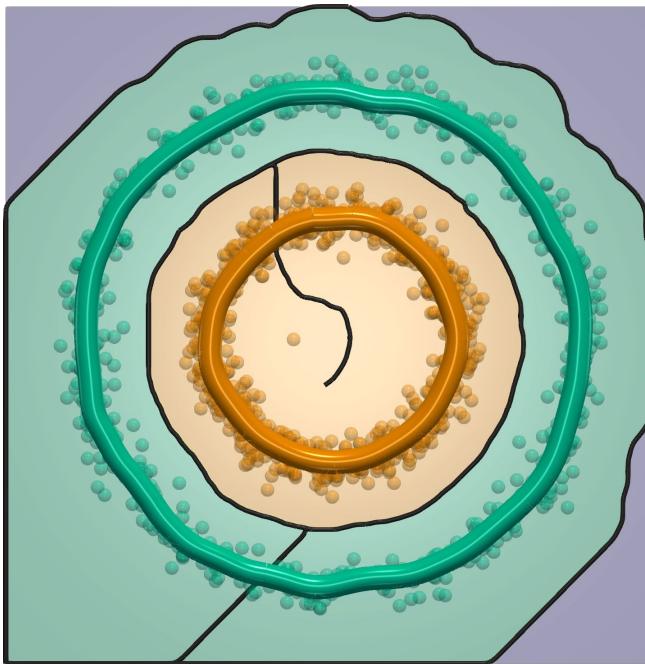
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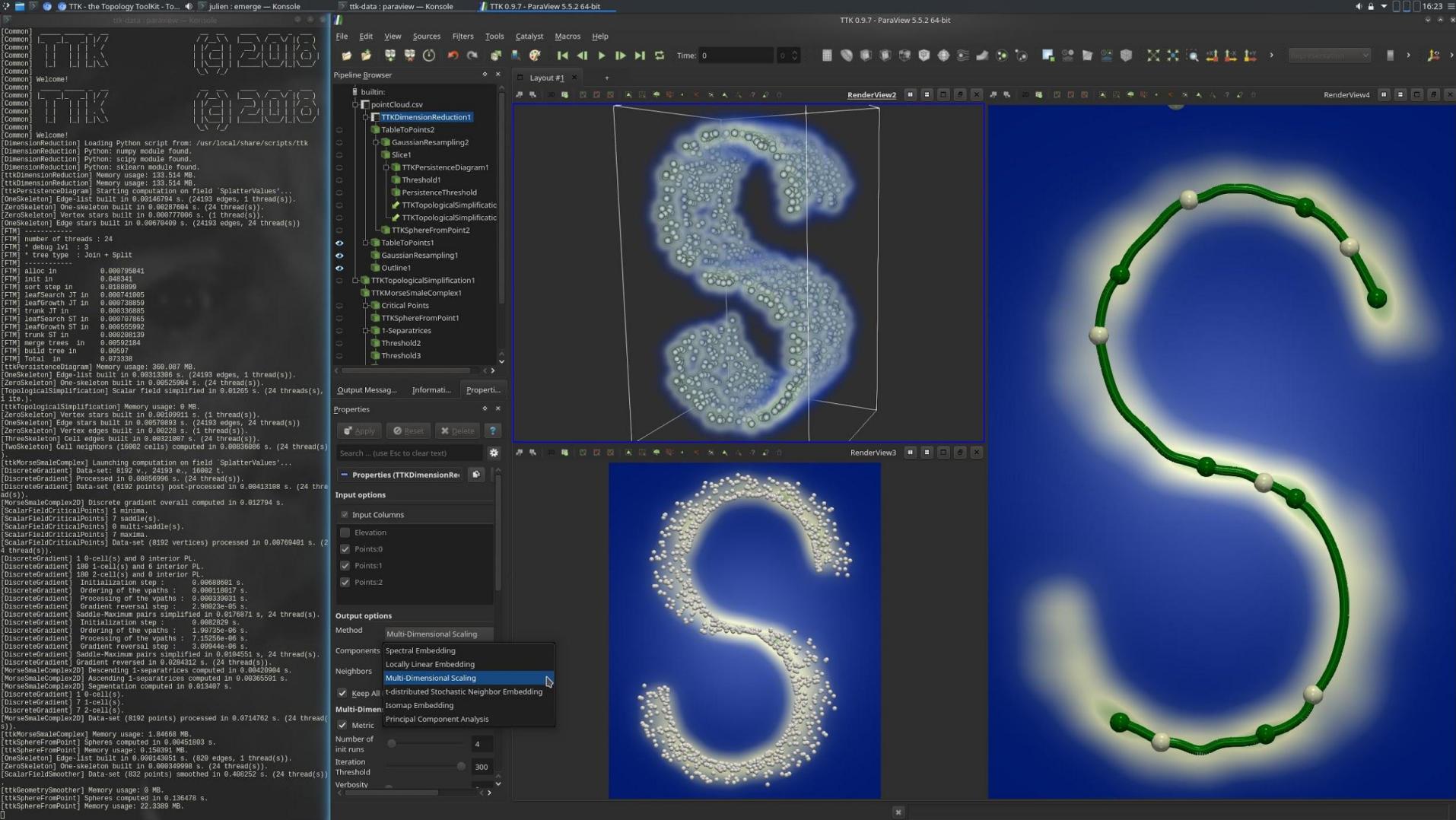


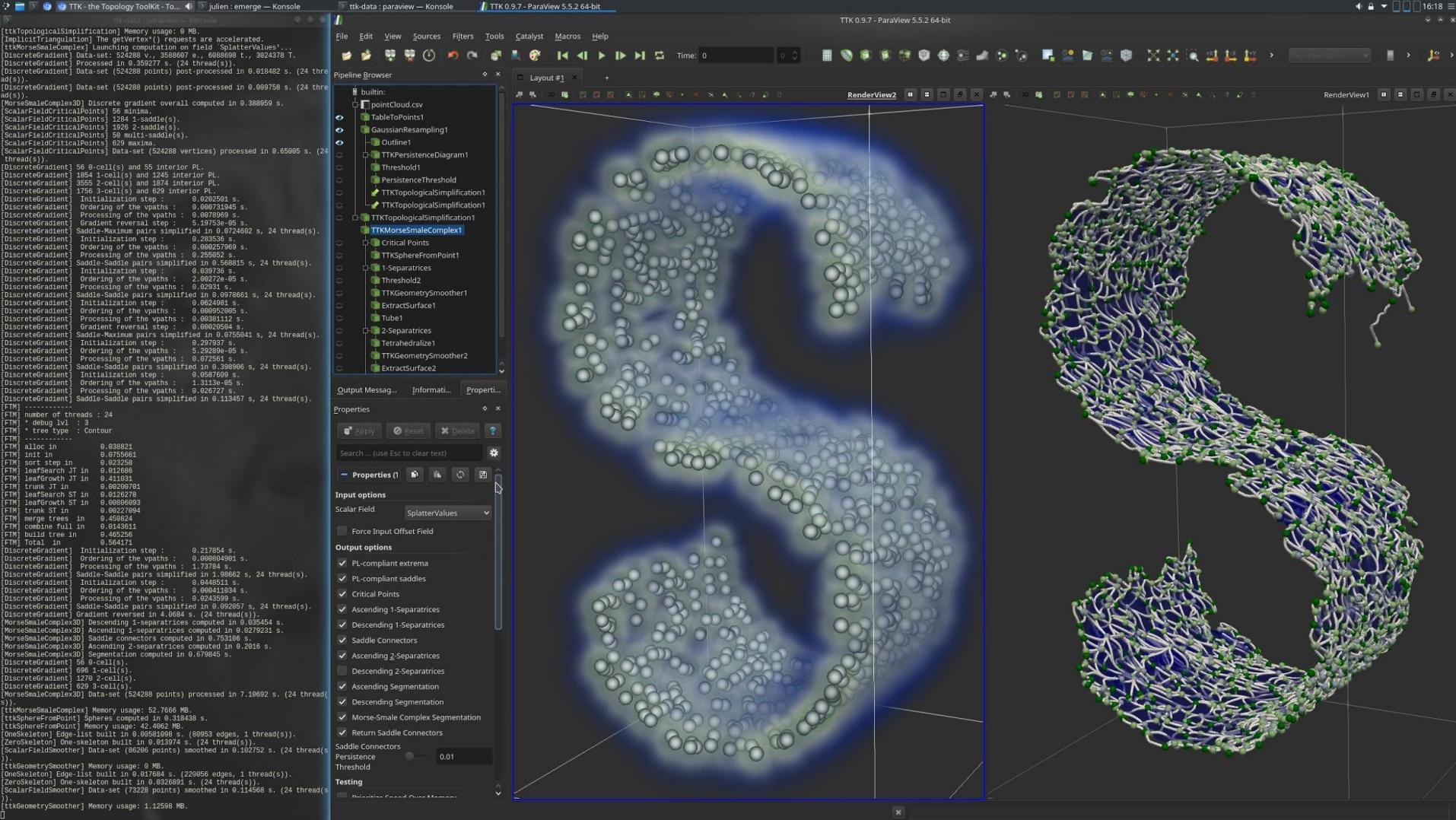




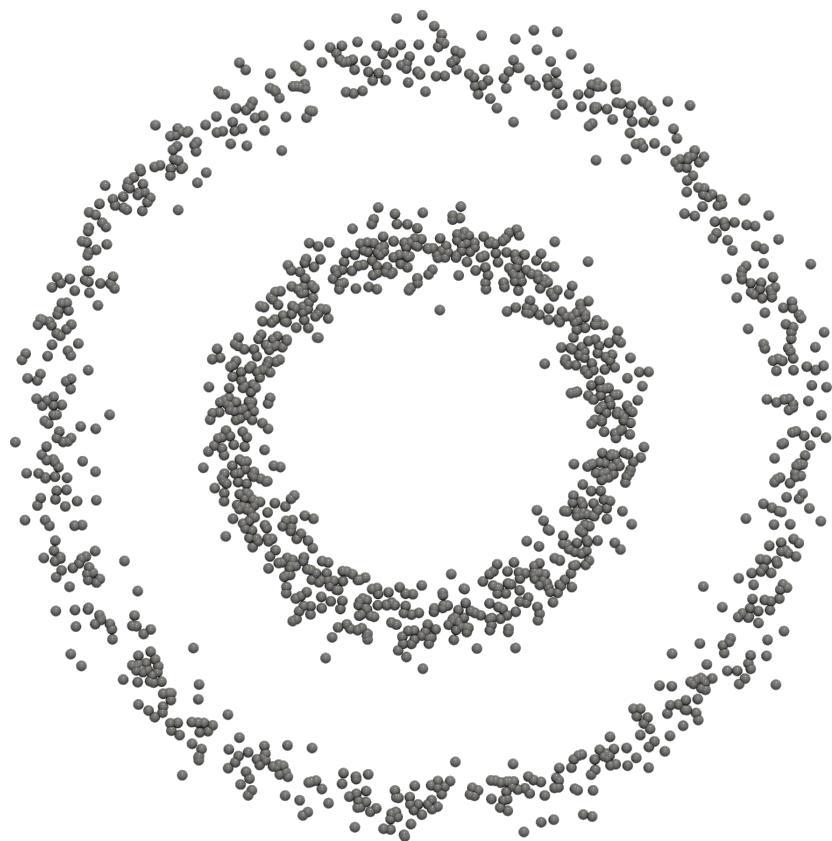
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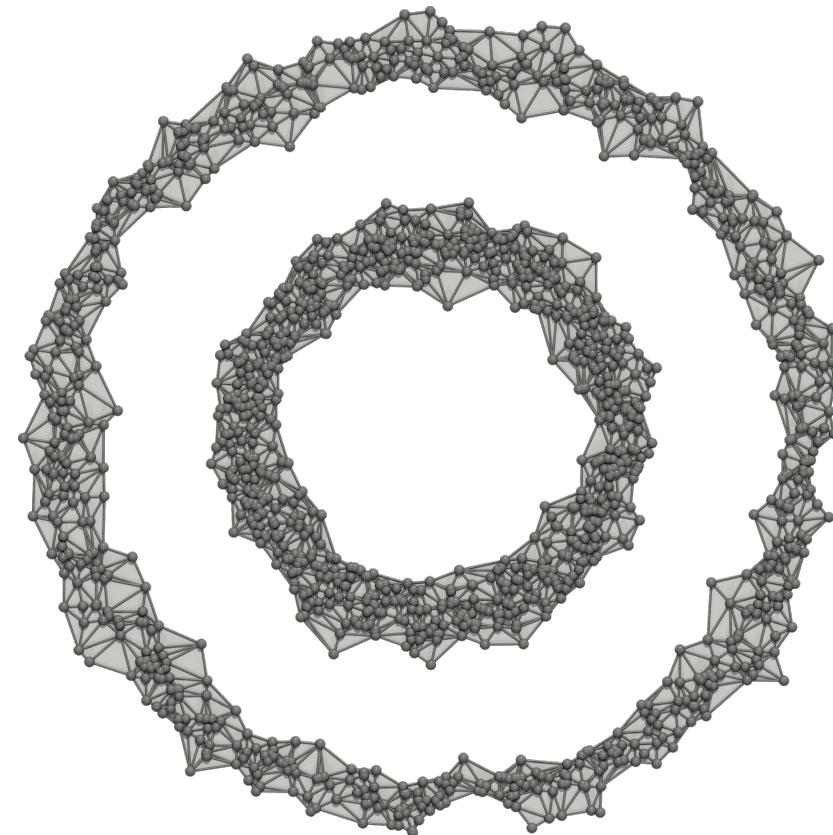
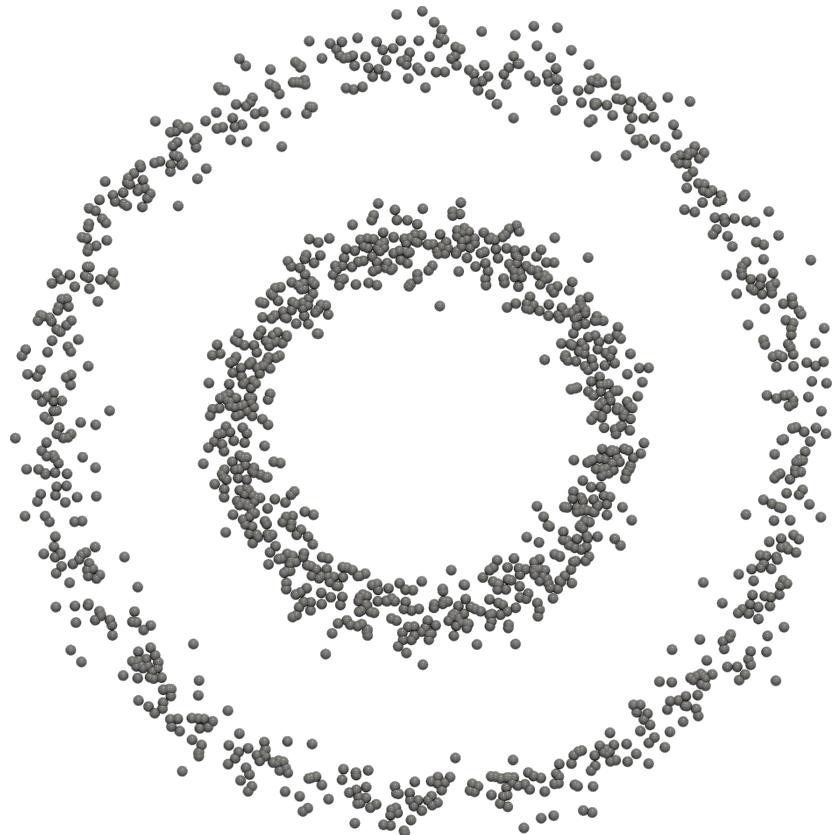




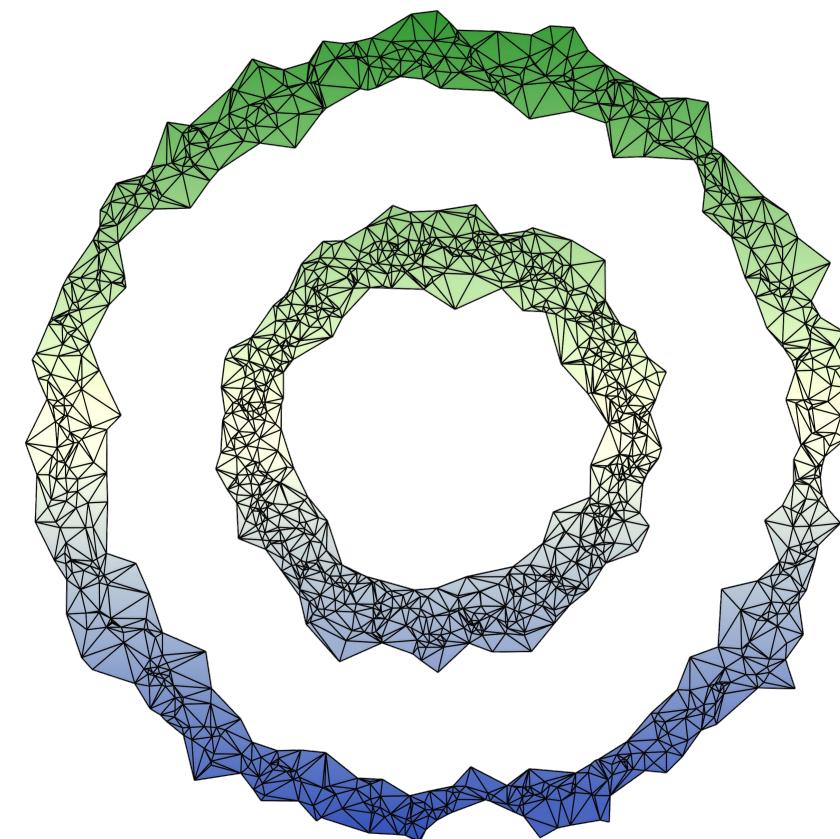
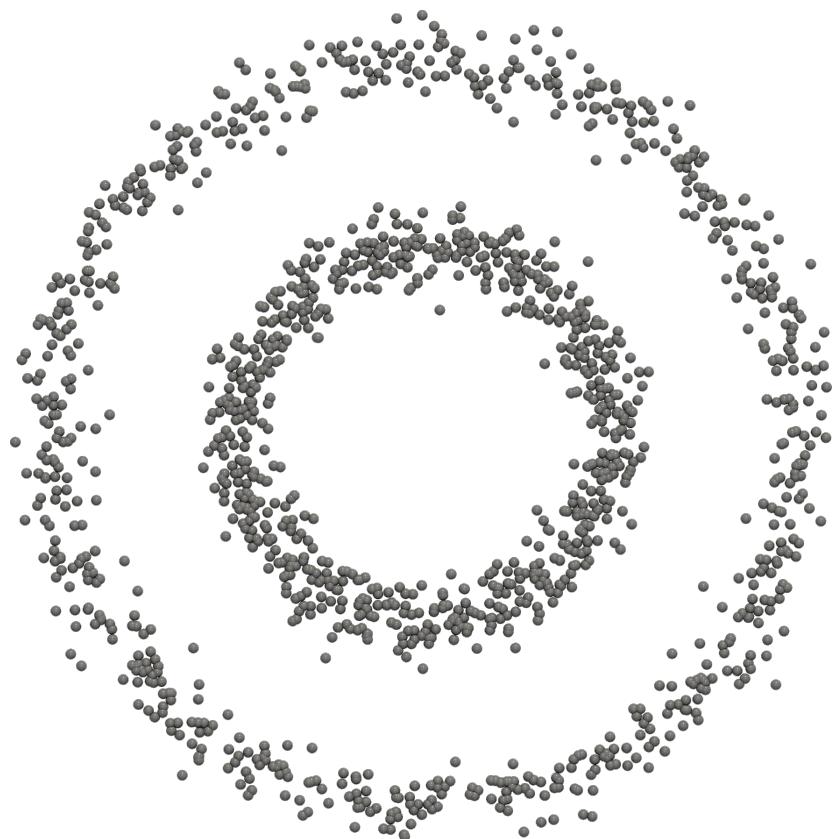
Mapper



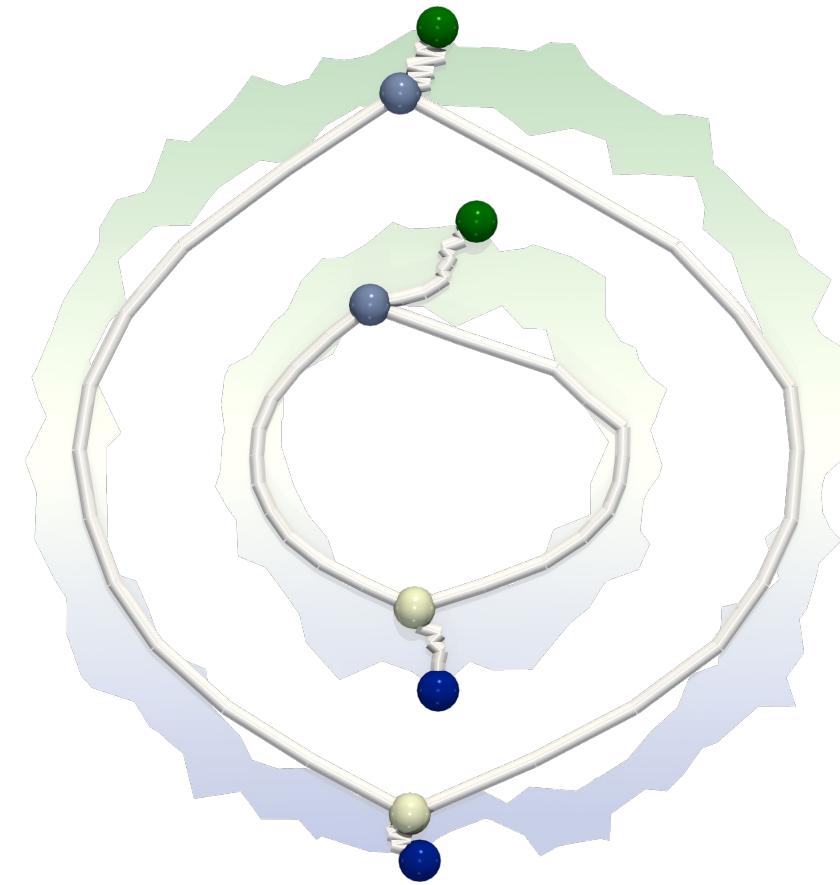
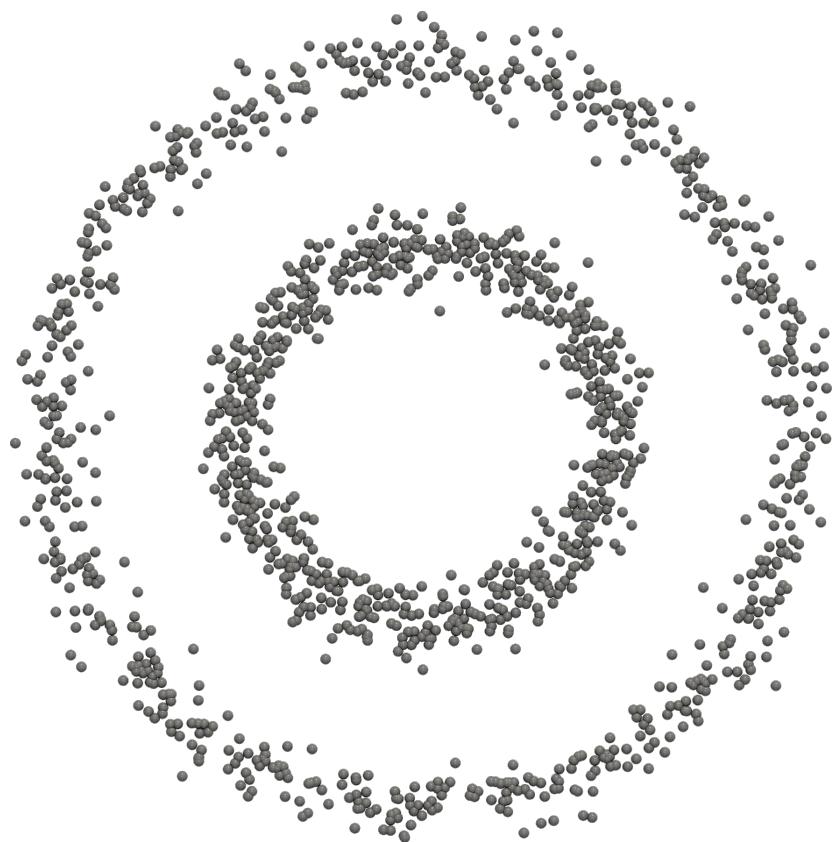
Mapper



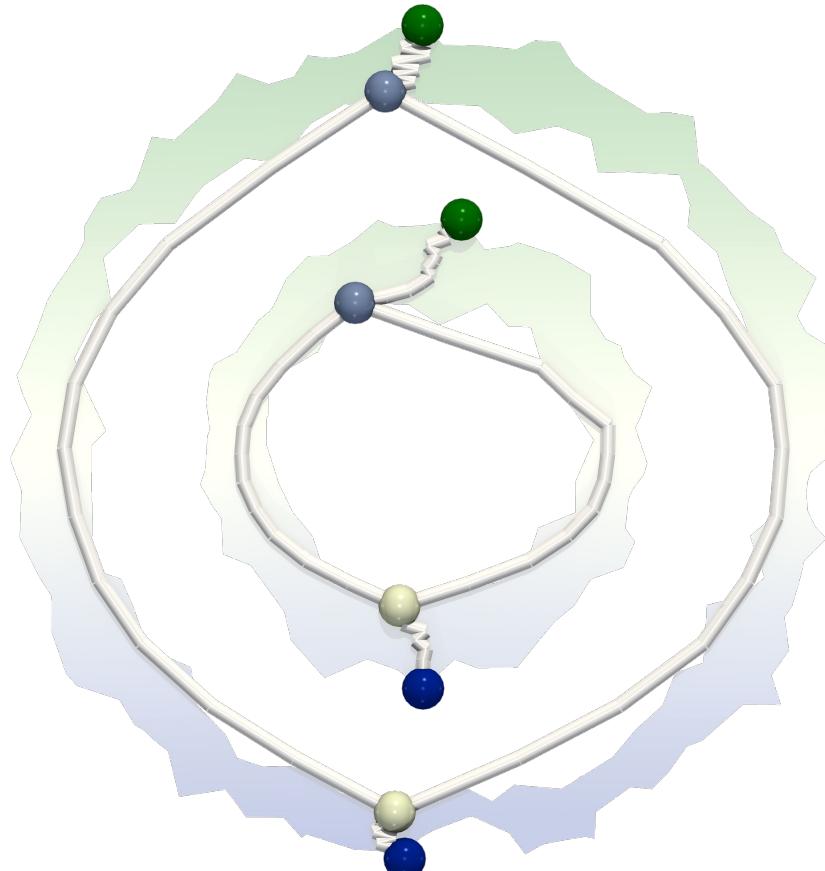
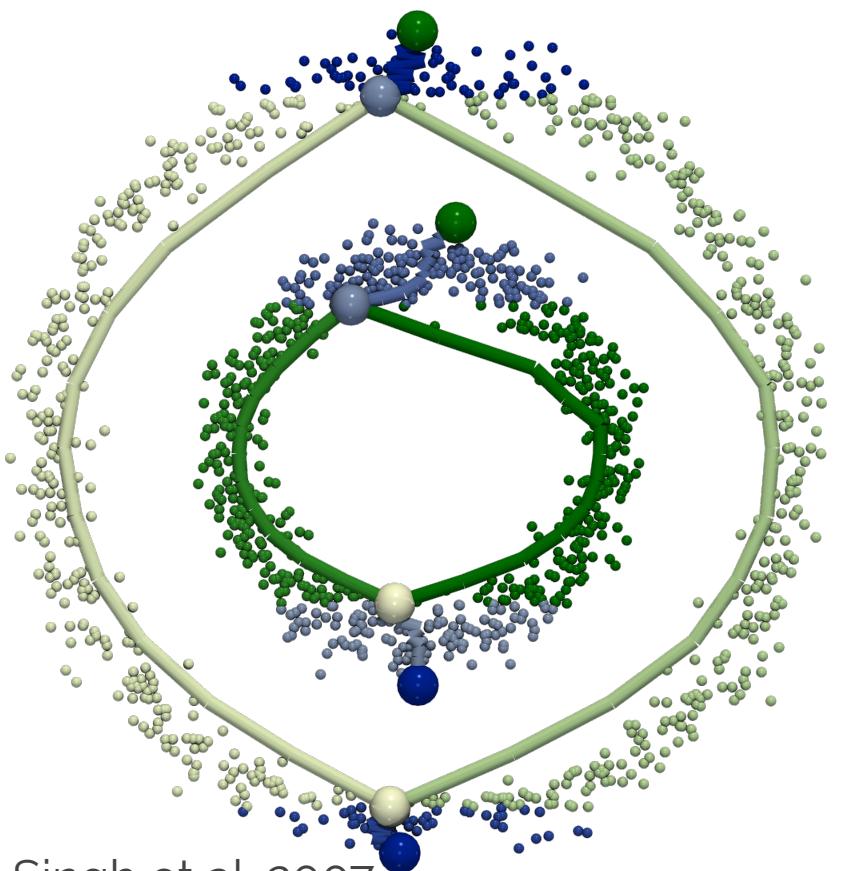
Mapper



Mapper



Mapper



• Singh et al. 2007

The Topology ToolKit (TTK)

- History

- Developed since 2014
- Released on April 1, 2017 (BSD license)
- <http://topology-tool-kit.github.io>



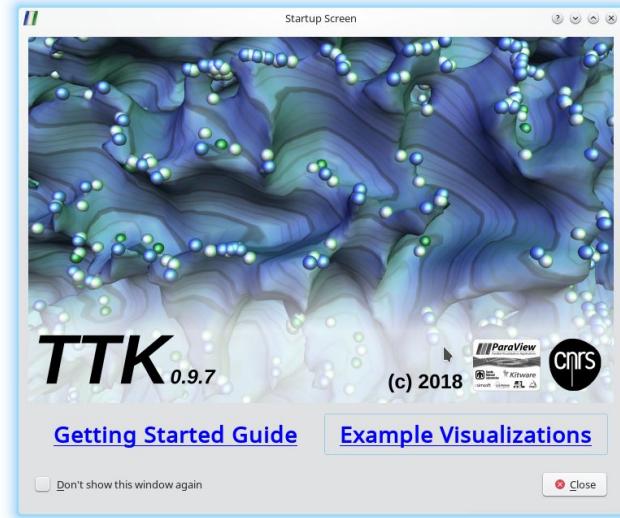
The Topology ToolKit (TTK)

- **History**

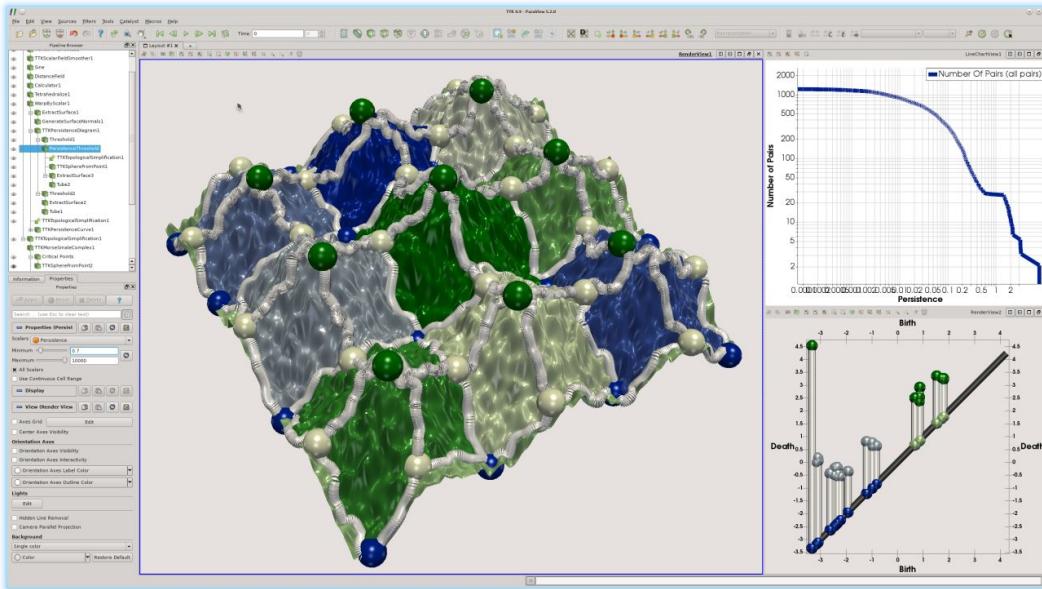
- Developed since 2014
- Released on April 1, 2017 (BSD license)
- <http://topology-tool-kit.github.io>

- **Since the release**

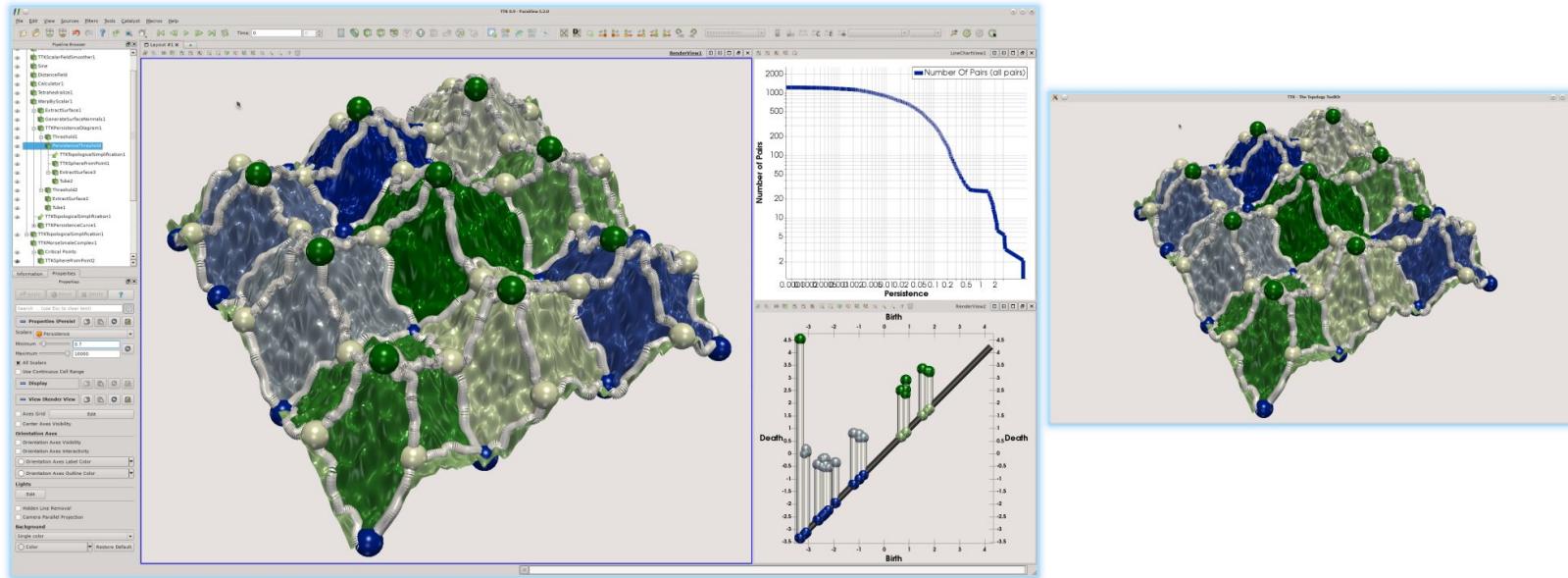
- ~2k commits, ~100k lines of C++
- 11 contributing institutions: Caboma Inc., CNRS, INRIA, Kitware, Linkoping University, Sorbonne Universite, Total, TU Kaiserslautern, U. of Arizona, U. of Utah, Zuse Institute Berlin
- >9k YouTube Views
- >14k unique visitors



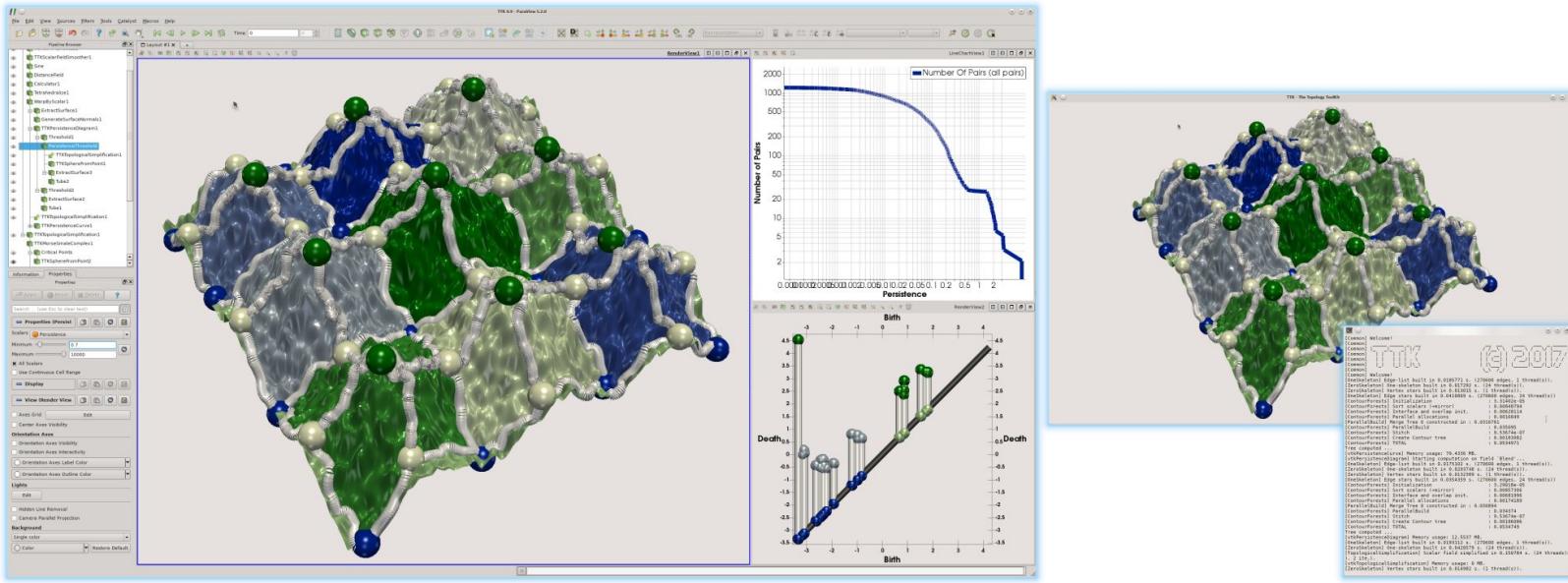
TTK Usage



TTK Usage



TTK Usage

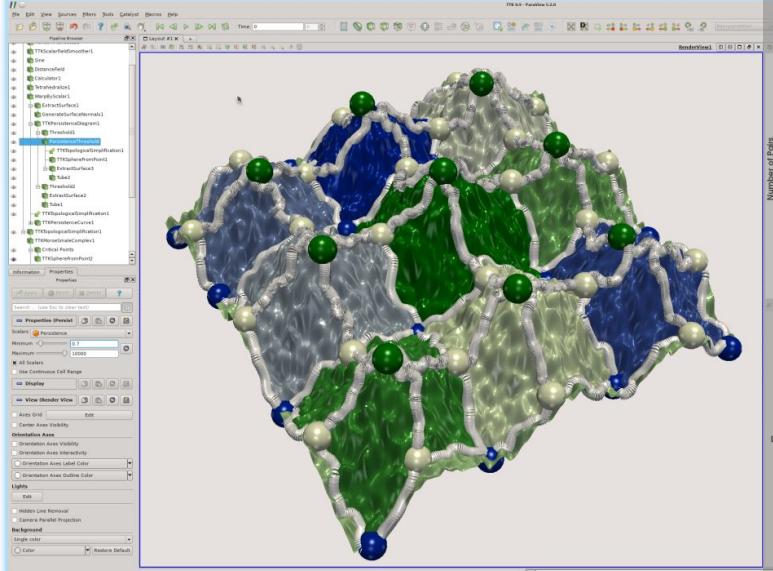


TTK Usage

The image displays three windows illustrating the TTK (Topological Toolkit) usage:

- Left Window:** Persistence 3D visualization. It shows a complex surface with a hierarchical structure of persistence pairs. The visualization includes a 3D view of the surface with colored points (blue, green, yellow) representing critical points, and a 2D contour plot below it. A Python script is running in the background.
- Middle Window:** A terminal window titled "Python (21 lines)" showing the execution of a Python script named "script4.py". The script performs topological analysis on an input dataset, identifying critical points and persistence pairs. The output provides detailed statistics about the analysis, such as the number of vertices, edges, and faces, as well as the distribution of persistence pairs across different scales.
- Right Window:** Persistence 3D visualization showing the same surface as the left window, but with a different color scheme or highlighting. It also displays a small "G2017" logo.

TTK Usage



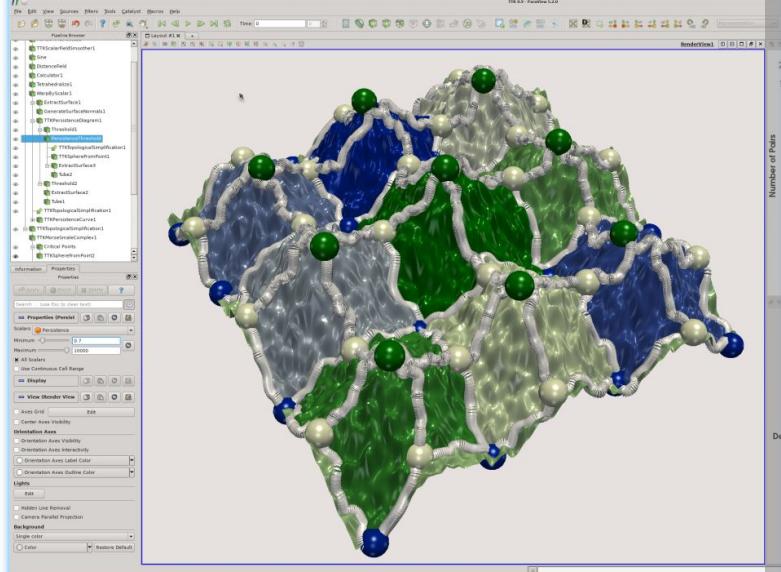
Python (21 lines)

```
1 #!/usr/bin/python
2
3 import vtk
4
5 from vtkmodules.vtkImagingCore import *
6 from vtkmodules.vtkIOXML import *
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65
66
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74
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```

VTK/C++ (37 lines)

```
1 //include <vtksys/vtkObject.h>
2 //include <vtksys/vtkThreadSafeObject.h>
3 //include <vtksys/vtkXMLDataFileReader.h>
4 //include <vtksys/vtkXMLDataFileWriter.h>
5 //include <vtksys/vtkXMLData.h>
6 //include <vtksys/vtkXMLDataElement.h>
7 //include <vtksys/vtkXMLDataElementCollection.h>
8 //include <vtksys/vtkXMLDataElementCollection.h>
9 //include <vtksys/vtkXMLDataElement.h>
10 //include <vtksys/vtkXMLData.h>
11 //include <vtksys/vtkXMLDataFileReader.h>
12 //include <vtksys/vtkXMLDataFileWriter.h>
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25 //include <vtksys/vtkXMLDataFileReader.h>
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27 //include <vtksys/vtkXMLData.h>
28 //include <vtksys/vtkXMLDataElementCollection.h>
29 //include <vtksys/vtkXMLDataElementCollection.h>
30 //include <vtksys/vtkXMLDataElement.h>
31 //include <vtksys/vtkXMLData.h>
32 //include <vtksys/vtkXMLDataFileReader.h>
33 //include <vtksys/vtkXMLDataFileWriter.h>
34 //include <vtksys/vtkXMLData.h>
35 //include <vtksys/vtkXMLDataElementCollection.h>
36 //include <vtksys/vtkXMLDataElementCollection.h>
37 //include <vtksys/vtkXMLDataElement.h>
38 //include <vtksys/vtkXMLData.h>
39 //include <vtksys/vtkXMLDataFileReader.h>
40 //include <vtksys/vtkXMLDataFileWriter.h>
41 //include <vtksys/vtkXMLData.h>
42 //include <vtksys/vtkXMLDataElementCollection.h>
43 //include <vtksys/vtkXMLDataElementCollection.h>
44 //include <vtksys/vtkXMLDataElement.h>
45 //include <vtksys/vtkXMLData.h>
46 //include <vtksys/vtkXMLDataFileReader.h>
47 //include <vtksys/vtkXMLDataFileWriter.h>
48 //include <vtksys/vtkXMLData.h>
49 //include <vtksys/vtkXMLDataElementCollection.h>
50 //include <vtksys/vtkXMLDataElementCollection.h>
51 //include <vtksys/vtkXMLDataElement.h>
52 //include <vtksys/vtkXMLData.h>
53 //include <vtksys/vtkXMLDataFileReader.h>
54 //include <vtksys/vtkXMLDataFileWriter.h>
55 //include <vtksys/vtkXMLData.h>
56 //include <vtksys/vtkXMLDataElementCollection.h>
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58 //include <vtksys/vtkXMLDataElement.h>
59 //include <vtksys/vtkXMLData.h>
60 //include <vtksys/vtkXMLDataFileReader.h>
61 //include <vtksys/vtkXMLDataFileWriter.h>
62 //include <vtksys/vtkXMLData.h>
63 //include <vtksys/vtkXMLDataElementCollection.h>
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65 //include <vtksys/vtkXMLDataElement.h>
66 //include <vtksys/vtkXMLData.h>
67 //include <vtksys/vtkXMLDataFileReader.h>
68 //include <vtksys/vtkXMLDataFileWriter.h>
69 //include <vtksys/vtkXMLData.h>
70 //include <vtksys/vtkXMLDataElementCollection.h>
71 //include <vtksys/vtkXMLDataElementCollection.h>
72 //include <vtksys/vtkXMLDataElement.h>
73 //include <vtksys/vtkXMLData.h>
74 //include <vtksys/vtkXMLDataFileReader.h>
75 //include <vtksys/vtkXMLDataFileWriter.h>
76 //include <vtksys/vtkXMLData.h>
77 }
```

TTK Usage



```
ttk::PersistenceDiagram <--> PersistenceDiagram
```

This block contains the first 21 lines of Python code, which reads input data from an XML file, initializes a persistence diagram, and performs various operations like selecting critical points and computing persistence pairs.

```
ttk::PersistenceDiagram <--> PersistenceDiagram
```

This block contains the second 21 lines of Python code, which continues the process by setting up a simplicial complex, calculating its persistence curve, and executing it.

```
ttk::PersistenceDiagram <--> PersistenceDiagram
```

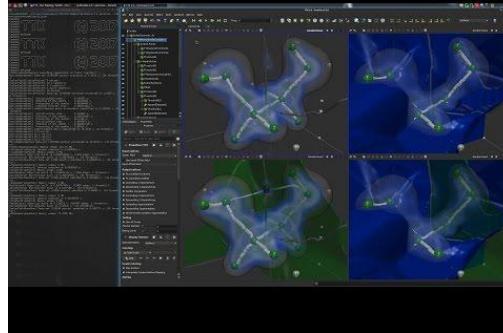
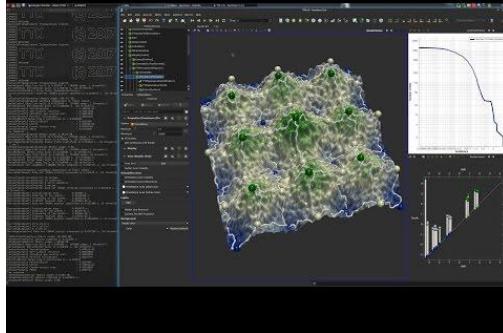
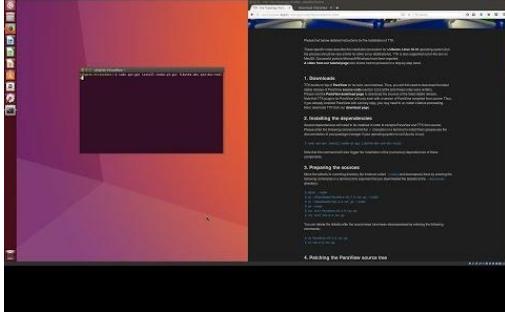
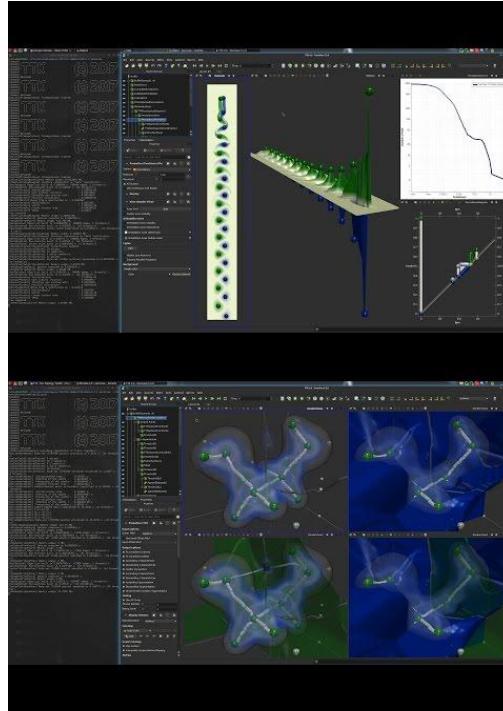
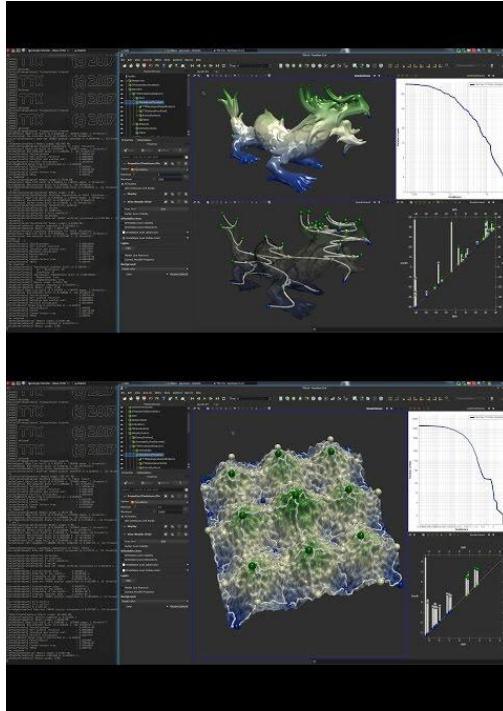
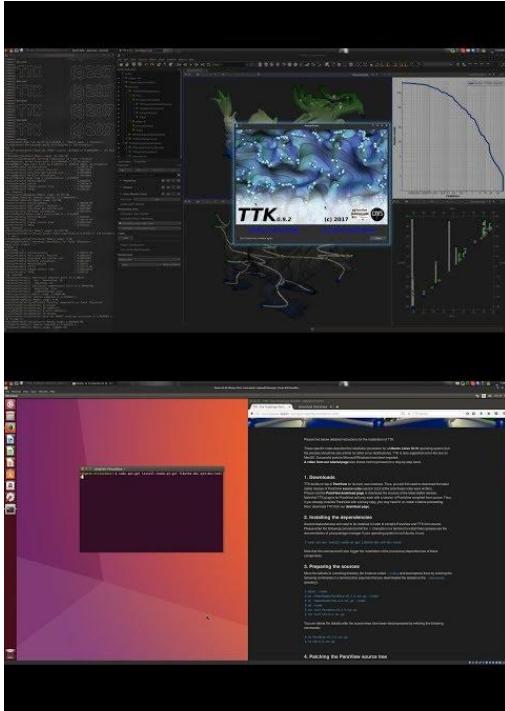
This block contains the final 101 lines of Python code, which handles the persistence curve, sets up a simplicial complex, calculates its persistence curve, and executes it.

Python
(21 lines)

VTK/C++
(37 lines)

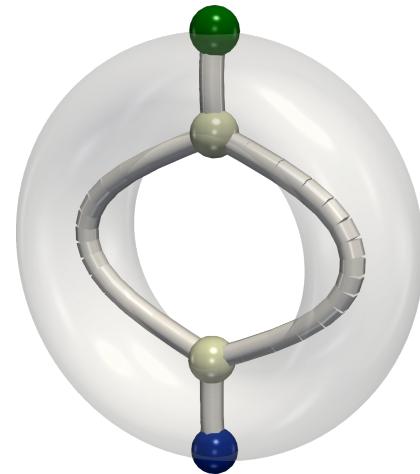
Pure C++
(101 lines)

TTK Tutorials



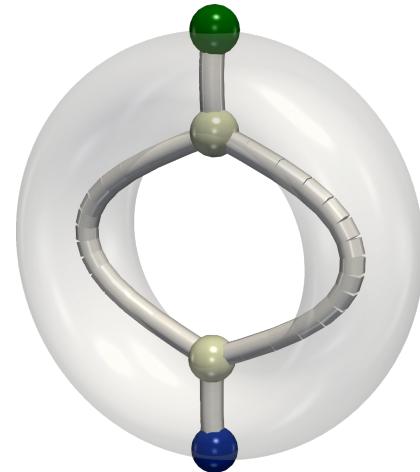
Take-home messages

- Data on meshes, or meshable data?
 - Features of interest?
 - ⇒ Topological Data Analysis
 - Robust, multiscale, successful in applications



Take-home messages

- Data on meshes, or meshable data?
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- The Topology ToolKit
 - <http://topology-tool-kit.github.io>
 - Efficient and easy-to-use implementations
 - ParaView, Python, VTK/C++, C++



Take-home messages

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 - Efficient and easy-to-use implementations
 - ParaView, Python, VTK/C++, C++
- Perspectives
 - Statistical/topological analysis of data ensembles
 - Towards high performance topological data analysis



Thanks!

- Papers, video, code, teaching material, exercise packages, tutorials...
 - <http://lip6.fr/Julien.Tierny>
 - <http://topology-tool-kit.github.io>
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- Questions?

Main publications

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