An Equilibrium Analysis of Risk-Hedging Strategies in Decentralized Electricity Markets

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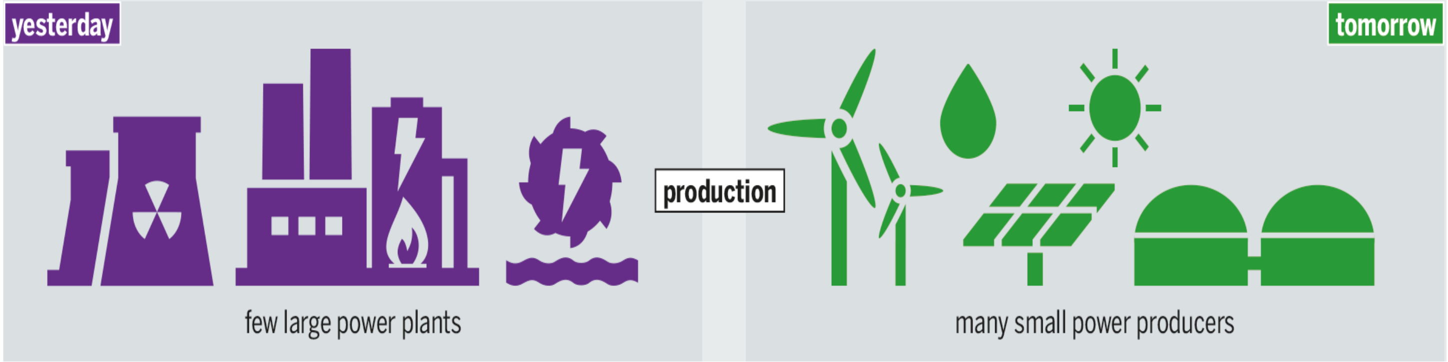
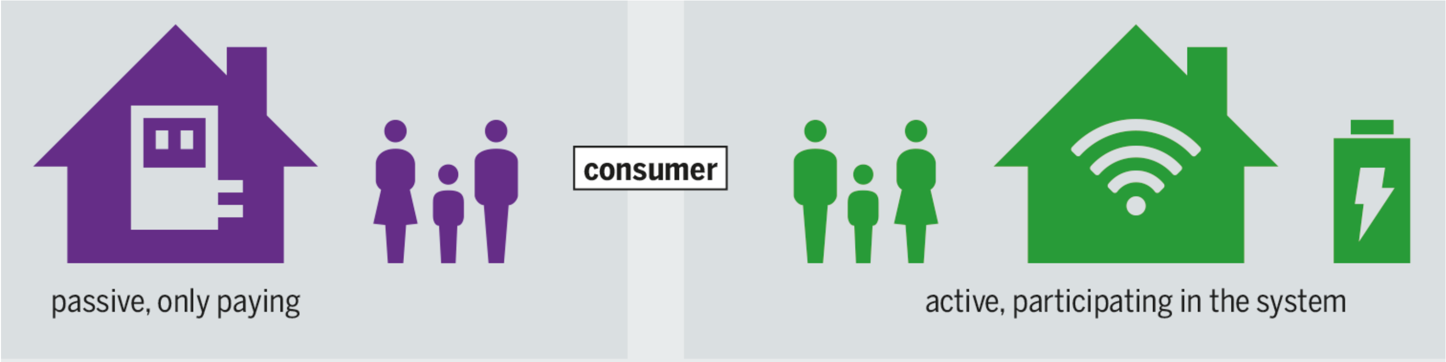
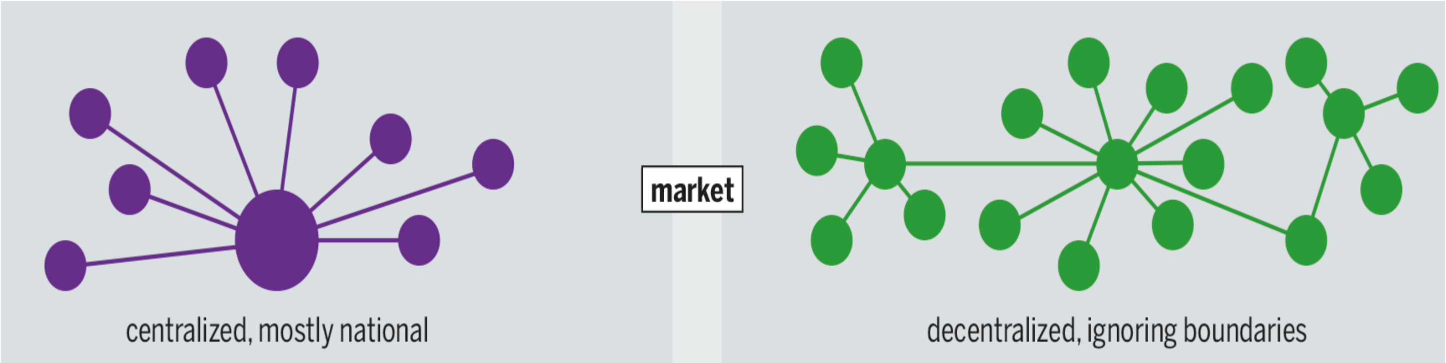


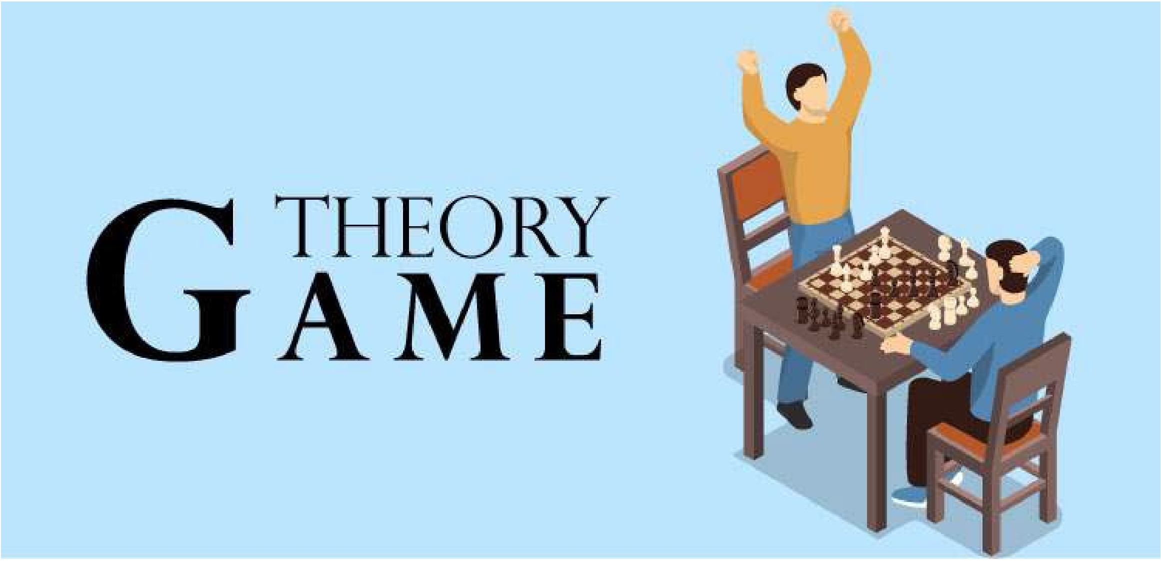
- Renewable energy sources (RES)
- Flexibility sources (batteries, consumption adjustment etc.)

- Active participation
- Strategic behavior





- Decentralized electricity markets
- Peer-to-peer, hybrid, community-based



- Outcome of player's choice of action depends on the actions of other players
- Non-cooperative game - conflicting interest over the outcome
- Static game - the players take their actions only once

Game in strategic form

- $\mathcal{N}$ – set of players
- $(S_n)_{\{n\in N\}}$ - set of strategies of each player
- $(\Pi_n)_{\{n\in N\}}$ - cost of each player
- $\{\mathcal{N}, (S_n)_{\{n\in N\}}, (\Pi_n)_{\{n\in N\}}\}$ – game in strategic form

Nash equilibrium

- s^* - Nash equilibrium if

$$\Pi_n(s^*) \leq \Pi_n(s, s^*_{-n}) \; \forall \, n \in \mathcal{N}$$

- No player can improve the outcome by deviating from s^* if other agents stick to s^*

- Always exists in a non-cooperative game with mixed strategies

- One game can may also have multiple Nash equilibria

- Strategy set may depend on other players' actions (generalized NE)

Prisoner A \ Prisoner B	Remain silent	Confess
Remain silent	A gets 2 years B gets 2 years	A gets 8 years B gets 1 year
Confess	A gets 1 year B gets 8 years	A gets 5 years B gets 5 years

Game theory in decentralized electricity markets

Advantages

- Includes users' strategic behaviour
- Models interactive trading between players
- Integrates pricing and incentive designs

Limitations

- Hard to directly involve human subject in the optimization process
- Dependent on the performance of the communication network

Electricity trading problem

$$\min_{x_n} \Pi_n(x_n, x_{-n})$$

$$s.t. \quad x_n \in \mathcal{K}_n(x_{-n})$$

$$x_n = (D_n, G_n, q_n)$$

- $C_n(G_n)$ quadratic generation cost
- $U_n(D_n)$ quadratic usage benefit function
- $\widetilde{C_n}(q_n)$ linear trading cost ($q_n = (q_{nm})_{\{m \in \Gamma_n\}}$)

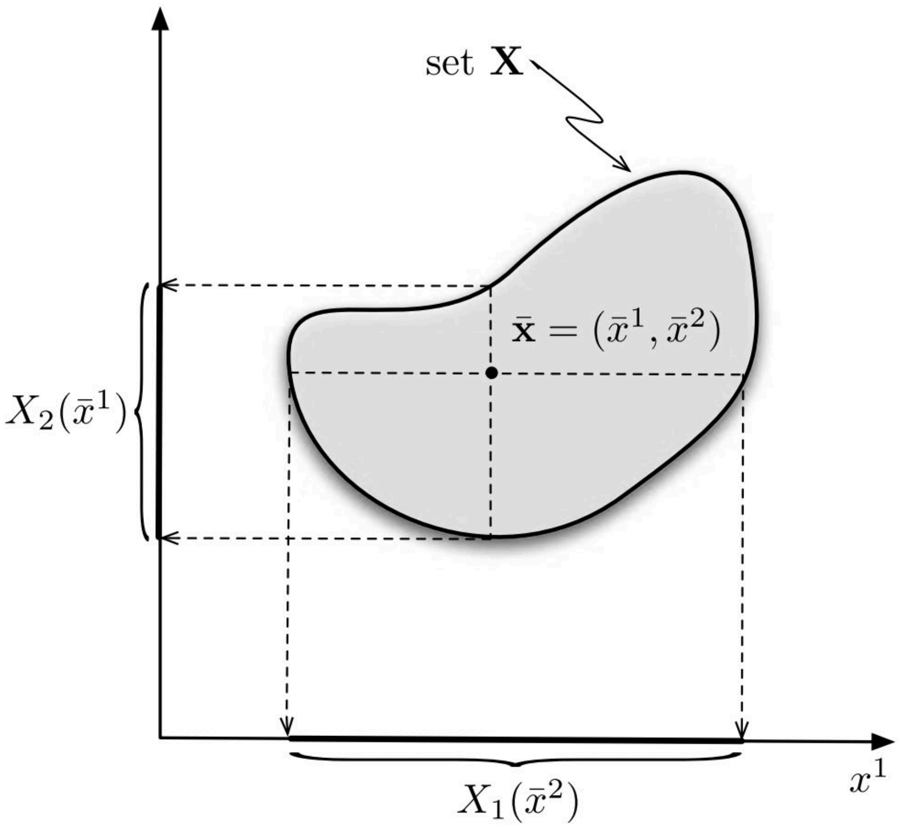
- $\mathrm{G_n^l} \le \mathrm{G_n} \le \mathrm{G_n^u}$, $\mathrm{D_n^l} \le \mathrm{D_n} \le \mathrm{D_n^u}$ flexible demand and generation bounds
- $D_n = G_n + \Delta G_n + \sum_{m \in \Gamma_n} q_{nm}$ supply-demand balance (λ_n)
- $q_{nm} \le \kappa_{nm}$ trading capacity bounds
- $q_{nm} + q_{mn} = 0$ bilateral trading bounds (ζ_{nm}) (coupling)

Generalized Nash Equilibrium

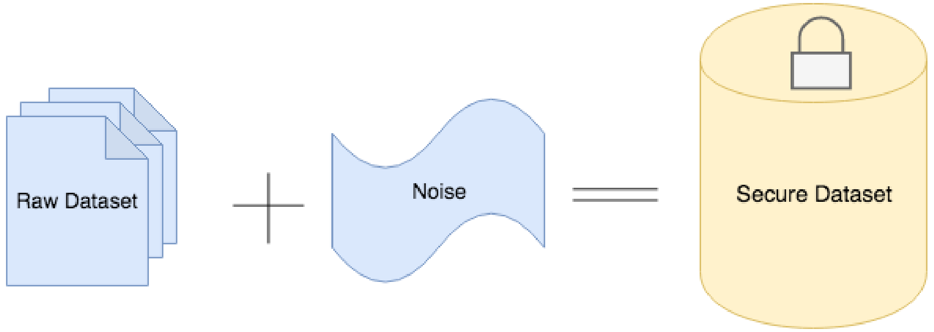
- A Generalized Nash Equilibrium (GNE) is a vector $x_n = (D_n, G_n, q_n)_n$ that solves the maximization problems above or, equivalently, a vector $x_n = (D_n, G_n, q_n)_n$ such that x_n solve the system KKT_n for each n

- A Variational Equilibrium (VE) is a GNE such that, in addition, the Lagrangian multipliers associated to the coupling constraints are equal

$$\zeta_{nm} = \zeta_{mn} \ \forall\, n \in \mathcal{N}, \forall\, m \in \Gamma_n$$

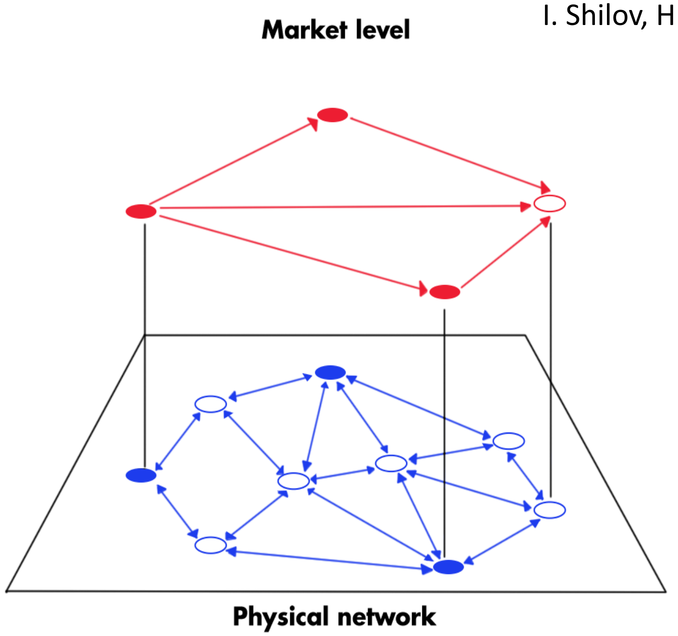


Privacy



I. Shilov, H. Le Cadre and A. Bušić, “Privacy Impact on Generalized Nash Equilibrium in Peer-to-Peer Electricity Market”, OR letters, 2021

Including network constraints



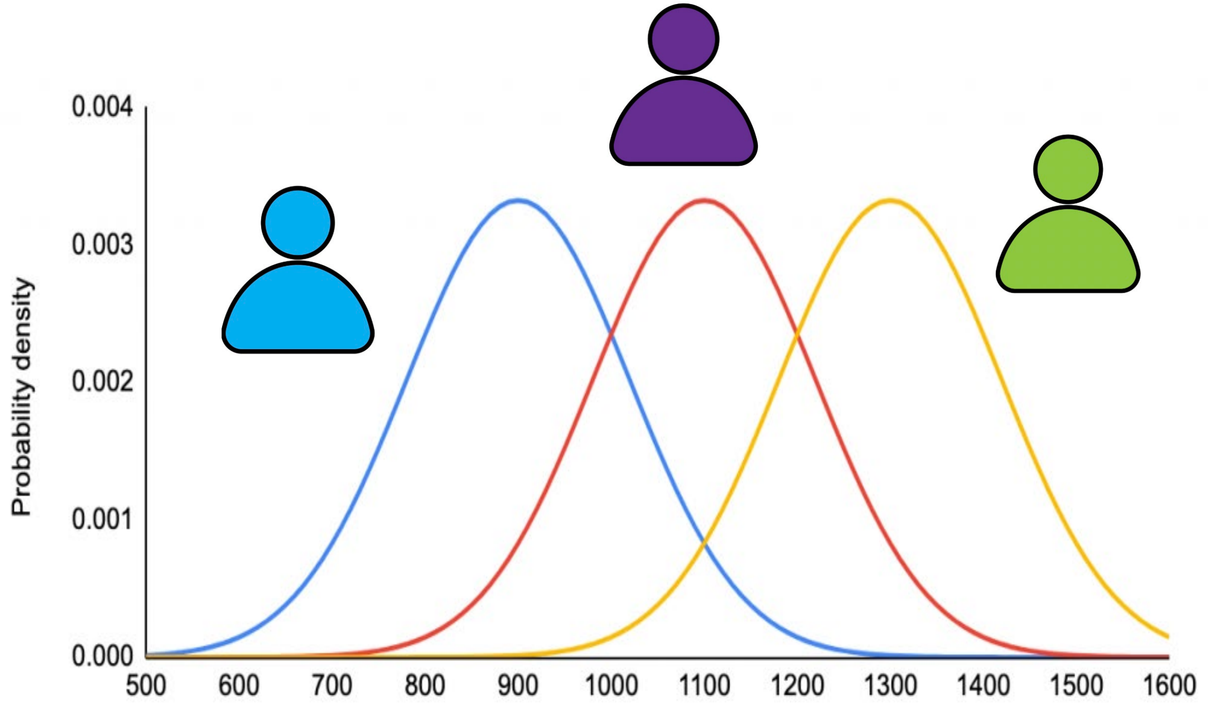
I. Shilov, H. Le Cadre, Ana Bušić ``A Generalized Nash Equilibrium analysis of the interaction between a peer-to-peer financial market and the distribution grid'', Proceedings of IEEE SmartGridComm 21', 2021

Risk-Hedging

Ilia Shilov, Hélène Le Cadre, Ana Bušić, Gonçalo de Almeida Terça. ``An Equilibrium Analysis of Risk-Hedging Strategies in Decentralized Electricity Markets'', submitted to Operations Research 2022

Heterogeneous Risk-Aversion

- Risk-neutral market design and risk-averse model.
- Heterogeneity in agents' risk attitudes
- In the latter prosumers are endowed with coherent risk measures reflecting



Coherent Risk Measure, CVaR

χ_n - risk attitude of n, $VaR_n = \min_{\eta_n}\{\eta_n \,|\, \mathbb{P}[\Pi_n^t \leq \eta_n] = \chi_n\}$.

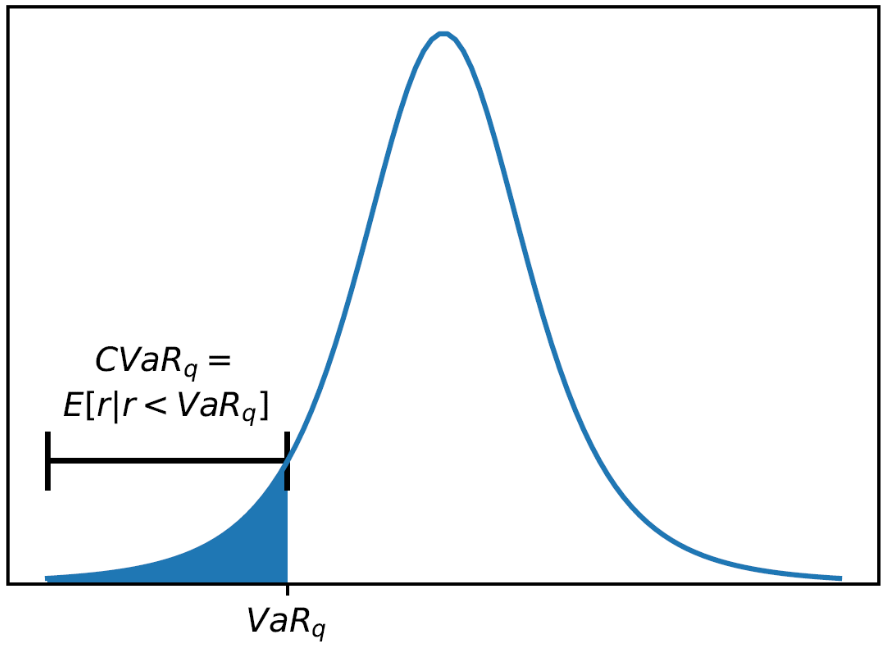
For our problem we write cVaR as follows:

$$R[\Pi_n(t)] = \eta_n + \frac{1}{(1-\chi_n)} \sum_{t \in t} p_t [\Pi_n(t) - \eta_n]^+.$$

The epigraph form to overcome non-differentiability:

$$R[\Pi_n(t)] = \eta_n + \frac{1}{(1-\chi_n)} \sum_{t \in t} p_t u_n^t,$$

with $u_n^t \geq 0$ and $\Pi_n(t) - \eta_n \leq u_n^t$ with dual variables π_n^t and τ_n^t respectively.



Risk-Hedging

In Decentralized Electricity Markets

1. One-stage design with inter-agent contract trading

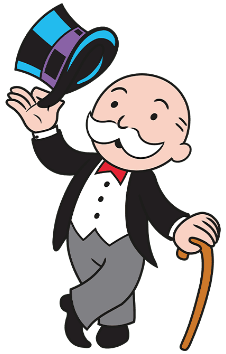
Endogenous risk trading: the price γ^t and the quantity bought(sold) by agent n is W_n^t.

$$R_n[\Pi_n^t] = \eta_n + \sum_{t \in \mathcal{T}} \gamma^t W_n^t + \frac{1}{(1-\chi_n)} \sum_{t \in \mathcal{T}} p^t [\Pi_n^t - W_n^t - \eta_n]^+.$$

2. **Stackelberg game** where the insurance company acts as a leader and prosumers are followers

Exogenous agent: fixed prices α_n^t per scenario t and contracts J_n^t

Stackelberg game



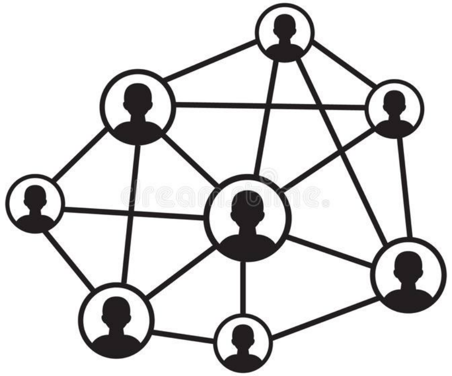
$$\min_{(\boldsymbol{\alpha}_n^t, \overline{J}^t)_{n \in \mathcal{N}}} \quad \sum_{n \in \mathcal{N}} \Big[-\sum_{t \in \mathcal{T}} \alpha_n^t J_n^t + \sum_{t \in \mathcal{T}} p^t J_n^t \Big]$$

$$s.t. \quad 0 \leq \alpha_n^t \quad \forall n \in \mathcal{N}$$

$$\forall n \in \mathcal{N} \quad J_n^t \in \arg\min_{J_n^t, \boldsymbol{x}_n^t} \overbrace{\sum_{t \in \mathcal{T}} \alpha_n^t J_n^t + \eta_n + \frac{1}{(1-\chi_n)} \sum_{t \in \mathcal{T}} p^t u_n^t}^{\Pi_n}$$

$$s.t. \quad x_n \in \tilde{\mathcal{K}}_n(\boldsymbol{x}_{-n}) \quad \forall n \in \mathcal{N}$$

$$0 \leq J_n^t \leq \overline{J^t} \quad \forall n \in \mathcal{N}$$

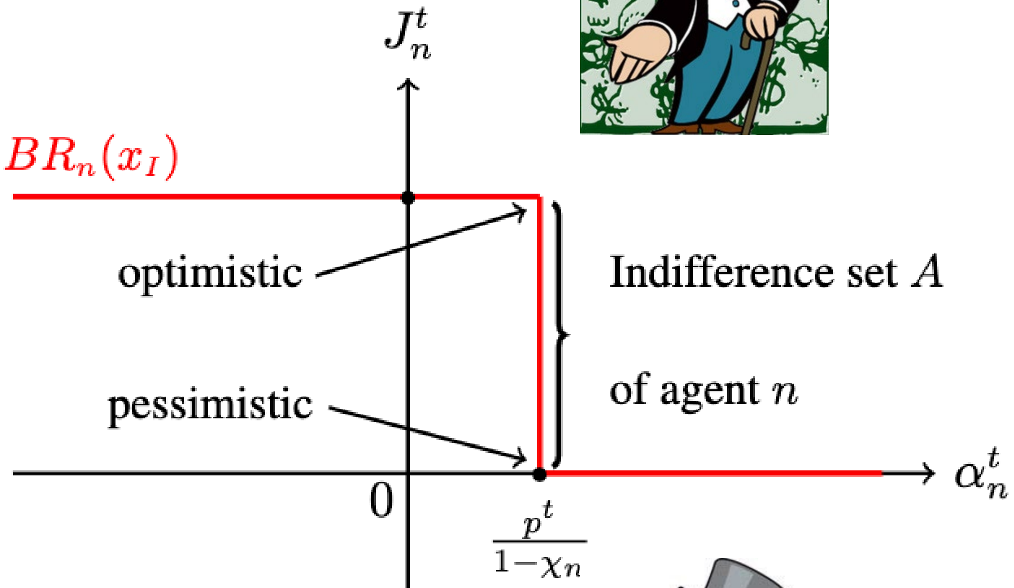


Stackelberg game

1. **Optimistic** – agents cooperate with the insurance company

2. **Pessimistic** – agents are reluctant to act in favor of insurance company

3. Might be no solution in **pessimistic** framework!



LEMMA 1. *The price α_n^t of the insurances J_n^t for agent n and scenario t does not exceed $\frac{p^t}{1-\chi_n}$.*

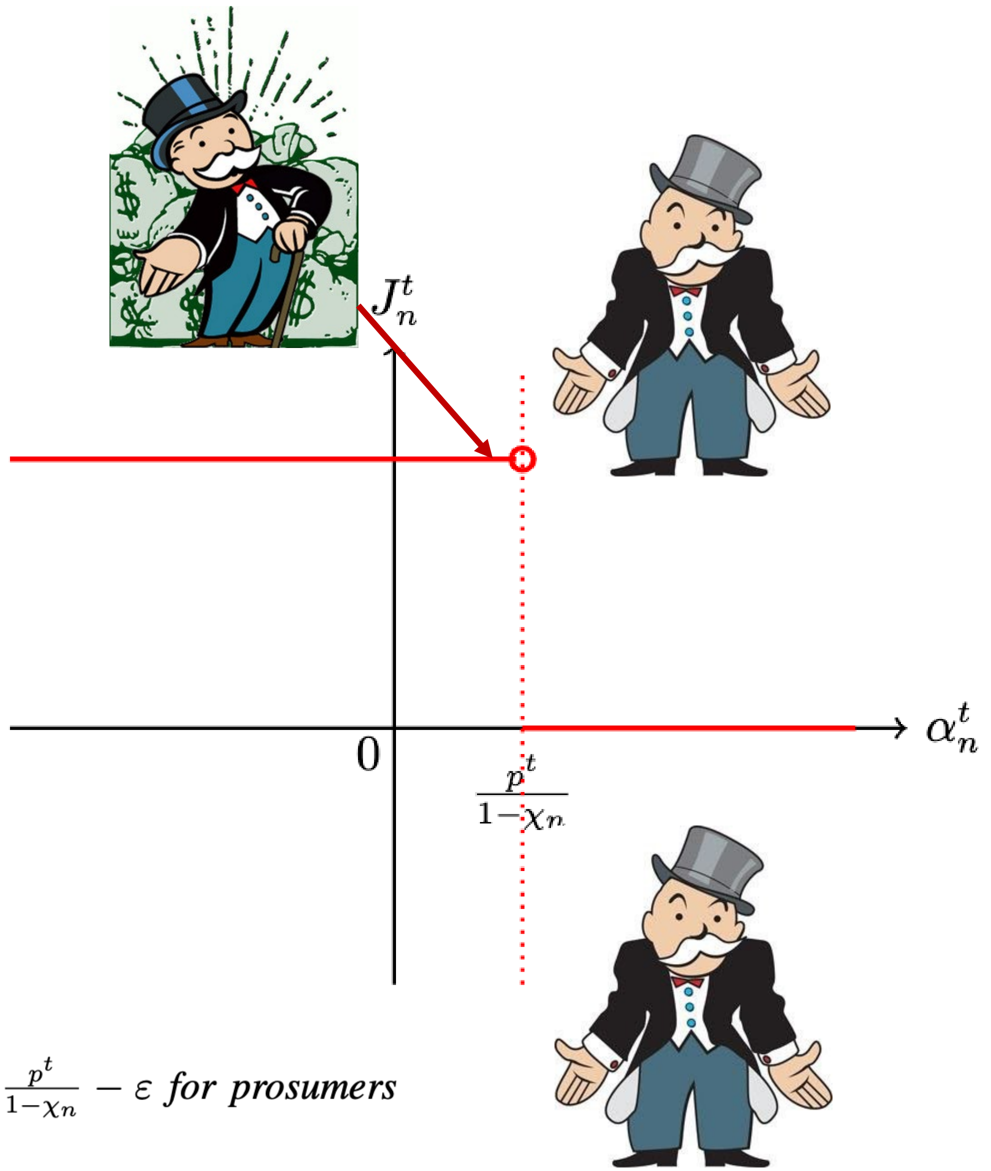
Stackelberg game

1. **Optimistic** – agents cooperate with the insurance company

2. **Pessimistic** – agents are reluctant to act in favor of insurance company

3. Might be no solution in **pessimistic** framework!

4. **Price incentives** should help!



PROPOSITION 4. *For any given ε, if insurance company sets the prices $\alpha_n^t = \frac{p^t}{1-\chi_n} - \varepsilon$ for prosumers $n \in \mathcal{N}'$, then the problem (14) has a solution.*

Pessimitic reformulation

$$\begin{aligned} \min_{x_I, (\boldsymbol{x}_n^L, \boldsymbol{z}_n^L)_n} \quad & \Pi_I(x_I, \boldsymbol{x}_n^L) \\ s.t. \quad & x_I \in X_I \\ & (x_n^L, z_n^L) \in E(x_I) \quad \forall n \in \mathcal{N} \end{aligned}$$

where $E(x_I)$ is the equilibrium set of the following GNEP:

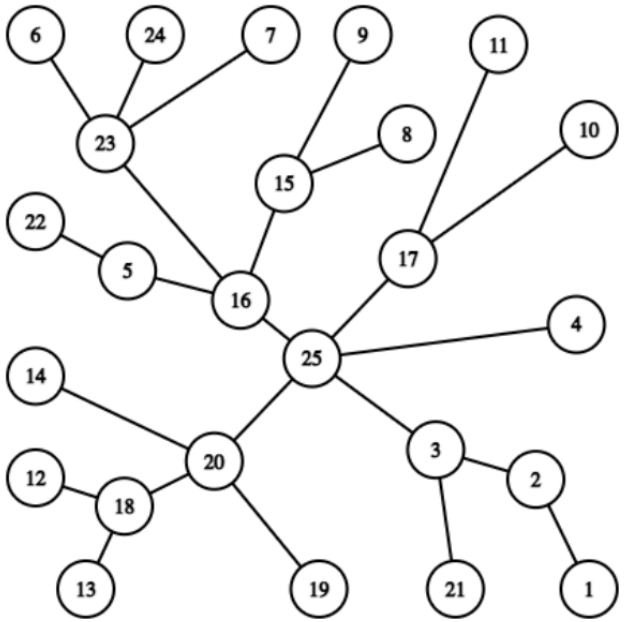
$$\begin{aligned} \min_{x_n^L} \quad & -\Pi_I(x_I, \boldsymbol{x}_n^L) \\ s.t. \quad & x_n^L \in X_n(x_I, \boldsymbol{x}_{-n}^L) \\ & \Pi_n(x_I, \boldsymbol{x}_n^L) \leq \Pi_n(x_I, \boldsymbol{z}_n^L) \end{aligned}$$

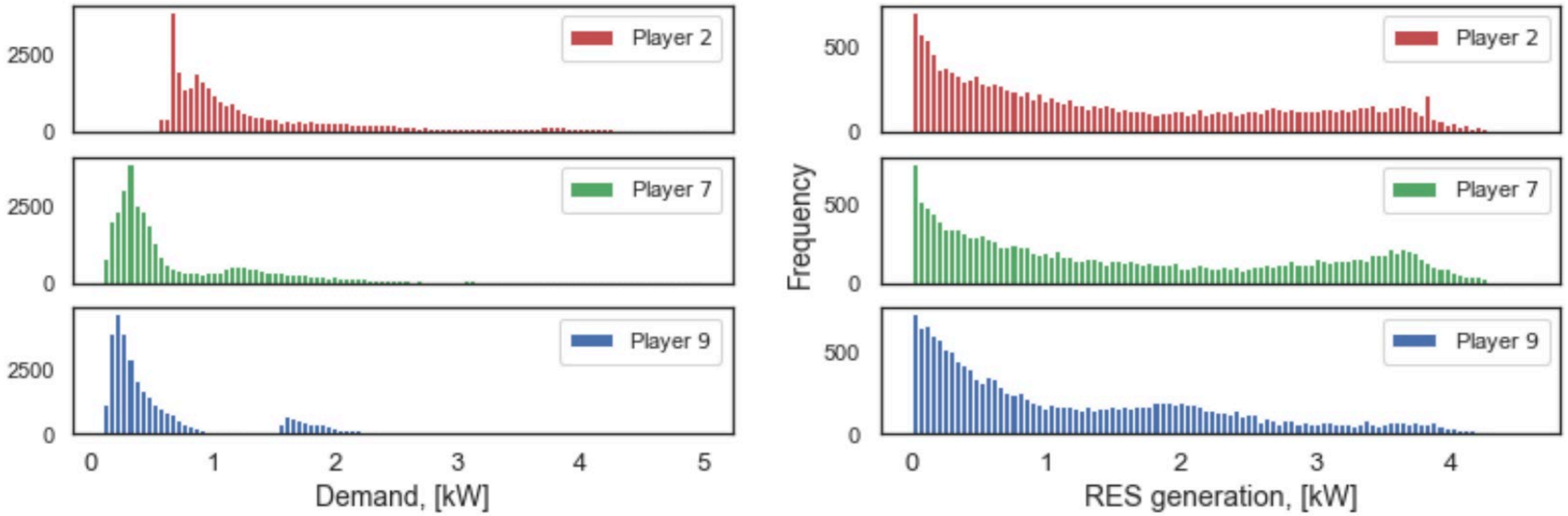
$$\begin{aligned} \min_{z_n^L} \quad & \Pi_n(x_I, \boldsymbol{x}_n^L) \\ s.t. \quad & z_n^L \in X_n(x_I, \boldsymbol{x}_{-n}^L) \end{aligned}$$

Some numerical experiments

- We use residential data provided by Pecan Street Pecan Street (2022) for Austin, Texas.

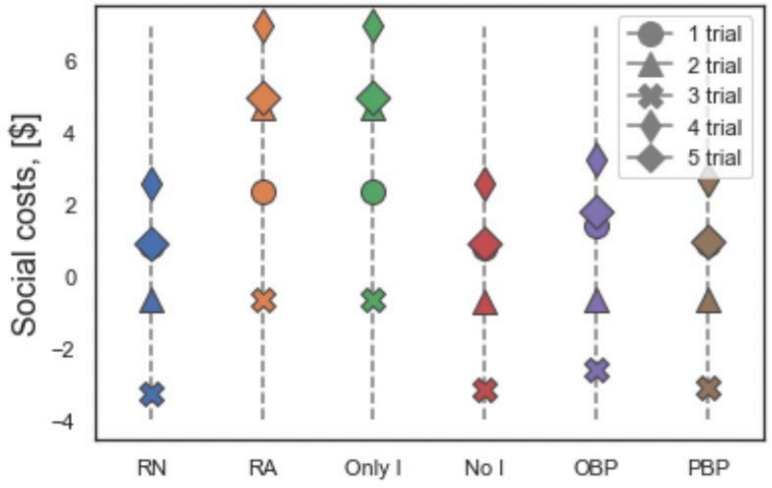
- The data consists of 15-minutes intervals specifying renewable generation, load and facilities energy consumption for 25 individual homes

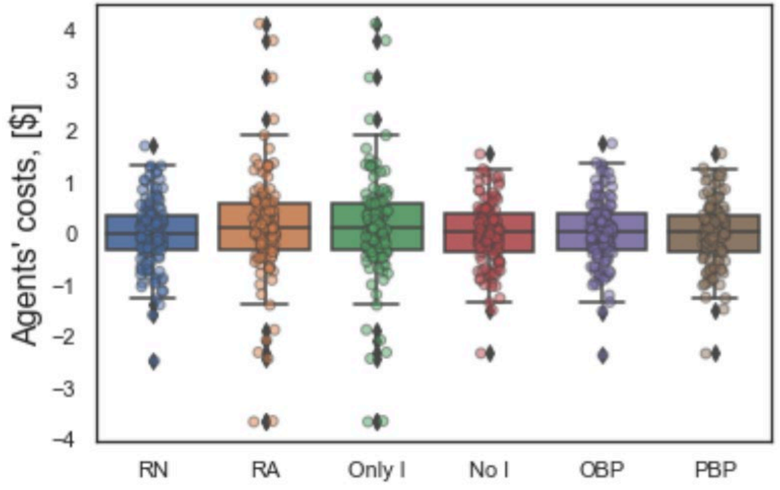




Some numerical results

- (RN) - risk-neutral
- (RA) – risk-averse
- (Only I) - two level with only I
- (No I) - one level with risk-hedging
- (OBP) - two-level optimistic
- (PBP) - two-level pessimistic





	RN	RA	Only I	No I	OBP	PBP
SC [$]	0.101	3.686	3.686	0.186	0.192	0.162
I's cost [$]	-	-	-1.41	-	-0.437	-0.018
Fairness		-	+	-	-	-
Equity		-	-	+	+	+

Conclusions

- Inclusion of **Insurance Company** leads to a **Stackelberg Game**
- In it’s **pessimistic formulation** there might be no solution
- This problem can be overcome by designing **price-based incentives**
- These incentives slightly decrease the profits of the insurance company
- But also allow prosumers to decrease their costs